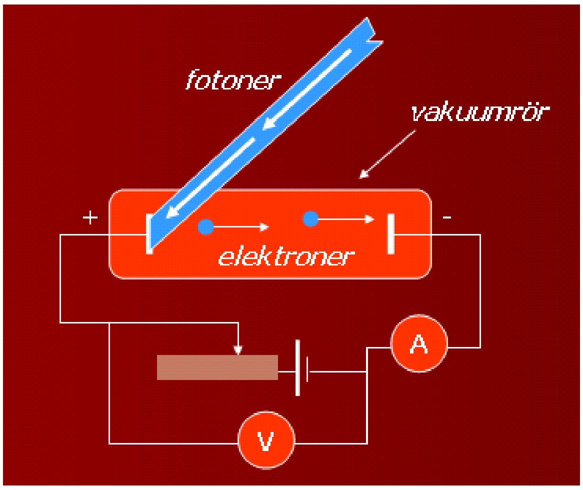
Chapter 3

The original quantum theory



The photoelectric effect

3.1 Introduction

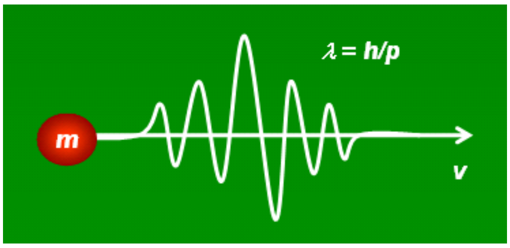
One can regard light as an electromagnetic wave with the energy E, wavelength λ and frequency f. But according to Planck light can be regarded as a stream of particles with the energy

$$E = hf$$

, where h = Planck's constant = 6.626 10^{-34} Js, $f = c/\lambda$ and c is the speed of light. These particles, *photons*, have a momentum p and the corresponding energy $E = pc$.

3.2 deBroglie's material waves

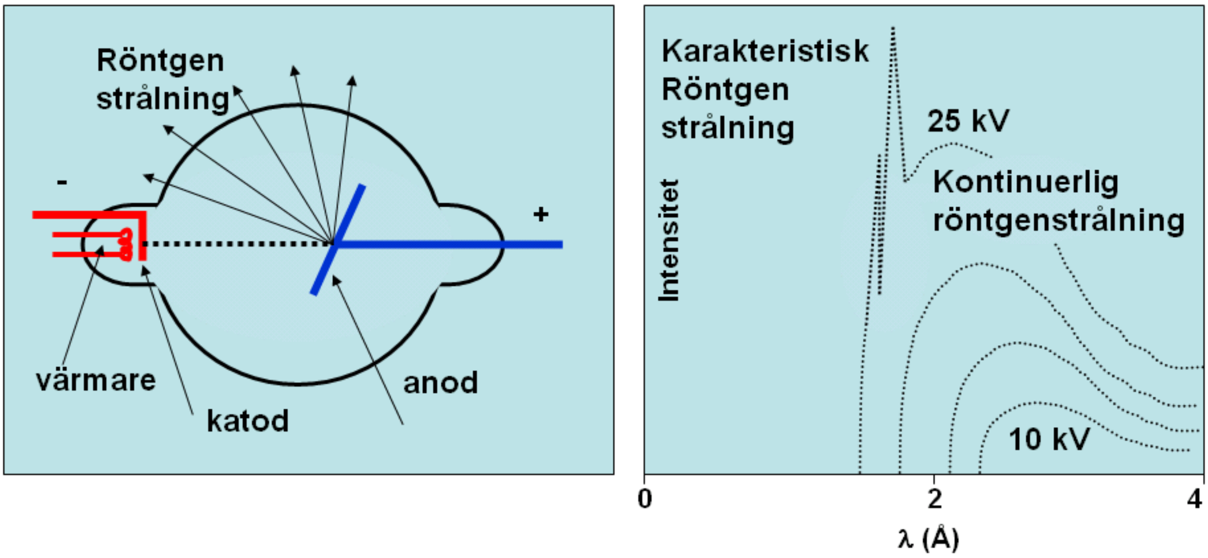
A crystal works as a three-dimensional grating. The distance between the atoms is of the order of 0.1-0.2 nm. In order to achieve an observable scattering, the wavelength of the radiation should be about the order which means that we are in the X-ray regime. ($E=hc/\lambda = 1240/\lambda$, E i eV , λ i nm)



deBroglie wave associated with the particle with the momentum $p = mv$

The crystal does not need to be a single crystal, but can be made of small crystals in powder form. In that case the atomic planes of the crystals of the powder are randomly oriented in space. (The X-ray radiation should contain some wavelengths or at least a few).

Only those atomic layers oriented in a way so that the Bragg condition $2d\sin\theta = m\lambda$ is fulfilled gives a contribution to the intensity. On a screen one gets a system of circles. One circle for each set of *d, m* and λ.
X-ray radiation can be made by accelerating electrons in an X-ray tube. When the anode stops them very rapidly, a continuous spectrum is obtained (so called Bremsstrahlung).



X-ray tube and X-ray spectrum where sees both the continuous and the characteristic part

Louis de Broglie suggested in his doctorial thesis (1924) that the photon's wave/particle dualism could be applied on particles with *rest mass*. Since these show wave behaviour one can associate them with a wavelength. To interpret E in the equation $E=hf$ is not possible since we does not know if we shall interpret E as the total relativistic energy or the kinetic energy (that is the same for photons), but with the momentum p there is no such double meaning. deBroglie suggested (without any experimental support) that each material particle with momentum p can be associated with a wavelength λ where:

$\lambda=h/p$

where h ($h = 6.62610^{-34}$ Js) is Planck's constant and λ the particles wavelength. de Broglie's hypothesis was confirmed later by the Davisson-Germer experiment (1927).

Example
Let us calculate the deBroglie-wavelength for an electron accelerated over a voltage of 120 V.

With de Broglies postulate $\lambda = h/mv$ can we determine the wavelength λ. The electron gets the kinetic energy

$$K = \frac{mv^2}{2} \quad \Rightarrow \quad v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 120 \times 1.602 \times 10^{-19}}{9.102 \times 10^{-31}}} = 6.5 \times 10^6 \, m/s$$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6{,}63 \cdot 10^{-34}}{9{,}11 \cdot 10^{-31} \times 6{,}5 \cdot 10^6} \approx 1{,}12 \times 10^{-10} m = 1{,}12 \mathring{A}$$

The wavelength will thus be 1.12 Å, which is in the X-ray region.

When light interacts with other particles one uses the photon concept and the conservation laws in order to successfully describe its interaction with matter in processes like the photoelectric effect, the Compton effect and pair production.

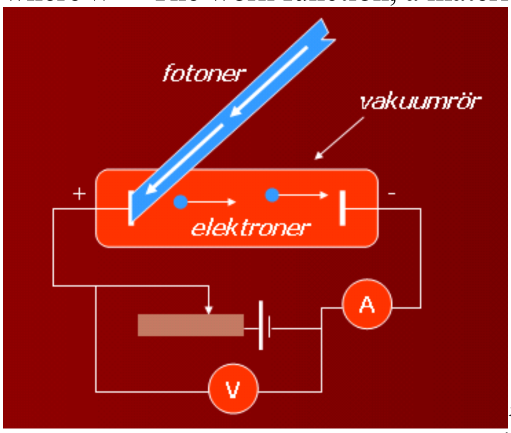
4. The photon

4.1 The photoelectric effect

The photoelectric effect, formulated by Einstein in 1905, can be explained if one consider the electromagnetic radiation from the source is radiated as light quanta or photons. The photons travel with the speed of ligh c and can completely be absorbed by an electron of the surface of a metal, thereby giving the electron kinetic energy E_k:

$hf = W + E_k$

where W = The work function, a material constant.



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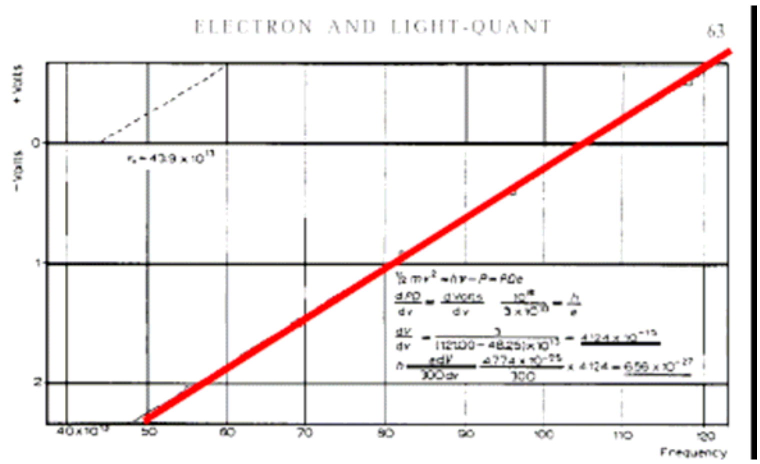
One created an experimental setup where one applied a stopping voltage V making it possible to determine the electrons kinetic energy E_K. One gets the following $E_K = eV$.
We divide Einstein's relation $hf = W + E_k$ with e and obtains

$$\frac{h}{e}f = \frac{W}{e} + \frac{E_K}{e} = \frac{W}{e} + V$$

Now we have the equatin describing the figure below, which is the Millikan experimental determination of e and h:

$$V = \frac{h}{e}f - \frac{W}{e}$$



Millikan's experimental curve describing the photoelectric effect.

Example
I the figure above describing the Millikan experiment he found at the crossing of the curve with the x-axis (when $V_0 = 0$ volt) that $f = 4.39\text{x}10^{14}$ Hz. Calculate the work function W for the substance in question in both eV and in Joule.

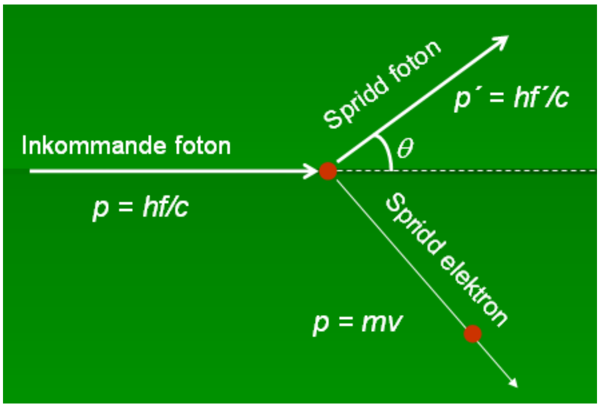
At the crossing with the f-axis we get $hf_0 = W$, and:
$W = hf_0 = 6.63\text{x}10^{-34}$ x $4.39\text{x}10^{14}$ J = $2.9\text{x}10^{-19}$ J = $2.9\text{x}10^{-19}/1.60\text{x}10^{-19}$ eV = 1.8 eV

Example
Also make a calculation of the Plank constant h using the Millikan graph.
The graph can be described by $V = \frac{h}{e} f - \frac{W}{e}$ where the slope of the curve is h/e.

$$h = \frac{e\Delta V}{\Delta f} = \frac{(2{,}0 - 0{,}0) \times 1{,}602 \times 10^{-19}}{(105 - 56) \times 10^{13}} Js = 6{,}54 \times 10^{-34} Js$$

The correct value of the Plank constant is $h = 6.625\text{x}10^{-34}$ Js.

4.2 The Compton effect



Energy conservation gives
$E + m_ec^2 = E' + m_ec^2 + Ek$
that can be written as
$E = E' + (p^2c^2 + m_e{}^2c^4)^{1/2} - m_ec^2$
The conservation of momentum gives
$p = p' + p_e$
We rearrange and take squares leading to
$p^2c^2 + p'^2c^2 \text{ -}2pp'c^2 = pe^2c^2$
but E = pc and E'= p'c so we get
$E^2 + E'^2 \text{ -}2EE'cos\theta = pe2c2$
If we use energy conservation solve for pe^2c^2 we get
$E = E' + (pe^2c2 + me^2c4)^{1/2} - m_ec^2$
Rearranging again and taking squares we get
$(E - E' + m_ec^2)2 = pe^2c2 + me^2c^4$
$E^2 + e'^2 \text{ -}2EE' + 2(E\text{-}E')mec2 = pe^2c^2 = E^2 + E'^2 - 2EE'cos\theta$
Simplification gives

$(E\text{-}E')\, m_e c^2 = EE'(1\text{-}\cos\theta)$

and after rearranging again we get

$(E\text{-}E') = EE'(1\text{-}\cos\theta)/\, m_e c^2$

that can be expressed in wavelength since we are dealing with photons:

$E = hf = hc/\lambda$

By inserting this expression above we get the Compton equation

$(\lambda' - \lambda) = (h/\, m_e c)(1\text{-}\cos\theta)$

$h/m_e c$ is called the electrons Compton wavelength and has the value 0.00243 nm.

Example

A photon of wavelength λ = 0.449 Å is Compton scattered at an angle of 60° against a free electron. Calculate the wavelength of the scattered photon.

We have the Compton expression:

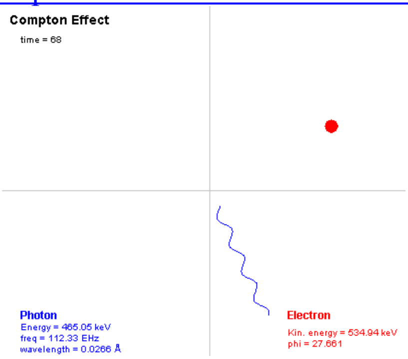
$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

The Compton wavelength is $h/m_e c$ = 0,0243 Å

$$\lambda' = \lambda + 0{,}023(1 - 1/2)\mathring{A} = 0{,}449 + 0{,}023/2\mathring{A} \approx 0{,}460\mathring{A}$$

Compton scattering – Applet simulation can be found on**:**

http://www.student.nada.kth.se/~f93-jhu/phys_sim/compton/Compton.htm



The program describes how the scattering occurs for different photon energies and incoming directions.

4.3 Pair production

The photon has the energy E_γ and the momentum p_γ. The electron and the positron have the corresponding entities E_e, p_e E_p and p_p respectively.

The energy and momentum conservations give the following equations

$$E_\gamma = E_e + E_p$$

$$p_\gamma = p_e + p_p$$

Observe that the charge also is conserved. The conservation of the momentum directly gives (in the direction of the photon)

$$p_\gamma c. = E_\gamma = [(p_e c)^2 + (mc^2)^2]^{1/2} + [(p_p c)^2 + (mc^2)^2]^{1/2} > p_e c + p_p c$$

This (strict) expression always holds why it is not possible to fulfil both the law of energy conservation and the law of momentum conservation without a fourth part participating in the process. It can only be fulfilled in the presence of a (heavy) nucleus overtakes the excessive momentum. If the nucleus is heavy it just takes a small part of the (kinetic) energy.

Example:
Let the incoming photon have the energy 3.0 MeV. The corresponding momentum then becomes 3.0 MeV/c. The electron/positron energies can have random energies (> 0,511 MeV) as long as the total energy is conserved.
Let for example E_{e-} be equal to 1.80 MeV and then we have E_{e+} = 1,20 MeV
The corresponding momentum will be p_{e-} = 1,73MeV/c and p_{e+} = 1,09 Mev/c
This leads to the momentum given to the nucleus must be
3.0 MeV/c – 1.73 MeV/c – 1.09 MeV/c = 0.18 MeV/c
Since the nucleus is heavy we *suppose* that we can do non-relativistic calculations (but of course we have to check this at the end) and the corresponding kinetic energy E_k becomes
$E_k = p^2 /2M = (0.18 \text{ MeV/c})^2 /2M = 0.0324 (\text{MeV})^2 /2Mc^2$

In order to get a view of E_k we let the nucleus be the lightest one we can think of, i.e. a proton with rest energy
$Mc^2 = 938.3$ MeV

Then we obtain E_k = 0.000 017 MeV = 17eV, which is a very small value why our assumption was correct.

Contents Chapter 3-4

Learning goals

Definie de Broglie's postulate
Be able to discuss matter waves, momentum and energy
Be able to discriminate between particles with mass and without (photon)
Explain what the photoelectric effect leads to compared to the classical description
Calculate the photoelectric work function and the electrons kinetic energy
Discuss the Compton effect
Determine the Compton wavelength
Calculate the wavelength of the scattered photon
Determine the scattered electrons energy and velocity
Discuss the conditions of pair production
Calculate the particle energy at pair production

Advices for reading

Think of the fact that de Broglie's hypotheses lead to the explanation of many unexplainable phenomena easily could be described and the dualism between particles and waves. Also think of the phenomena, photoelectric effect, Compton effect and pair production are responsible for the interaction between light and matter.

Reading advices

- Thornton, Rex, Modern Physics, Saunders
- Krane, Modern Physics, Wiley
- Beiser, Concepts of Modern Physics, McGraw-Hill
- Serway, Moses, Moger, Modern Physics, Saunders
- Eisberg, Resnick, Quantum Physics of Atoms, Molecules, Solids and Particles, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Halliday and Resnick, Fundamentals of Physics, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Benson, University physics, Wiley

WEB-advices

- http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1929/broglie-bio.html
- http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Broglie.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1927/compton-bio.html
- http://www.student.nada.kth.se/~f93-jhu/phys_sim/compton/Compton.htm