EP2200 Queuing Theory and Teletraffic Systems

Tuesday, March 18$^{\text{th}}$, 2014, 14.00-19.00, Q21,Q24.

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1. Customers arrive at a service desk following a Poisson process with intensity 20 arrivals/hour. There are 3 assistants in the service desk. Each customer requires a service time that is exponentially distributed with a mean of 5 minutes, and selects one of the 3 assistants uniformly at random. Customers arriving when all assistants are busy wait in a common line on a FIFO fashion.
a) Give the Kendall notation of the above system and draw the system diagram. (2p)
b) Consider an arbitrary assistant. Calculate the percentage of time this assistant is busy with serving customers. (2p)
c) Calculate the probability that an arriving customer: i) has to wait to be served, ii) finds the service desk completely idle, iii) will need to wait for more than 2 minutes. (4p)
Assume now that the customers arriving at the service desk leave immediately, if they estimate that their expected waiting time will be larger than 4 minutes. Give the average number of customers that are served within 1 hour. (2p)

2. In the Laboratory for Communication Networks 4 PhD studens are sharing a set of 3 servers for running simulation tasks. The students prepare (program and debug) a simulation task for an exponentially distributed amount of time, with a mean of 1 hour. When they are done, they dispatch the task to the servers and wait until its execution is completed. Then, they start preparing a new task. Each task runs on one available server. A task dispatched to an idle server needs an exponentially distributed amount of time to complete, with a mean of 20 minutes. A task that arrives when all servers are busy needs to wait for a vacant server. Consider the system at steady state.
a) Give the Kendall notation of the above system and draw the state diagram. (1p)
b) Calculate the average number of tasks that are completed within 4 hours. (2p)
c) Calculate the utilization of a server and the average completion time of an arbitrary task. (2p)
d) Calculate the percentage of time when no server is idle. (1p)
e) Calculate the probability that the execution of a task can not start immediately. (2p)
Assume now, that a task that arrives when all servers are busy, chooses a random server and shares its processing power with the task under execution in the same server. This task moves to an idle server as soon as a server becomes idle.
f) What is the probability that an arriving simulation task can not run at full processing speed? (2p)

3. Consider a queuing system with a single server with random service time X, such as $X = X_1 + X_2$ and X_1 and X_2 are Exponentially distributed random variables with $\mu = 2$. The system has one queuing position. Customers arrive

according to a Poisson process with $\lambda = 1$, blocked customers leave the system immediately.
a) Give the Kendall notation of the queuing system. Represent a system with a Markov-chain, and calculate the state probabilities in steady state. (3p)
b) Calculate the probability that the system is empty and the probability that an arriving customer is blocked. (2p)
c) Calculate the average number of customers in the server, in the queue and in the queuing system. Calculate the average time an accepted customer spends in the system. (3p)
d) Consider a customer that arrives when the server is busy, but no customer is waiting. Calculate the expected waiting time of this customer. (2p)

4. Two kinds of jobs arrive to a job processing system with one server. Jobs of Type 1 require a fixed service time of 1 time unit, while jobs of Type 2 have Exponential service time with a mean value of 5 time units. Jobs arrive to the system according to a Poisson process with one job per 10 time units in average. 30% of the jobs are Type 1, the others are Type 2. Jobs arriving when the server is busy wait in an infinite buffer.
a) Give the average service time, the second moment of the service time, its variance and the coefficient of variation. (2p)
b) Give the Kendall notation of the system. Calculate the portion of time when the server is idle, the average waiting time and the average system time of an arbitrary job. (3p)
Consider now that Type 1 jobs are served with preemptive resume priority.
c) Calculate the expected waiting time of the Type 1 and of the Type 2 jobs in this case and compare the results to the one in part b). (3p)
d) Consider a Type 2 job that arrives to an empty system. Calculate the expected time this jobs spends in the system until its service is finally completed. (2p)

5. a) Consider a sensor node with a transmission capacity of 240 kbit/s. Packets are generated at the node according to a Poisson process with 100 packets per second on average. The packet lengths are Exponential with a mean of 150 Bytes. Whenever there is no packet to transmit, the node goes to sleep state to save energy. It wakes up after a fixed time of 0.1 second. If there is still no packets to transmit, the node goes to sleep state again. Give the expected waiting time of a packet. (2p)
b) A queuing network consists of two nodes, each with a single server and infinite buffer capacity. The service times are Exponential at both of the nodes, with parameters $\mu_1 = 3$ jobs/second and $\mu_2 = 2$ jobs/second. Jobs arrive to Node 1 according to a Poisson process with parameter $\lambda = 1.6$ jobs/second. After getting served at Node 1, jobs proceed to Node 2 with probability 0.8. Otherwise they return to Node 1. Give the probability that the queuing network is empty. Calculate the average time jobs spend in the network. (3p)
c) Consider an M/M/10/10 system with $\lambda = 10, \mu = 1$. Calculate the average durations for the blocking and for the non-blocking time. (3p)
d) Consider an M/M/1 system with $\lambda = 5, \mu = 8$. Assume that a last-in-first-served policy is used, i.e. an arriving customer that finds the server busy is placed on the first queuing position. Calculate the average waiting time of an arriving customer that needs to wait. (2p)