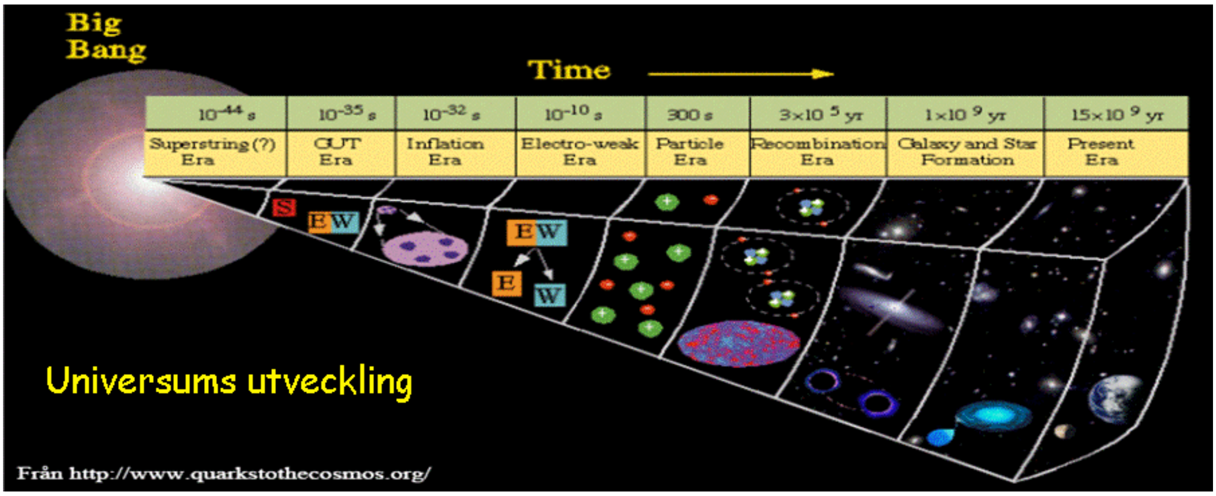
Modern Physics 9p ECTS



The course in Modern Physics 9p is divided into three parts of 3p each. *Part I* deals about special relativity, quantum theory,the photon, statistical physics and the Schrödinger equation. They make the foundation for further studies of modern physics. *Part II* describes atoms, molecules, solid-state-and nanophysics. Finally, *Part III* deals with nuclear physics and energy, particle physics and astrophysics. Within each chapter there are numerous examples with solutions and Multiple Choice-questions, as well as two. One can do an exam of 3p of each part separately.

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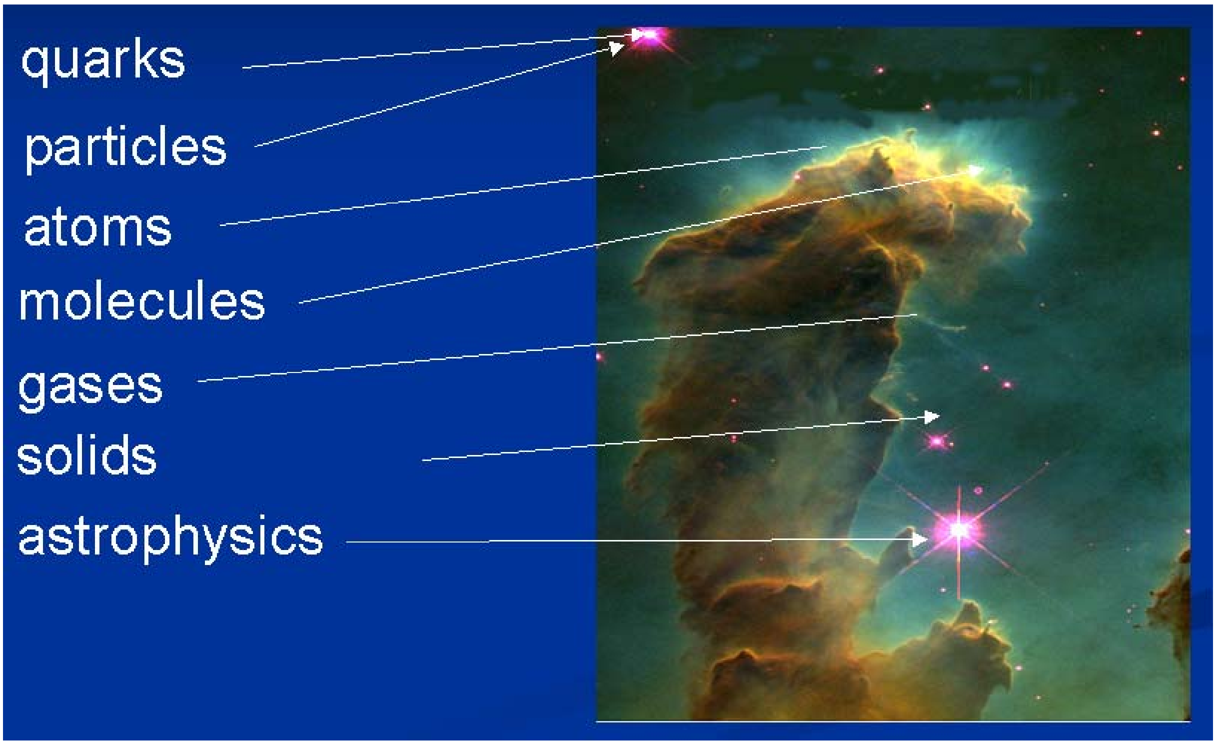
Part I

Chapter 1

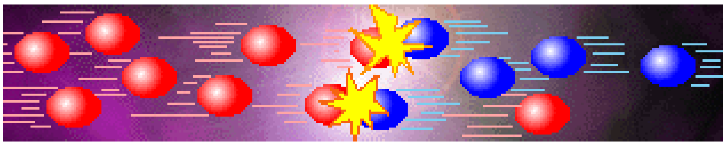
Introduction

The classical mechanics, developed from Newton, had successively been applied to many dynamic problems in various areas as thermodynamics, electricity and magnetism and astronomy. At the end of the 19:th century, together with the support from Maxwell's equations, this lead to the belief that all kinds of problems in physics could be solved. However, there were questions that had no good answers. One example is the photoelectric effect; when light is hitting a polished surface of Zn in vacuum, electrons are pushed out of the surface and one can measure the current in a circuit of these *photoelectrons*. One then sees that this is impossible to achieve a current with red or infrared radiation, but with green, blue or ultraviolet light we have a photocurrent. Maxwell's equations showed that it would be possible also with red light or infrared radiation, if just the intensities were high enough. The effect did not show up. Einstein, however, could show that the photoelectric effect could be explained by the photon concept, where quanta of light $E = hf = hc/\lambda$ have a higher energy if the frequency is higher. There is a threshold for the effect to occur, and a threshold for the wavelength of the light. Many other effects have also been able to be explained with modern quantum physics.
In this course, we look out into the Universe and at the many phenomena taking place. We will try to explain many of these effects in the course, which are dealing bout solid materials, gases, atoms and molecules, nuclei, particles and quarks.



Chapter 2

The theory of special relativity

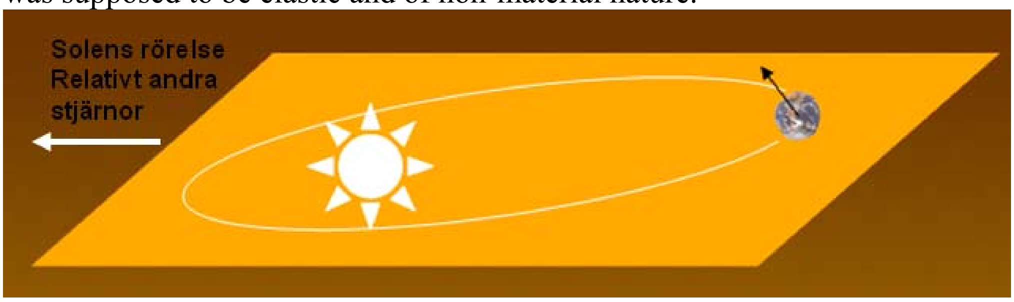


2.1 Relativity
Let us suppose that Newtons laws hold in a coordnate system S and that another system S' is moving along the x-axis. It is a so-called inertial system that is moving at a constant velocity relative the other system. The three position coordinates and a time coordinate that have different values for the two systems can then characterize an event, P. If their origins coincides at $t = 0$ the following relations hold between the two coordinate systems:

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$
$$t' = t$$

It is easy to check that the Newtonian mechanics laws are not changed after a Galilean transformation. This is a consequence of that v is constant, which leads to that the acceleration (second derivative of x and x' respectively as a function of the time t) are the same in both systems. When it shows that both A and A' observe the same force and the same mass, Newton's second law (***F***=***am***) will hold in both systems S and S'. A closer investigation shows that whole the Newtonian mechanics is invariant under a Galilean transformation. Unfortunately, it shows that the important laws of electromagnetic radiation, the Maxwell's equations, not are invariant under the Galilean transformation. Einstein tried to find a transformation, where all laws of physics were invariant. The search for such a transformation lead to the theory of special relativty.

Einstein started with the statement that the speed of light is independent of the observations in the two coordinate systems as well as the speed of the light sources. He was supported by the famous Michelson-Morleys experiment, which was performed during the later part of 1800. Their experiment was a search for the speed of the Earth in the so-called *Ether*. At this time, people thought that light not could expand or move in empty space, a medium for its propagation, like air) This medium was called Ether (after the Aristotle's fifth element) and was supposed to be elastic and of non-material nature.

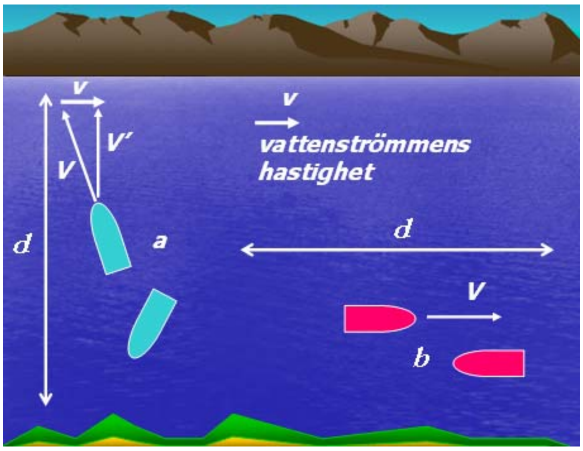


The Earth's orbit around the Sun in the "Ether"

In the Michelson-Morleys experiment one supposed that the speed of light measured on Earth must be dependent on its movement in respect to the Ether. Since the Earths movement relative the Ether was not known, on tried the following: The Earth moves in its orbit around the Sun at a speed around 30 km/s. By using a special experimental setup, a Michelson interferometer, one compared the lights velocity parallel to the Earths orbit around the Sun as well as perpendicular to its orbit. The experiment was repeated at different positions of the orbit around the Sun, since the earth could not be at rest with respect to all points of its orbit in the Ether. At least at some occasions one should get different values of the speed of light in the two perpendicular measurements. However, no such observations were made.

2.2 Michelson-Morley's experiment

Two boats are supposed to cross a river flowing at a velocity of $\boldsymbol{v}$. Both boats have the same speed, $\boldsymbol{V}$. Boat ***a*** crossing directly in the y-direction by steering to the left making its velocity in the y-direction to become $\boldsymbol{V'}$;



$$V^2 = V'^2 + v^2$$

We can thus obtain the velocity of the boat in the y-direction:

$$V' = \sqrt{V^2 - v^2} = V\sqrt{1 - \left(\frac{v}{V}\right)^2}$$

If the boat travels the direction d back and through, i.e. $2d$, we can derive the time it takes, $t_a = 2d/V'$ or:

$$t_a = \frac{2d}{V\sqrt{1 - \left(\frac{v}{V}\right)^2}}$$

The other boat moves back and through the distance $2d$ against the stream and with the stream. The over all time will be

$$t_b = \frac{d}{V + v} + \frac{d}{V - v} = d\frac{(V - v) + (V + v)}{(V - v)(V + v)} = \frac{2dV}{V^2 - v^2} = \frac{2d}{V\left(1 - \left(\frac{v^2}{V^2}\right)\right)}$$

We can divide these two times between each other and get.

$$\frac{t_a}{t_b} = \sqrt{1-\left(\frac{v}{V}\right)^2}$$

Now, we can determine the rivers streaming velocity *v* if we know *V*.
The difference in time $\Delta t = t_a - t_b$ can be determined by the expressions above:

$$\Delta t = t_a - t_b = \frac{2d/V}{1-\left(\frac{v}{V}\right)^2} - \frac{2d/V}{\sqrt{1-\left(\frac{v}{V}\right)^2}}$$

By using the binomial theorem, we get

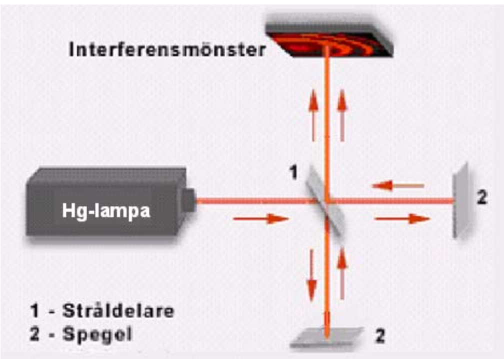
$$(1 \pm x)^m = 1 \pm mx + \frac{m(m-1)x^2}{2!} \pm \frac{m(m-1)(m-2)x^3}{3!} + \ldots$$

This expression holds, if $x^2 < 1$. However, if $x \ll 1$ we can make an approximation:

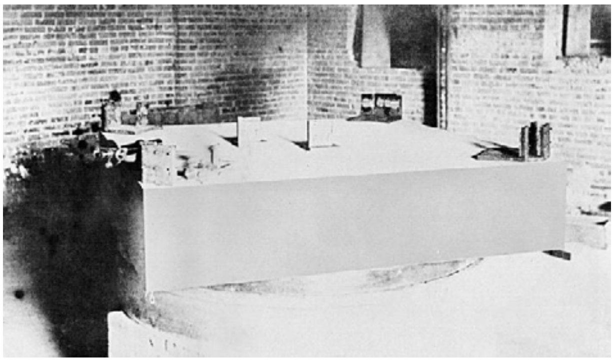
$$(1 \pm x)^m \approx 1 \pm mx$$

In our case we have $(v/V)^2 = (30/300\,000)^2 = 10^{-8}$, where we have been using $c = 300\,000$ km/s and the speed of the Ether wind $v = 30$ km/s. Thus, we can make an approximation of Δt above, with:

$$\Delta t = \frac{2d}{c}\left[\left(1+\frac{v^2}{c^2}\right) - \left(1+\frac{1}{2}\frac{v^2}{c^2}\right)\right] = \frac{d}{c}\frac{v^2}{c^2}$$



Michelson interferometer


Michelson-Morely's experimental setup

In the Michelson interferometer one can think of the Ether moving parallel with one axis and perpendicular to the other. By rotating the interferometer and study the interference pattern it would be possible to determine the velocity of the Ether wind. One was able to measure with an accuracy of around 0.5 m/s.
If one in the interferometer measures the difference in length between the arms, ℓ, and the corresponding difference in time Δt, we get

$$\ell = c\,\Delta t$$

If we suppose the change in the number of fringes, n, of the interference pattern corresponds to a difference in distance, ℓ, we obtain

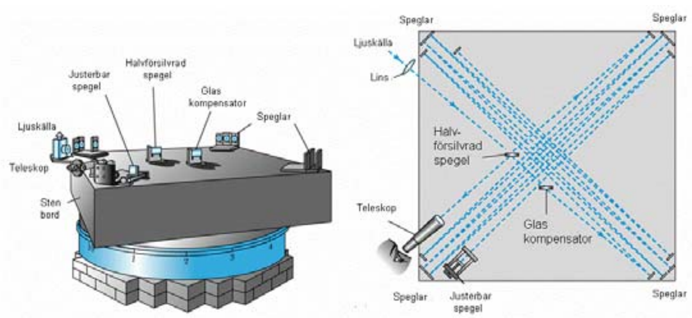
$\ell = n\lambda$,

where λ is the wavelength of the light. In Michelson´ s case it was a mercury lamp with a filter giving the wavelength λ = 546 nm. The effective length of the interferometer was *2d=2x5.0* m.

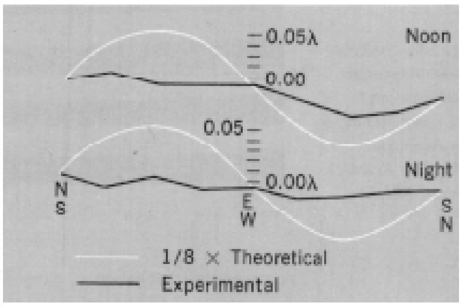
We can deduce the number of fringes n to be

$$n = \frac{c\Delta t}{\lambda} = \frac{dv^2}{\lambda c^2} = \frac{2\times 5{,}0\times\left(30\times 10^3\right)^2}{546\times 10^{-9}\times\left(3{,}00\times 10^8\right)^2} = 0{,}18 \text{ fringes}$$

Since both arms of the interferometer add to give the total change in fringe shift. we will get 0.4 fringes, which is easy to observe. However, Michelson did not observe any change in fringes! No Ether wind could be detected. One could summarize, that the speed of light was the same everywhere, disregarding the observer's velocity. Below is shown schematically the experimental setup and the experimental result.



The rotatable setup with multiple reflexions.



The measuring result (Michelson, Studies in Optics)

The results of these experiments showed that the velocity of light was the same in all directions, independent of the observer's own movements. Since people had difficulties in accepting this, many attempts were made to explain the results by another method. One explanation was the so-called Lorentz contraction hypothesis, where objects moving in the Ether were shortened in the direction of the movement. However, this hypothesis was rejected due to stellar observations.

2.3 Einstein's two postulate

1. The laws of physics are the same (have the same mathematical form) in all-inertial systems. One can say that an object, not influenced by forces, is moving at a constant velocity.

2. The speed of light in vacuum has the same value (299792458 m/s) in all inertial systems.

These postulates lead to the Lorentz transformation between the systems S and S'. The systems S and S' move at the constant velocity v relative each other along the x-x´ axis (standard configuration) and the coordinates for an event (x, y, z, t) in S are related to the coordinates (x', y', z', t') for the same event in S' according to the formulae below:

$x' = \gamma(x - vt)$

$y' = y$
$z' = z$
$t' = \gamma(t - x\, v/c^2)$

$\gamma(v) = 1/\sqrt{1-v^2/c^2}$

Einstein´s postulat leads to length contraction, time dilatation and relativistic mass.

2.4 Length contraction
Since both observers agree about the relative velocity, v (although with different signs), $v = l/t = l' / t'$

$l = l' / (1 - v^2 / c^2)^{1/2}$

The one who moves will look shorter - Mr Tompkins can be of help here. (The formula is easy to learn – the important thing is to remember when to multiply or to divide).

Example
We have two velocities of 10^6 m/s and 0,9c. Calculate the length contraction in both cases.

Solution

$$\frac{L'}{L} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\frac{L'}{L} = \sqrt{1 - \left(\frac{10^6}{3 \times 10^8}\right)^2} = 0{,}99999444$$

, that is 99,99944 percent! In spite of the high velocity, the length contraction is very small. In the next case we get

$$\frac{L'}{L} = \sqrt{1 - \left(\frac{0{,}9c}{c}\right)^2} = 0{,}4359$$

, that is 44%, almost half the original length.

2.5 Time dilatation
It follows from the ”light clock” (look at. "Thinking Physics is Gedanken Physics") and results in
$t = t' / (1 - v^2 / c^2)^{1/2}$
An observer always thinks that a moving clock goes to slow.

Example
We can look at the famous example with μ-mesons, elementary particles entering our atmosphere at high velocities, 0.998c. These particles decay with a lifetime of 2.0 μs. During this time they move the distance
$s = vt = 0{,}998 x 3 x 10^8 x 2{,}0 x 10^{-6}\ m = 600\ m$.
However, these particles are formed at high altitudes above Earth, at around the height of 10 km and it should be impossible for them to reach the surface of the Earth. In spite of this,

many particles reach the surface. How can this be possible? We use the theory of relativity to see if we can explain this. In the mesons own coordinate system the life-time is the same. However, their distance to the Earth is shortend by a factor

$$\frac{s}{s_0} = \sqrt{1 - v^2 / c^2}$$

The mesons will see the distance as *s* that is experienced as 600 m, why observers on the Earth experiences it as s_0. The distance we believe the μ-mesons travel will then become

$$s_0 = s\frac{1}{\sqrt{1 - v^2 / c^2}} = 600\frac{1}{\sqrt{1 - (0{,}998c)^2 / c^2}} = 9490m$$ **,** that is around 9.5 km

In this way, the mesons can travel down to the face of the Earth in spite of their small lifetime, since they travel so fast. You can also look at the problem in another way. You can try to determine their lifetime in the Earth's reference system. Their lifetime in the Earth's system will be 32 μs instead of 2 μs, which is enough to travel the longer distance 9.5 km.

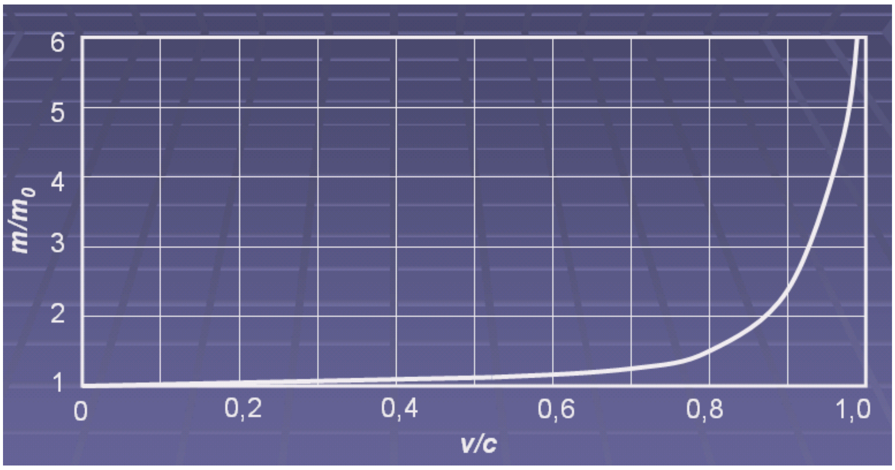
2.6 Relativistic mass

If an observer watches an object moving relative him, the observed mass *m* will be greater then the rest mass m_0 as the observer experiences himself.

Equation:

$$m = \frac{m_0}{\sqrt{1 - v^2 / c^2}}$$

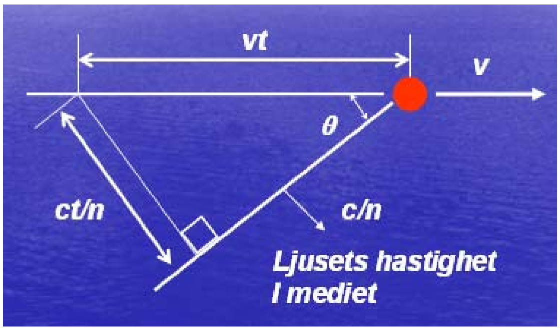
Studying the law of conservation of momentum one can derive this expression. One simply uses the expression of time dilatation.



At the construction of bending magnets in large accelerators you have to compensate for this effect. In nuclear power plants one can observe in the water basins surrounding the reactor, a blue like light appearing when relativistic particles travel at speeds greater than the speed of light in the medium, the water. In the water the speed of light is $c/n = c/1{,}33$ where *n* is the refractive index. This is the so-called Cerenkov effect. The particle velocity can be derived from

$$v = \frac{c}{n \cdot \sin\theta}$$

In the figure below, the Cerenkov effect is illustrated:



Example
A particle is moving at the velocity 0.9c. How large is the particles relativistic mass?

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}} = \frac{m_0}{\sqrt{1-(0{,}9c)^2/c^2}} = \frac{m_0}{\sqrt{1-0{,}81}} \approx 2{,}3m_0$$

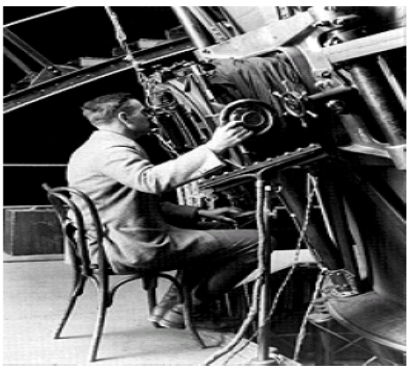
2.7 Relativistic Doppler shift
One example of the Doppler shift is the frequency changes caused by the stars movement relative to the Earth. This effect makes it possible for astronomers to measure their velocities relative to the Earth. This also makes it possible to discover planets close to stars. The effect also makes it possible to measure the red shift of distant stars and galaxies travelling away from us at high velocities, v, due to the expansion of the Universe. The frequency change that is caused by the Doppler effect can be described by the formula:

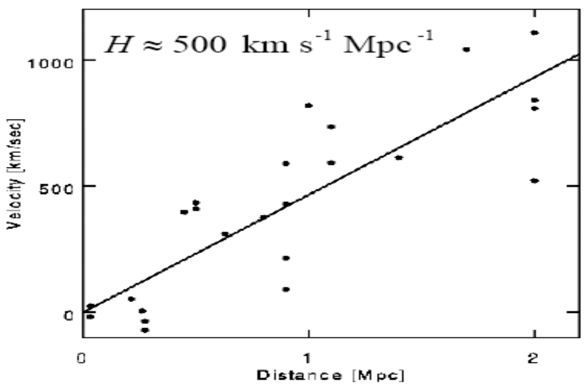
$f = f_0\,((1-v/c)/(1+v/c))^{1/2}$

where f is the frequency that the observer measures, and f_0 is the frequency of the emitter (star) and its velocity v.
The astronomer Edwin Hubble observed spectra at the end of 1920 of light from a great number of galaxies, looking at certain combinations of spectral lines one can identify elements and Hubble found by estimating the distance to the objects that the longer away from Earth the galaxies were, the more were their red shifts. Hubble explained this phenomenon as due to the Doppler effect. His conclusion was that if the galaxies moved away from the Earth at a velocity v, it was proportional to the distance mot r, to the galaxy:
$v = H_0\, r$, *where* H_0 is around 20-24 km/s /(M light years)

Edwin Hubbles observationer 1929

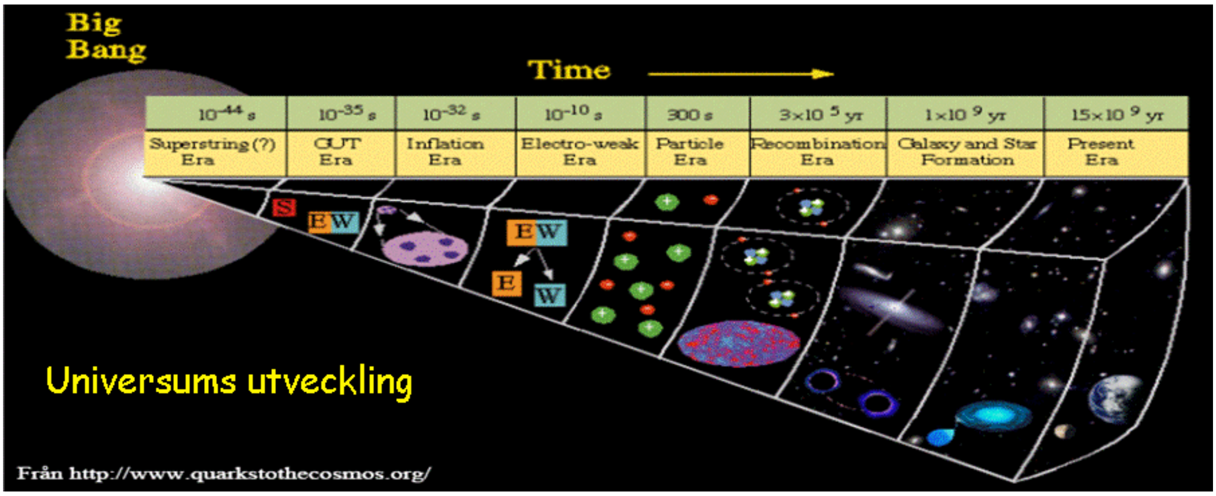
Hubbles lag: $v = H \cdot d \quad ; \quad H = \frac{da/dt}{a}$





1 Mpc = 3.3×10^{6} ljusår = 3.1×10^{22} meter

to från Mt. Wilson: http://www.mtwilson.edu/History
Från http://www.astro.ucla.edu/~wright/cosmolog.htm



When the theory appeared it was not the only cosmological theory, but now most researchers claim that there is one cosmological standard model meaning that the Universe is homogeneous and isotropic and that in the beginning the Universe was hot and dense. One reasonable question is if the Universe will expand forever. This question and similar questions lead to fascinating proposals:

- there are only a few percent of ordinary matter in the
- there is about one fourth of ”dark matter”
- there is about three fourth of ”dark energy”

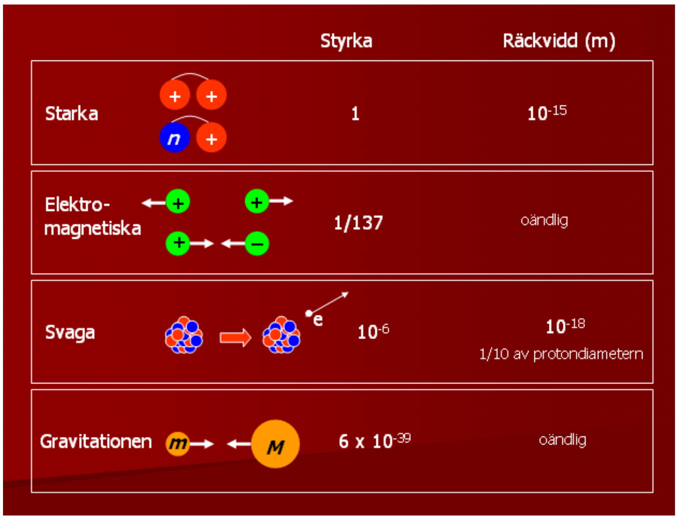
which results in that both some new exotic materia and energy is needed in the Universe and that it also leads to the expansion of the Universe and that it accelerates!

2.8 Force and momentum

We can describe Nature by four kinds of forces:

- The Gravitational force

- The electromagnetic force
- The strong force
- The weak force



The Gravitational force is always present and for instance the electromagnetic force is being used when we make phone calls and sends SMS messages, e-mail or look at TV. Both forces work at large distances, while the strong and weak forces work at short distances. Both forces work on distances of the order of femto-meters, around 10^{-15} m.
Mechanical forces, tension and electromagnetic forces, Coulomb forces are often responsible for the Forces of Nature.

When one in modern physics wants to describe events where high velocities of particles, for instance electrons, are at hand, one has to do the calculations relativistically. Newtons laws can be re-defined:

$$F = dp/dt = d/dt(mv) = d/dt(m_0 v/(1-v^2/c^2)^{1/2})$$

Since the mass depends on the velocity according to the formula above, the kinetic energy, $E_{k,}$ becomes

$$E_k = \int F\,ds = \int d/dt(mv)\,v dt = \int v\,d(mv) = \int (v^2 dm + mvdv)$$

since

$$E_k = \int c^2 dm = (m-m_0)\,c^2 = m_0 c^2(1/(1-v^2/c^2)^{1/2} -1)$$

This leads to the famous Einstein relation

$$E = mc^2 = m_0 c^2 + E_k$$

By definition we have
$$p^2 = (mv)^2 = m_0^2 v^2/(1-v^2/c^2)$$

the total energy *E* expressed in momentum p can be written

$E^2 = m_0^2c^4(1 - v^2/c^2 + v^2/c^2) /(1-v^2/c^2) = m_0^2c^4 + p^2c^2$

At low velocities, v << c an expansion in a power series of γ gives, when only two terms has been taken into account in the power series:

$E_k = m_0c^2(1-(-½)(v^2/c^2) +½ (-½)(-3/2)(-v^2/c^2)^2 - 1) = ½m_0v^2 - 3m_0^2v^4/8c^2$

Notice that this is not the same as $½mv^2$ (where m is the relativistic mass)
This leads to the well-known expression

$E_k = ½ m_0v^2$

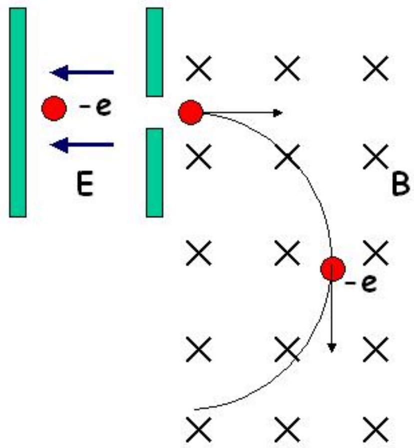
that of course holds for low velocities. In order to get a view of when one should use relativistic calculations one can look at this expression:
$[E_{k(rel)} – E_k]/E_k = ½m_0v^2 - 3m_0^2v^4/8c^2 - ½m_0v^2/1/2m_0v^2 =3v^2/4c^2$

This gives a 1% difference whence $v/c = 0.1$. The classical energy $½m_0v^2$ is then $m_0c^2/200$ that can be used when one has to decide when to do the calculation relativistically.

At high velocities when $v \to c$ we have $E \approx E_k \approx pc$ that holds *exactly* for particles moving at the speed of light, that have the rest mass = 0. Examples of these particles are photons (also being its own anti-particle), three neutrinos and their anti-neutrinos.

In order to investigate the movements of a charged relativistic particle, one often applies homogeneous magnetic fields. In this field the momentum p can be determined. Such setups are often used in many modern particle detectors. In a plane perpendicular to the magnetic field lines the following holds:

$qvB = mv^2/r$ that can be written as $p = mv = qBr$



Electrons cicular movements in a homogeneous magnetic field.

We can notice that the mass m in the expression above concerning p is the relativistic mass.

Contents Chapter 2

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2.2 Michelson-Morely's experiment

2.3 Einstein's two postulates
2.4 Length contraction
2.5 Time dilatation
2.6 Relativistic mass
2.7 Relativistic Doppler effect
2.8 Force and momentum

Learning goals

Define the Galilei transformation
Discuss the Ether concept and the speed of light in vacuum
Differences between the Galilei transformation and the Lorentz transformation
Explain what the Einstein's postulates lead to
Perform calculations about the Doppler shifts
Analyse what supports Hubble's observations and the Universe expansion

Advices for reading

Think about the fact that such a simple hypothesis that the speed of light is constant and the maximal velocity in vacuum, leads to the Lorentz transformation explaining time dilatation, relativistic mass, relativistic Doppler effect etc.

Reading

- Thornton, Rex, Modern Physics, Saunders
- Krane, Modern Physics, Wiley
- Beiser, Concepts of Modern Physics, McGraw-Hill
- Serway, Moses, Moger, Modern Physics, Saunders
- Eisberg, Resnick, Quantum Physics of Atoms, Molecules, Solids and Particles, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Halliday and Resnick, Fundamentals of Physics, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Benson, University physics, Wiley

WEB-readings

- WEB-book: Hyper Physics Internet book
  http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html