KTH Electrical Engineering

## Complementary test in EG2050 System Planning, 23 April 2014, 10:00-12:00, the seminar room

## I nstructions

Only the problems indicated on the attached answer sheet have to be answered (the score of the remaining problems is kept from the exam). Motivations and calculations do not have to be presented.

The maximal score of the complementary test is 40 points including the points that are kept from the exam. You are guaranteed to pass if you get at least 33 points.

## Allowed aids

In this complementary test you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One handwritten, single-sided A4-page with your own notes (original, not a copy), which should be handed in together with the answer sheet.


## Problem 1 (4 p)

Answer the following theoretical questions by choosing one alternative, which you find correct.
a) (1 p) Which players are economically responsible that during a particular trading period (for example one hour) the system is supplied as much energy as the consumptionton?

1. The system operator.
2. The balance responsible players.
3. The consumers.
b) (2 p) We use the notion "ahead trading" to describe all the trading which occurs before the hour of delivery (or any other trading period). In the ahead market it is possible to trade with the following contracts: I) Take-and-pay contracts, i.e., when the customer subscribes to a specific maximal power, and during the duration of the contract is allowed to buy any amount of energy per trading period as long as the maximal power is not exceeded, II) Firm power, i.e., when the seller is committed to deliver a specific amount of energy per trading period during the duration of the contract, III) Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).
4. None of the statements is true.
5. Only $I$ is true.
6. Only II is true.
7. Only III is true.
8. I and II are true but not III.
c) (1 p) We use the notion "post trading" to describe all the trading which occurs after the hour of delivery (or any other trading period). Which of the following contracts can be traded in a post market?
9. Balance power, i.e., when a balance responsible player is selling any surplus in their balance to the system operator, or when a balance responsible player is buying from the system operator to cover for any deficit in their balance.
10. Firm power, i.e., the customer buys the same amount of energy in each trading period as long as the contract is valid.
11. Regulation power, i.e., when a player at request from the system operator is supplying more power to the system (up-regulation) or when a player at request from the system operator is supplying less power to the system (down-regulation).

## Problem 2 ( 6 p)

Assume that the electricity market in Land has perfect competition, perfect information and that there are neither capacity, transmission nor reservoir limitations. Data for the power plants in Land are shown in table 1. The variable operation costs are assumed to be linear within the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the power plants in Land.

| Power source | Production capability <br> $[T W h / y e a r]$ | Variable costs <br> $[\mathrm{\alpha} / \mathrm{MWh}]$ |
| :--- | :---: | :---: |
| Hydro power | 66 | 5 |
| Nuclear power | 60 | $90-100$ |
| Biofuel | 20 | $200-400$ |
| Fossil fuels | 10 | $300-500$ |

a) (2 p) In addition to the domestic generation, Land is importing 6 TWh from Maa. How large is the electricity consumption in Land if the electricity price during a certain year is $360 \mathrm{~d} / \mathrm{MWh}$ ?
b) ( $\mathbf{1} \mathbf{p}$ ) Consider the same system as in part a, but assume that the nuclear power producers have to pay a waste deposit fee of 10 for each generated MWh. Which electricity price will there then be in Land?
c) (1 p) Consider the same system as in part a, but assume that there are also 6 TWh wind power with a negligible variable cost. What will the electricity price then be in Land?
d) (2 p) If capacity limitations in the power plants and transmission limits are considered, the electricity prices in Land and Maa will vary from hour to hour. Consider an hour when the electricity price in Land is $400 \mathrm{a} / \mathrm{MWh}$ and the electricity price in Maa is $300 \mathrm{o} / \mathrm{MWh}$ Assume that the interconnections between the two countries have a capacity of 800 MW . How much is Land importing from Maa during this hour?

## Problem 3 ( 6 p)

Consider a power system divided in four areas. Data for the primary control in the system are given in table 2. Data for the transmission lines between the countries are shown in table 3. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. The power flow on the HVDC line are not affected by the frequency of the system, but can only be controlled manually..

Table 2 Data for the primary control.

| Area | Gain <br> (available between 49.0 and 51.0 Hz ) <br> $[\mathrm{MW} / \mathrm{Hz}]$ |
| :---: | :---: |
| A | 2500 |
| B | 3000 |
| C | 1500 |
| D | 1000 |

Table 3 Data for the interconnections.

| Connection | Type | Current transmission (at 10:15) <br> $[M W]$ | Maximal <br> capacity [MW] |
| :---: | :---: | :---: | :---: |
| $\mathrm{A} \leftrightarrow \mathrm{B}$ | Alternating current | 1600 MW from A to B | 2000 |
| $\mathrm{~A} \leftrightarrow \mathrm{C}$ | Alternating current | 600 MW from A to C | 1000 |
| $\mathrm{~B} \leftrightarrow \mathrm{D}$ | Alternating current | 1000 MW from B to D | 1500 |
| $\mathrm{C} \leftrightarrow \mathrm{D}$ | Direct current (HVDC) | 500 MW from C to D | 600 |

a) (1 p) At 10:00 there is balance between production and consumption in the system, no transmission lines are overloaded and the frequency in area $A$ is 50.3 Hz . What is the frequency in area D at this occasion?
b) (2 p) At the occasion described in part a, 800 MW generation is lost in area B and shortly thereafter another 400 MW generation is disconnected in area D. How large is the transmission from area $A$ to area $B$ then the primary control has stabilised the frequency in the system after these two events? (Answer 0 MW if the connection is disconnected due to overloading.)
c) (1 p) What is the frequency in area $A$ when the primary control has stabilised the frequency in the system after the events in part b?
d) (2 p) What is the frequency in area $B$ when the primary control has stabilised the frequency in the system after the events in part b?

## Problem 4 ( 12 p)

Stads energi AB owns a thermal power plant with three blocks. Assume that the company has formulated their short-term planning problem as a MILP problem and that the following symbols have been introduced:

Indices for the power plants: Block I-1, Block II - 2, Block III - 3 .

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\begin{aligned}
\beta_{G g}= & \text { variable operation cost in power plant } g, g=1,2,3, \\
C & C_{g}^{*}= \\
C_{g}^{* *}= & \text { start-up cost of power plant } g \text { after one hour down-time, } g=1,2,3, \\
& g=1,2,3, \\
G_{g, t}= & \text { generation in power plant } g \text { after at least two hours down-time, } \\
G_{g}= & \text { installed capacity in power plant } g, g=1,2,3, \\
\underline{G}_{g}= & \text { minimal generation when power plant } g \text { is committed, } g=1,2,3, \\
\lambda_{t}= & \text { expected electricity price hour } t, t=1, \ldots, 24, \\
s_{g, t}^{*}= & \text { start-up of power plant } g \text {, hour } t, \text { after one hour down-time, } g=1,2,3, \\
& t=1, \ldots, 24, \\
s_{g, t}^{* *}= & \text { start-up of power plant } g, \text { hour } t, \text { after at least two hours down-time, } \\
& g=1,2,3, t=1, \ldots, 24, \\
u_{g, 0}= & \text { unit commitment of power plant } g \text { at the beginning of the planning period, } \\
& g=1,2,3, \\
u_{g, t}= & \text { unit commitment of power plant } g, \text { hour } t, g=1,2,3, t=1, \ldots, 24 .
\end{aligned}
$$

a) (4 p) Stads energi AB sells power to the local power exchange ElKräng.. Formulate the objective function if the aim of the planning problem is to maximise the income of sold electricity minus the costs of the thermal power plant. Use the symbols defined above.
b) ( $\mathbf{4} \mathbf{p}$ ) Formulate the constraint that sets the relation between maximal generation and unit commitment in power plant $g$, hour $t$. Use the symbols defined above.
c) ( $\mathbf{3} \mathbf{p}$ ) Formulate the limits of the optimisation variables defined above for the planning problem of Stads energi AB. To get full score for this problem, you will also have to state the possible index values for each limit.
d) (1 p) The best production equivalent in the hydro power plant Strömmen is $0.75 \mathrm{MWh} / \mathrm{HE}$ and is attained at the discharge 160 HE . The relative efficiency at the discharge 200 HE is $96 \%$. How much is generated in Forsen at the discharge 200 HE?

## Problem 5 ( 12 p)

Consider an electricity market with a normally distributed load, which is supplied by three power plants having the installed capacities 300, 200 and 100 MW respectively (sorted according to increasing variable costs). Table 4 shows some partial results of a probabilistic production cost simulation of this electricity market.

Table 4 Results from a probabilistic production cost simulation of the electricity market in problem 5.

|  | $x=250$ | $x=300$ | $x=350$ | $x=400$ | $x=450$ | $x=500$ | $x=550$ | $x=600$ | $x=\infty$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\tilde{F}_{0}(x)$ | 0.841 | 0.159 | 0.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| $x$ <br> $\int \tilde{F}_{0}(\xi) d \xi$ <br> 0 | 247.917 | 272.917 | 274.990 | 275.000 | 275.000 | 275.000 | 275.000 | 275.000 | 275.000 |
| $F_{1}(x)$ | 0.849 | 0.201 | 0.051 | 0.050 | 0.050 | 0.050 | 0.042 | 0.008 | 0.000 |
| $x$ <br> $\int \tilde{F}_{1}(\xi) d \xi$ <br> 0 | 248.021 | 274.271 | 278.741 | 281.250 | 283.750 | 286.250 | 288.646 | 289.896 | 290.000 |
| $\tilde{F}_{2}(x)$ | 0.864 | 0.281 | 0.146 | 0.145 | 0.130 | 0.065 | 0.043 | 0.012 | 0.000 |
| $x$ <br> $\int \tilde{F}_{2}(\xi) d \xi$ <br> 0 | 248.219 | 276.844 | 285.867 | 293.124 | 300.177 | 305.052 | 307.655 | 309.031 | 310.000 |
| $F_{3}(x)$ | 0.878 | 0.352 | 0.218 | 0.158 | 0.132 | 0.073 | 0.052 | 0.017 | 0.000 |
| $x$ <br> $\int \tilde{F}_{3}(\xi) d \xi$ <br> 0 | 248.397 | 279.159 | 292.102 | 301.496 | 308.746 | 313.859 | 316.908 | 318.633 | 320.000 |

a) (1 p) How large is the probability that the load in the system is larger than 300 MW ?
b) (3 p) What is the LOLP of the system?
c) (2 p) How large is the expected generation per hour in the third power plant?
d) (2 p) Assume that the same system is simulated using Monte Carlo techniques and that the random value 250 MW has been generated for the total load. What is the complementary random number of this value?
e) (4 p) Assume that the same electricity market is simulated using a multi-area model. The results of 10000 scenarios are shown in table 5 . This simulation is using both control variates and stratified sampling. Which estimate of LOLP is obtained from this simulation?

Table 5 Results from a Monte Carlo simulation of the system in problem 5.

| Stratum, <br> h | Stratum weight, $\omega_{h}$ | Number of scenarios, $n_{h}$ | Results from multi-area model, $\sum_{i=1}^{n} x_{i, h}$ <br> (where $x_{i, h}$ is the observed value of LOLO in scenario $i$, stratum $h$ ) | Results from PPC model, $\sum_{i=1}^{n} z_{i, h}$ <br> (where $z_{i, h}$ is the observed value of $L O L O_{P P C}$ in scenario $i$, stratum $h$ ) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.913 | 4900 | 0 | 0 |
| 2 | 0.070 | 5000 | 900 | 0 |
| 3 | 0.017 | 100 | 100 | 100 |

## Answer sheet

Name:
Personal number:

## Problem 1

a) Alternative is correct.
b) Alternative ......................... is correct.
c) Alternative is correct.

## Problem 2

a)
TWh
b)
a/MWh
c)
a/MWh
d)
MWh

## Problem 3

a)
Hz
b)
MW
c)
Hz
d)
Hz

## Problem 4

a)
b)
c) $\qquad$
$\qquad$
$\qquad$
d)

MW

Problem 5
a)
\%
b)
\%
c)
MWh/h
d)
MW
e)
\%
Problem $\mathbf{4}$
a) maximise $\sum_{t=1}^{24} \sum_{g=1}^{s}\left(\left(\lambda_{t}-\beta_{G g}\right) G_{g, t}-C_{g}^{* s} S_{g, t}^{*}-C_{g}^{* *} s_{g, t}^{* *)}\right.$.
b) $G_{g, t} \leq u_{g, t} \bar{G}_{g}$.
c) The minimal and maximal generation for each hour is controlled by special constraints. The
only limits that have to be stated are then for the binary variables:
$s_{g, t}^{*} \in\{0,1\}, \quad g=1,2,3, t=1, \ldots, 24$,
$s_{g, t}^{* *} \in\{0,1\}, \quad g=1,2,3, t=1, \ldots, 24$,
$u_{g, t} \in\{0,1\}, \quad g=1,2,3, t=1, \ldots, 24$.
d) The generation is given by $H(Q)=\gamma_{\text {max }} \cdot \eta(Q) \cdot Q=0.75 \cdot 0.96 \cdot 200=144 \mathrm{MW}$.
d) The generation is given by $H(Q)=\gamma_{\text {max }} \cdot \eta(Q) \cdot Q=0.75 \cdot 0.96 \cdot 200=144 \mathrm{MW}$.

## Problem 5

a) The load duration curve states the probability that the load exceeds a certain level. In this case we are looking for $\tilde{F}_{0}(300)=\{$ read in table $\}=15.9 \%$.

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\text { c) } E G_{3}=E E N S_{2}-E E N S_{3}=\int_{500}^{\infty} \tilde{F}_{2}(\xi) d \xi-\int_{600}^{\infty} \tilde{F}_{3}(\xi) d \xi=
$$


$=(310.000-305.052)-(320.000-318.633)=3.581 \mathrm{MWh} / \mathrm{h}$.
d) The inverse transform method states that $D=F_{D}^{-1}(U)$, where $U$ is a $U(0,1)$-distributed random number. Since it is the duration curve that is given in the problem, we may as well use the
transform $D=\tilde{F}_{D}^{-1}(U)$. The original random number must then have been $U=F_{D}(200)=0.841$. Hence, $U^{*}=1-U=0.159$, which results in $D^{*}=\tilde{F}_{D}^{-1}\left(U^{*}\right)=300$ MW.
The problem can also be solved by observing that the normal distribution is symmetrical; hence,
if $D=\mu_{D}+X$ then $D^{*}=\mu_{D}-X$. In this case we have $\mu=E[D]=275$; thus, $D=275+(-25)$ and if $D=\mu_{D}+X$ then $D^{*}=\mu_{D}-X$. In this case we have $\mu=E[D]=275$; thus, $D=275+(-25)$ and
then we get $D^{*}=275-(-25)=300$ MW.
e) We start by computing the expected difference between the multi-area model and the PPC model in each stratum:

We can now combine the results of each stratum weighted by their stratum weights:
$m_{(X-Z), 1}=0$,
$m_{(X-Z), 2}=(900-0) / 5000=0.18$,
$m_{(X-Z), 3}=0$. ,
$m_{(X-Z)}=\sum \omega_{h} m_{(X-Z), h}=0+0.07 \cdot 0.18+0=0.0126$.
LOLP of the multi-area model is given by the expected difference plus the result of the PPC model
(which was computed in part b):
$L O L P=m_{(X-Z)}+L O L P_{P P C}=0.0126+0.017=2.96 \%$.

