9. Solid state physics

The ISS-station where the solar cells has a power of 250 kW

9.1 Introduction
When you want to describe solid-state physics, one can start with looking at the interaction between a large numbers of atoms in a crystal grating. On can start with looking at the electric and thermal properties of materials, such as conductivity.

Let us classify solids according to their electrical properties.

1) The resestivity ρ at room temperature, $\rho = RA/l\ (\Omega m) \leftrightarrow R = \rho l/A\ (\Omega)$

2) The temperature coefficient α defined as $\alpha = (1/\rho)(d\rho/dT)\ (K^{-1})$

3) The number of charge carriers per volume unit n. $(1/m^3)$

By measurements one finds that some material does not conduct currents, i.e. that have a very high resistivity, and are called *isolators* (diamond has a factor of 10^{24} higher resistivity than copper). Another isolator is polystyrene with a ρ-value around 10^5 Ωm.
One may use measurements of ρ, α and n in order to divide the non-isolators, at least at modest temperatures, into two main groups, *metals* and *semiconductors*.

- Semiconductors have much higher resistivity than metals

- Semiconductors have a negative and large temperature coefficient

- Semiconductors have much less charge carriers per volume unit than metals

Let us start with the **electrical conductivity** and examine how it changes with temperature. We start with a conductor and measure the voltage (V) and current (I). By applying Ohm's law we get $V = RI$ where R is the resistance (Unit: ohm, Ω) of the conductor.
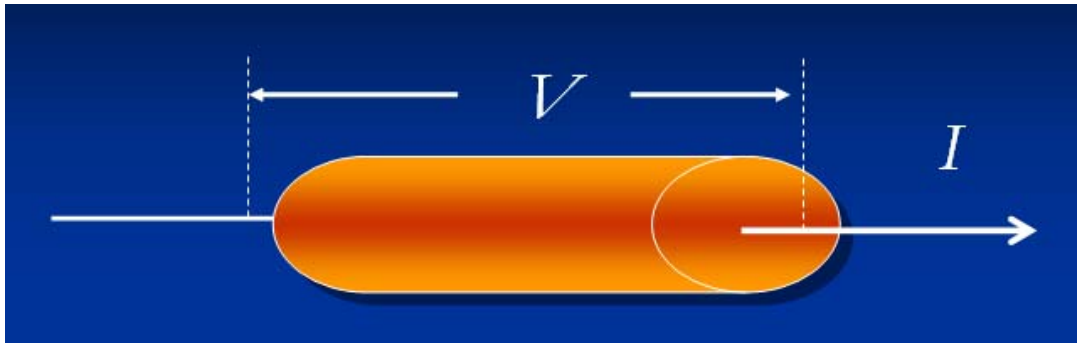
When studying the factors responsible for the resistance, one finds from basic electricity:
The resistivity $\rho = \rho_0 (1 + \alpha\Delta T)$ where ρ is measured in the unit [Ωm], and T is the temperature in the unit Kelvin and where α is a temperature constant measured in T^{-1} which is specific for the metal in question. For copper the constant has the value $\rho_0 = 1.7 \times 10^{-8}$ Ωm at $T_0 = 273$ K and $\alpha = 4 \times 10^{-3}$ T^{-1}. If we compare with a semiconductor as silicon (Si) the picture changes drastically as is shown in the picture below. Si has the resistivity $\rho_0 = 3 \times 10^3$ Ωm, $\alpha = -7 \times 10^{-2}$ T^{-1} within the right part of the curve for silicon.
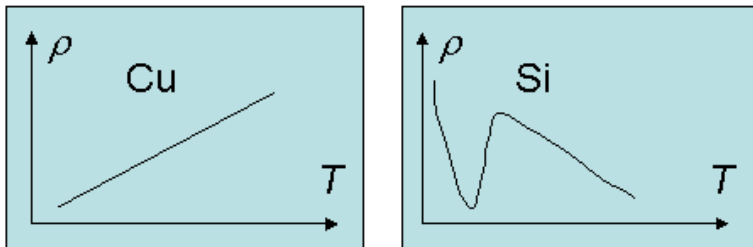
These macroscopic effects in crystals will be discussed later and be possible to explain with the theory of solid-state physics. We will look at different crystals, the band theory, isolators, semiconductors and the properties of the elements of solid-state physics.

The large variations in resistivity ρ (or conductivity $\sigma = 1/\rho$) , mainly depends on the large variation in n (i.e. the number of charge carriers/m^3)
This relation can define the conductivity:

$$j = \sigma E$$

$j = \xi\, v_d = \xi\, v_d = \pm|\,\xi\,|\, v_d =$ current density vector, $[j] = A/m^2$
$\xi =$ space charge $[\xi] = As/m^3$
$E =$ Electric field vector, $[E] = V/m$
$v_d =$ velocity of the space charge $=$ the mean drift velocity of the electrons in the electric field E , that builds up the current.

The mobility μ (>0) is defined by the relation

$$v_d = \pm \mu E$$

($\pm$ since the electrons get a velocity directed opposite to the electric field).
From these relations we also get the following condition

$\sigma = \xi\, \mu$

9.2 The forces within crystals
A crystal differs from other atomic constellations by its three-dimensional periodicity concerning the positions of the atoms. One can create a crystal by starting with a unity-cell and then let it be multiplied in all three directions. A cubic crystal is easy to expand, but if you have a pentagon, it becomes more difficult. The same holds for the creation of a two-dimensional crystal. In that case it is possible to build it with quadratic elements, other parallelograms, triangles, hexagons etc, but not pentagons as unit cells.
Crystal structures were examined in the beginning of 1900 by applying X-ray spectroscopy. X-ray is excellent to use since the wavelength of the radiation is of the same order as the distance between crystal planes.

X-ray diffraction. Solid-state physics is looking for a connection between materials macroscopic properties and its microscopic properties, as well as its atomic and nuclear structure. One also wants to study its composition, discuss various models and properties. One can ask what kind of atoms (chemical identification) we have in the crystal structure and how they are sitting in the crystal geometrically. In most materials, the atoms are arranged, at least locally, in well-defined crystal structures. In order to determine the crystal structure different forms of diffraction techniques are applied. The most common and most powerful technique is X-ray diffraction (XRD). In order to study crystal structures of atomic distances of a few Angstroms, one has to use wavelengths of the same order. One often applied X-ray source is Cu K_{α} with wavelength 1.54 Å. X-ray diffraction within crystals can be regarded as interference between reflexes from closely laying atomic planes of the same layer, according to the figure below.
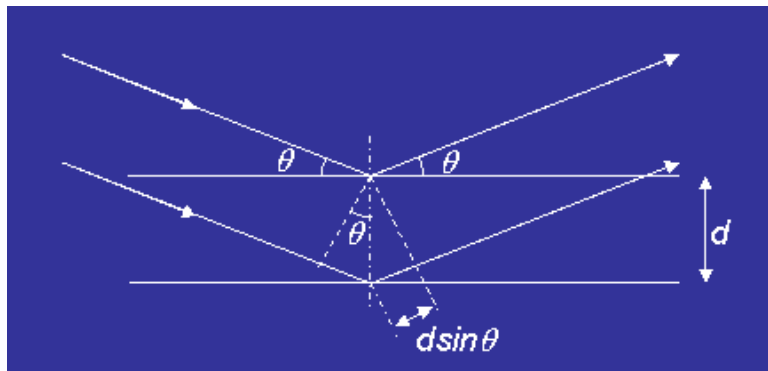
Bragg reflexion for X-rays between two crystal planes.
Constructive interference when *2d sinθ = pλ*, p = 1,2,3,….

The incoming ray can be described by the wave vector ***k***, and the reflected ray by the wave vector ***k'***. The wave vector change is *Δ**k** = **k** - **k'***. In order for constructive interference to occur, **Bragg's law** holds:

$$2dsin\theta = \lambda,$$

where *d* is the distance between two crystal planes. Observe that in this equation, θ is the *angle of grazing* incidence and not the angle of incidence.

When performing X-ray photography using a grating spectrometer with a cubic grating, one gets fourfold symmetry, as can be seen in the picture below. If the crystal is rotated along a diagonal one can achieve threefold symmetry and in that way orient the symmetry lines of the

crystal. In order to get a spectrum like this, the X-ray beam has to consist of a continuous spectrum (as in the case of Bremsstrahlung)
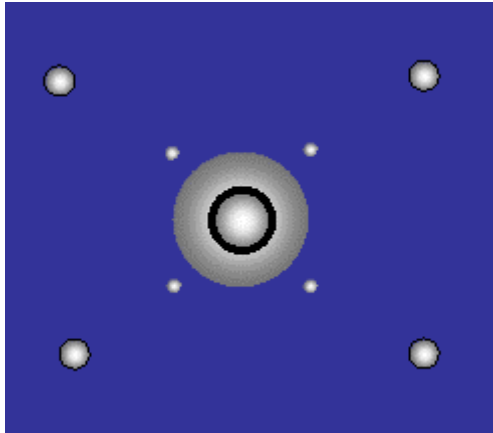
Laue diffraction pattern for a cubic crystal

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Example

An X-ray beam hits a cubic crystal along one of the cubes axis. One observes a diffraction spot at $90,0^o$ degrees with respect to the incoming beam. Earlier one has determined the wavelength of the X-ray radiation to be 2.5 Å (0.25 nm).
Calculate the grating constant of the cubic crystal (the distance between the atomic layers).

Solution

Since the angle of the incoming beam with respect to the reflected beam is $90,0^o$, the angle of grazing incidence must be 45° according to the figure below.
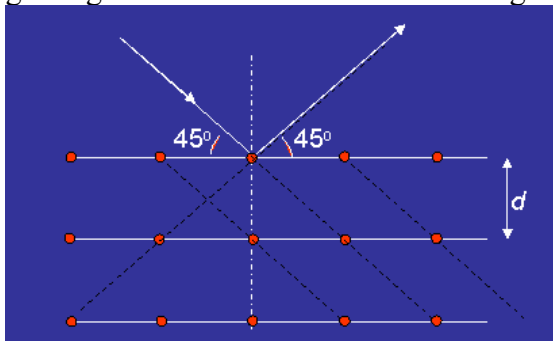

The distance between the atomic layers must be $\dfrac{d}{\sqrt{2}}$. With the Bragg formulae one gets

$$2\left(\frac{d}{\sqrt{2}}\right)\sin 45^o = 0,25nm$$

d = 0.25 nm

9.3 Ionic crystals

The Coulomb force is the dominating interaction between the charges of the ionic crystals in almost spherical symmetry. One normally takes NaCl as an example of an ionic crystal. It is a so-called "face centred cubic, fcc" crystal that can be seen in the figure at different angles.
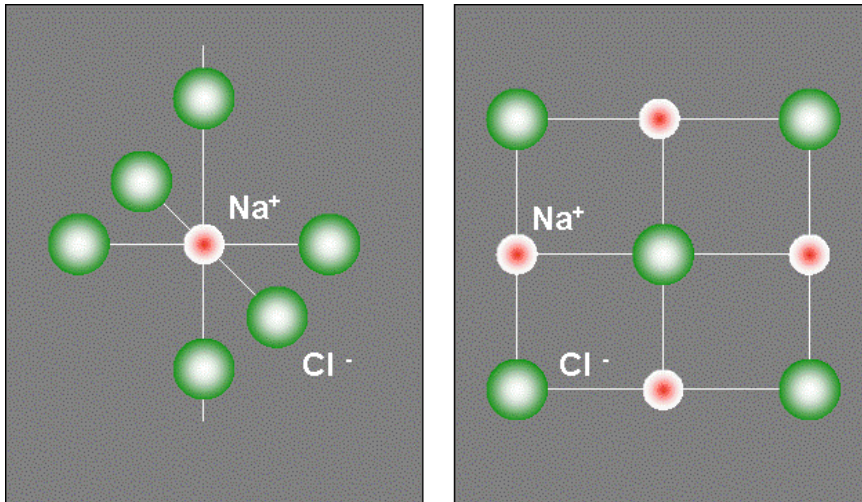
A NaCl crystal seen at 3-dim., and a cut through the atomic layers

We can also study the CsCl-crystal, a simple cubic crystal (simple cubic, sc). Here, the number of equidistant neighbours is 8 the so-called coordination number.
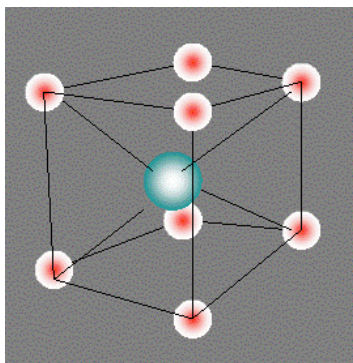
CsCl with a simple cubic form

Let us look at NaCl again. NaCl has the coordination number 6, i.e. there are 6 closest neighbour atoms. If we for a moment suppose that the interaction between the other neighbours can be neglected, the potential energy will become

$$V = -6 \frac{e^2}{4\pi\varepsilon_0 a}$$

Here, *a* is the distance to the closest neighbour atom. If we expand the study and also look at the 12 closest neighbour atoms that have the same sign, not looking at the 6 first, and the next 8 that have opposite sign, we get

$$V = -\sum_j \frac{e^2}{4\pi\varepsilon_0 r_j} = -\frac{e^2}{4\pi\varepsilon_0 a}\left(6 - \frac{12}{\sqrt{2}} + \frac{8}{\sqrt{3}} - ...\right) = \alpha \frac{e^2}{4\pi\varepsilon_0 a}$$

Here α is called the *Madelung*-constant. It thus depends on the build-up of the crystal and not on the type of ions.

Example
Suppose we have a fictive linear ionic crystal where our mission is to determine the constant α, the Madelung-constant for the crystal.
Solution
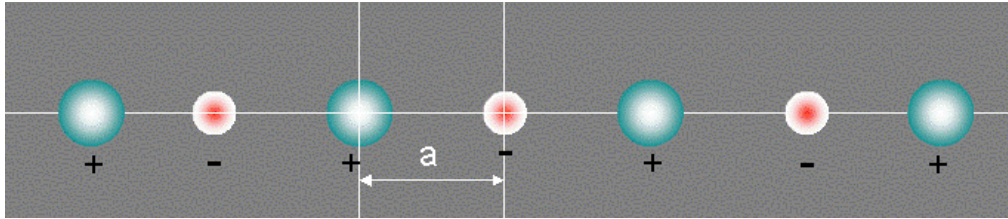
The potential energy for a charge is given by the following expression:

$$V = -\frac{q^2}{4\pi\varepsilon_0} \times 2x\left(\frac{1}{a} - \frac{1}{2a} + \frac{1}{3a} - \frac{1}{4a} + ...\right) = -\alpha\frac{q^2}{4\pi\varepsilon_0 a}$$

The factor 2 depends that we do the calculations in both the left and the right directions. Within the parenthesis, we have an alternating series, *1/1-1/2+1/3-1/4+...,* that has the sum *ln2*. The Madelung-constant gets the value *α = 2ln2 = 1,39*.

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9.4 Covalent crystals
The covalent forces are much weaker than the ionic crystal. Covalent crystals have low melt- and boiling temperature. H_2, CH_4, CO, CO_2, C_6H_6 are examples that can give rise to.
Let us take the hydrogen molecule as the simplest example to describe the covalent forces. The H_2-molecules two electrons move in a way that their charge distribution only is symmetric looking at a time average. The probability can be large for the electrons to be in a part of the molecule that makes it possible to attract another H_2 molecule etc. In such a way a weak bonding appears.
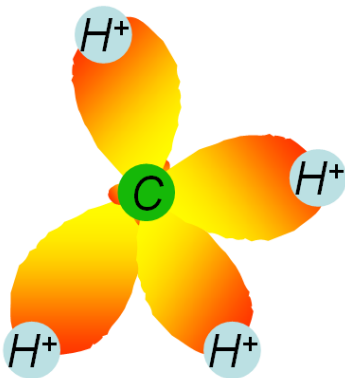

4 orbitals of the CH_4 molecule
In the figure we see that there are four electron orbitals that have their origin at the carbon atom, *C*, directed towards the corners of a tetrahedron. The valence bonds of CH_4 appears from each of those tetrahedral orbitals and one *1s* orbital from H. Also semiconductors as silicon and germanium are crystals with this tetrahedral structure.

9.5 Metallic crystals
One feature for metals is that they are good conductors for electric currents. In metals there are free electrons transporting the charge. The metals have in their outer shell weakly bound electrons. The electrons are moving just as molecules of a gas, an electron gas:
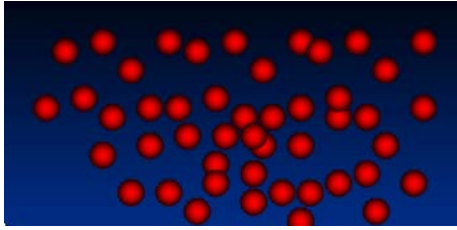
Electrons are moving in the metal just as a gas

9.6 Energy model

We can regard a crystal as a 3-dimensional box with the side *a*. The electron can move freely within the dimensions of the box. Earlier, we derived an expression for a "particle in a box" that only moved in the X-direction. Let us now generalise to three dimensions and we introduce three quantum numbers, n_1, n_2 and n_3, all integers n_i = 1,2,3,…with *i* = 1,2,3.
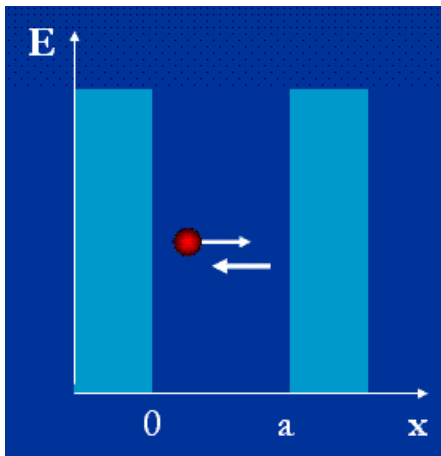
The discrete energy levels can be described by the quantum numbers:

$$E_n = \frac{h^2}{8ma^2} \cdot \left(n_1^2 + n_2^2 + n_3^2\right)$$

n_1, n_2, n_3, = 1,2,3,….

Here, the value of *a* is relatively large since the crystal >> atom.

9.7 The Fermi-distribution function

Let us consider a metal with an electron gas. We let the energy be *u*. The function *f(u)* describes how many electrons have the energy *u* of the metal. u_f is the so-called Fermi-energy. At the temperature T = 0, no electron has energies above the Fermi-energy. This is shown in the graph below:
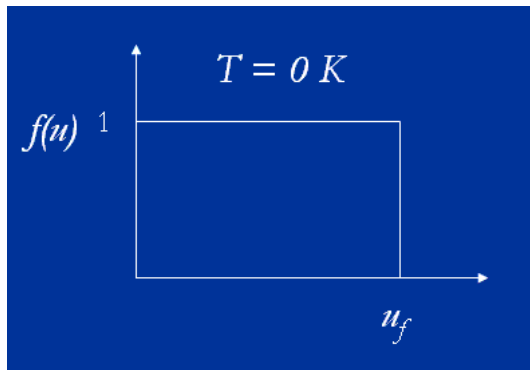
We let the temperature rise so T>0 K.
This means that $f(u) = 1/2$ when $u = u_f$.
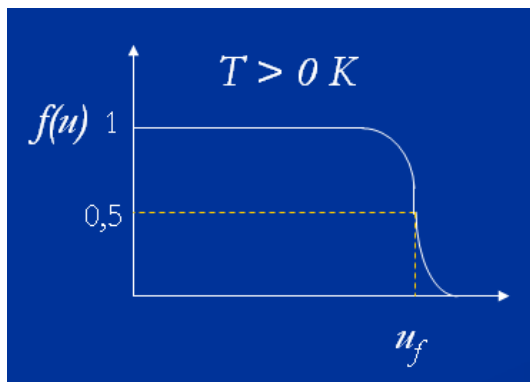
We observe that a small part of the electrons of the electron gas will have energies greater than the Fermi energy u_f. The Fermi function can be derived to the following rather simple expression:

$$u_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

Here N/V is the density of free electrons.

The Fermi distribution function contains components that can be found in the Plank's law of radiation. One can describe the distribution function $f(u)$ as the probability for a certain energy state with the energy u, *where u_f is the Fermi-enery*. In the function, there is a constant $k = 1.38 \ 10^{-23}$ *J/K* which is the *Boltzmann constant*, and the parameter T, which is the absolute temperature of the Kelvin scale.

$$f(u) = \frac{1}{e^{(u-u_F)/kT} + 1}$$

When T $\rightarrow$ 0 K then the Fermi function $f(u) \rightarrow 1$.

Example
At a certain energy *u* we have *f(u)* = 0.75. How many electrons in average do we have for 100 states in an energy interval *du* around *u*?
Solution
f(u) is the mean population number => The number of states will become *f(u)* x 100 = 0.75 x 100 = 75 electrons

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Example
Calculate *f(u)* for $u=u_F - 0,1$ eV and $u=u_F+0,1$ eV at *T*=0 and 300 K
Solution
Formula: $f(u) = \dfrac{1}{e^{(u-u_F)/kT} + 1}$

| T	u	f(u)	u	f(u)
0	u_F-0,1	1	u_F+0,1	0
300	u_F-0,1	0,979	u_F+0,1	0,021

The Fermi energy distribution function
Fermi also gave an expression for the energy distribution of an electron gas, where it has energies between *u* and *u+du* according:

$$n(u)du = \frac{8\sqrt{2}\pi V m^{3/2}/h^3}{e^{(u-u_F)/kT} + 1}\sqrt{u}\,du$$

We see that the Fermi-function $f(u) = \dfrac{1}{e^{(u-u_F)/kT} + 1}$ is implemented in the expression and below one can see a diagram that describes the distribution:
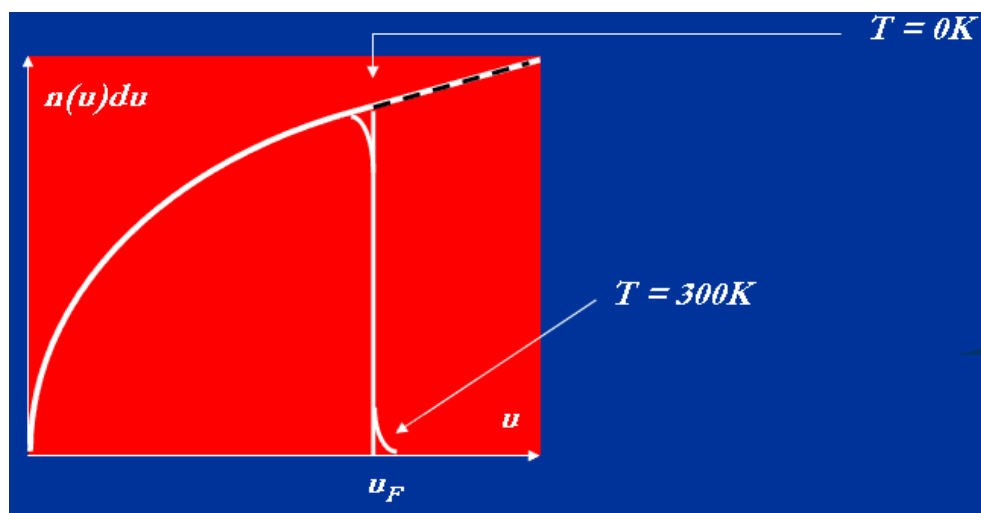
We see that at T = 300K, there are energies found above the Fermi energy u_f. The distribution at T = 0K is also seen in the diagram. The right part of the function is dominated by $\sqrt{u}$ and close to the Fermi energy of the Fermi function: $f(u) = \dfrac{1}{e^{(u-u_F)/kT} + 1}$

The Fermi energies of metals

| Metal	Fermi energy eV
Li	4.7
Na	3.1
K	2.1
Cs	1.5
Cu	7.0
Ag	5.5
Au	5.5

9.8 The Band theory

If you push free atoms together new electron levels will be created with somewhat different energies (due to the Pauli principle). The first excited *2s*-level split and the *2s*-orbitals will expand and the splitting will take place at larger nuclear distances for the *1s* orbital. Generally, a crystal with N atoms will get a splitting of every atomic energy level into N very tightly laying energy levels.

The difference between the highest and lowest levels varies with R (but not with N) and can reach around an electron volt. When a mole of a substance contains $6.022 \cdot 10^{23}$ atoms, one realizes that the levels are placed so tightly that they form a *band*.

Let us summarize:
- The valence electrons in a metal are close
- The Pauli-principle says that the outer electrons energies must differ somewhat
- An ENERGY BAND will form
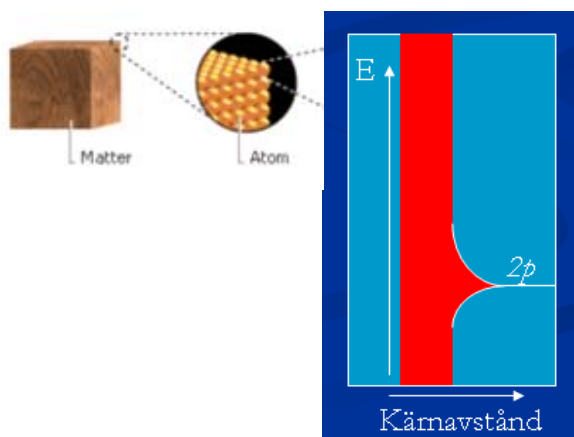
In the figure above you can see the energies of sodium and the chlorine atom. We see the *2p* state of the atom and the band of the crystal. The energies of the sodium crystal correspond to the energy levels of the atom and can just have the energies within the energy band. Some bands can overlap if they arise from different atomic states, since in that case the quantum numbers will be different. The electrons get a continuous energy distribution of allowed energies. In other crystals there are *forbidden bands*, which is energy areas not allowed for the electrons. The electric properties of a crystal are determined both of its energy band structure and how these bands are filled of electrons.
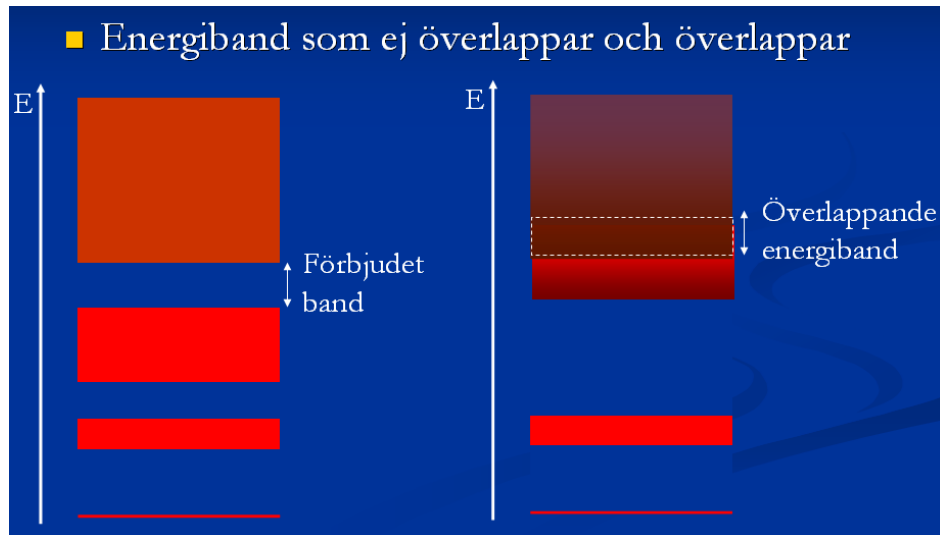
In the figure below one can see the energy crystalline and atomic sodium.
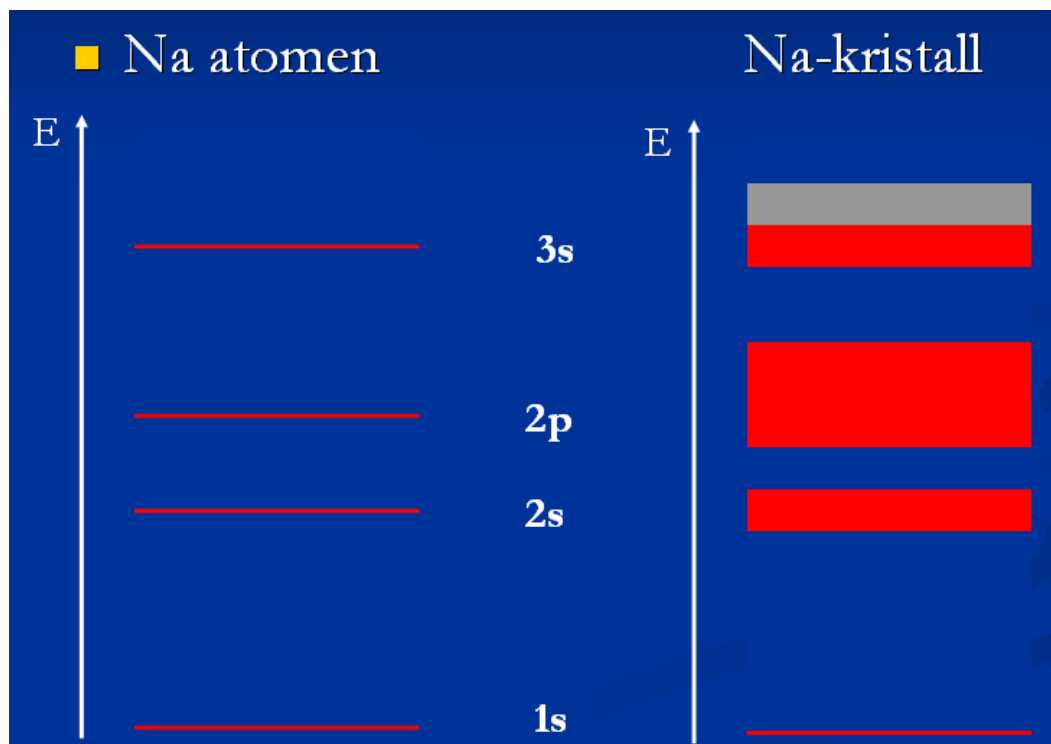
So, atomic sodium has one electron in its *3s* state. This means that the corresponding energy band of the crystal only can be half filled, since the atomic *3s* state can have two electrons. If one adds an external electric field over crystalline sodium, the electrons will gain extra energy in spite of being in their original energy band. This extra energy, of the form kinetic energy, will contribute to the electric current for electrons in motion. So, sodium is a good conductor for electric current, just as all metals with partially filled energy bands.

9.9 Metals, isolators and semi-conductors
Let us compare metals, isolators and semi-conductors with respect to the energy structure and draw conclusions about similarities and discrepancies by looking at the figure below:
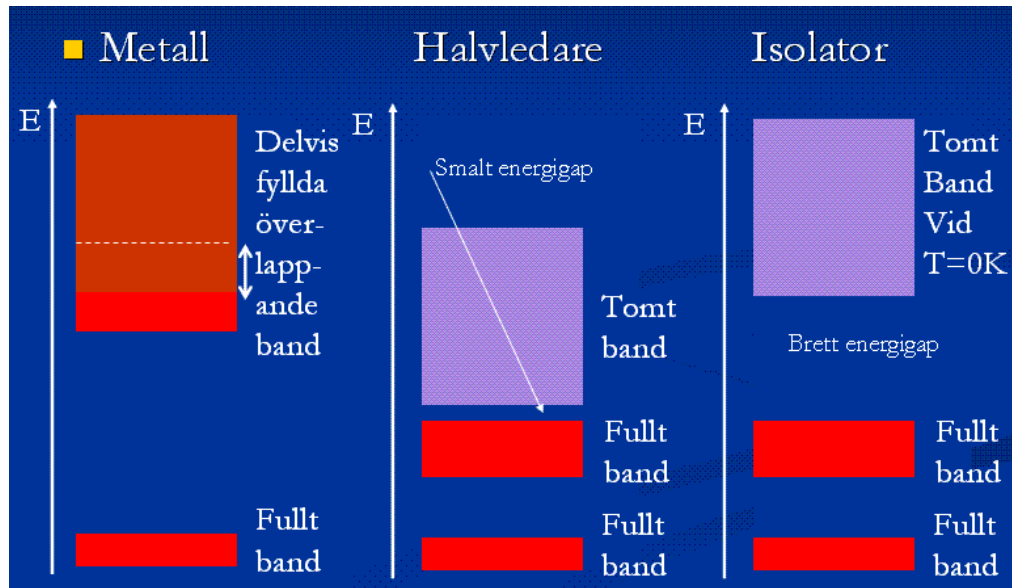
We see that metals have partly filled and overlapping bands, making the electrons move freely in the metal. This means that metals are good conductors with respect to electric conductivity and heat conductivity. Looking at semi-conductors there is only a small energy gap between the valence and conduction bands. Thus it is possible for the semi-conductors to conduct electricity by implanting atoms in the crystal, i.e. doping of the crystal. If we then look at the isolators, we see that the distance between the valence and conduction bands is large. We thus have difficulties to excite electrons from the valance band to the conduction band. That is why the isolator's properties of conduction differ from metals and semi-conductors.

We can examine how easy it is to move electrons from the valence band to the conduction band. Looking at an isolator the band gap is relatively large. For diamond the band gap 5.5 eV. Using the Boltzmann distribution function we can estimate how much thermal energy an electron can have at different temperature *T*.

$$N = N_0 e^{-(E-E_0)/kT}$$

Here $k = 8{,}62 \ 10^{-5}$ eV/K is the Boltzmann constant.

N/N_0 is the relative population of atoms at the energy E compared to the population N_0 with energy E_0.

 Example

What is the relative probability that an electron can jump from the valence band to the conduction band of diamond?

Solution

We put $(E-E_0)$ = band gap $E_g = 5.5$ eV for diamond and T = 293 K (room temperature) and

obtains $-\dfrac{E_g}{kT} = 5.5/(8.62 \times 10^{-5} \times 293) \approx -218$

The probability will become $P = N/N_0 = e^{-218} = 4.6 \times 10^{-94}$ which is an extremely small number. The probability is close to zero.
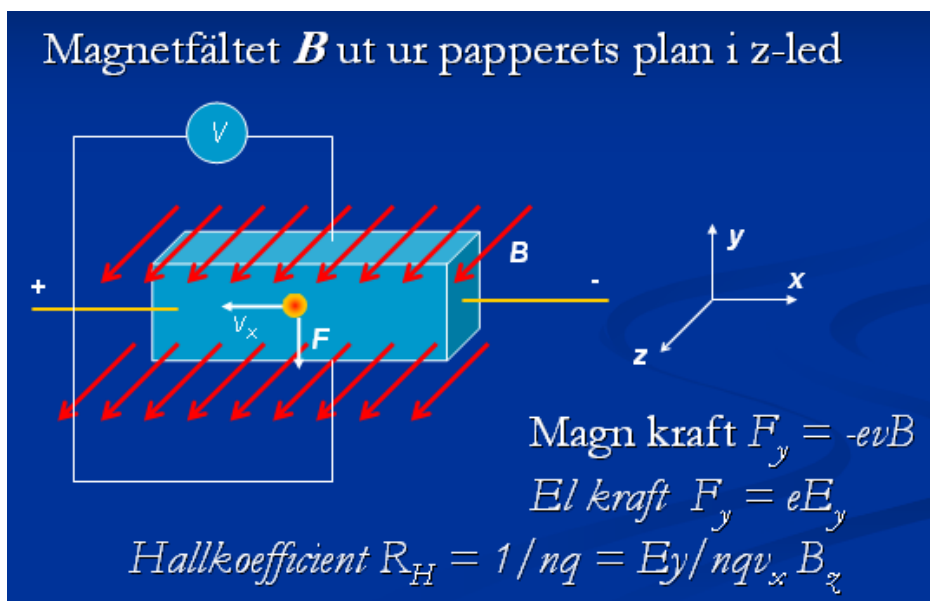
9.10 Intrinsic conductivity - doping

One observes a difference in conductivity where "pure" crystals of silicon and germanium with high purity are compared with doped materials. Often "pure" materials are doped with other atoms of the order of $1:10^7$. By choosing special dopants the crystal can get an excess of moving electrons or the lack of electrons, which is generating "holes". Ordinary semi-conductor materials are silicon and germanium. Germanium has 4 electrons in its outer shell, it has 4 valence electrons.
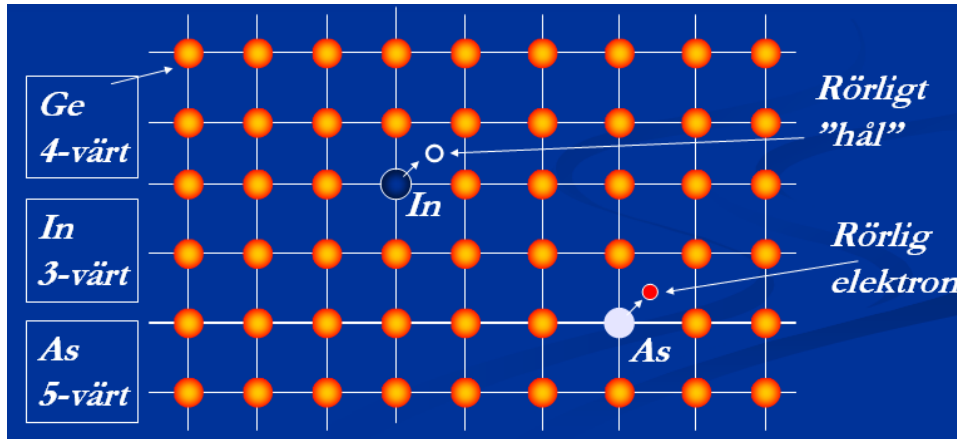
If one dope germanium with atomic As, with 5 valence electrons in small amounts, typically 1 As-atom on 10^7 Ge-atoms, then you produce a free electron moving in the crystal. If you instead implant 1 indium atom, with 3 valance electrons, there will be a lack of electrons, or we can say that a positive "hole" will be created.

What type of charge carrier a material has can be determined by using the Hall effect. One puts a homogeneous magnetic field perpendicular to a crystal (in our case with the direction out of the plane) and one studies what happens when one sends a current through the material. The magnetic field affects the charges making them move towards one side of the material and a lack of electrons on the other side. If electrons are collected on one side, holes are created on the opposite side, why an electric field between the different charges appear, E_y. The magnetic field B_z has its direction out of the plane.

Magnetfältet **B** ut ur papperets plan i z-led

Magn kraft $F_y = -evB$

El kraft $F_y = eE_y$

Hallkoefficient $R_H = 1/nq = E_y/nqv_x B_z$

At equilibrium between the magnetic and electric forces we get an expression for the so-called Hall coefficient R_H. The sign shows what type of charge carriers we have at hand.

Let us study the conductivity, σ, of semi-conductors. *n-doping* means an excess of electrons and p-*doping*, an excess of "holes". The conductivity can be generalised to:

$$\sigma = n \, e \, \mu_n + p \, e \, \mu_p$$

Here is *n, p* the number of charge carriers/m^3 and μ_n, μ_p is the mobility of the charge carriers.

9.11 Doped semi-conductors
By doing as in the former section we can dope a material with dopants in order to change the properties of the crystal. All modern materials are based on doping. In the figure above we see the different types, **n-doped** respectively **p-doped** materials.
The introduction of dopants in germanium and silicon crystals can be controlled in getting a material of the *n-type* and the opposite material *p-type*, with only a small area in between. The simplest way to produce such crystals is to pick out a growing crystal, for instance melted germanium containing a donor, and rapidly introduce an acceptor in the melt. The first part of the crystal created is of *n-type* and the rest of *p-type*.

One important property of a **p-n** transition in a crystal is that it conducts the electric current much better in one direction than in the other. The material works as a diode. One can show that the current of a circuit with a diode, where the applied voltage is *V,* can be described by:

$$I = I_0 \left(e^{eV/kT} - 1 \right)$$

This equation describes the diode's characteristics. For *V* > 0 the diode is connected in the forward direction and the current grows exponentially with *V*. An applied alternating current will give a pulsating direct current. When V < 0, the diode is connected in the backward direction an only a weak current will be able to pass.
The characteristics of the diode is shown in the figure below:
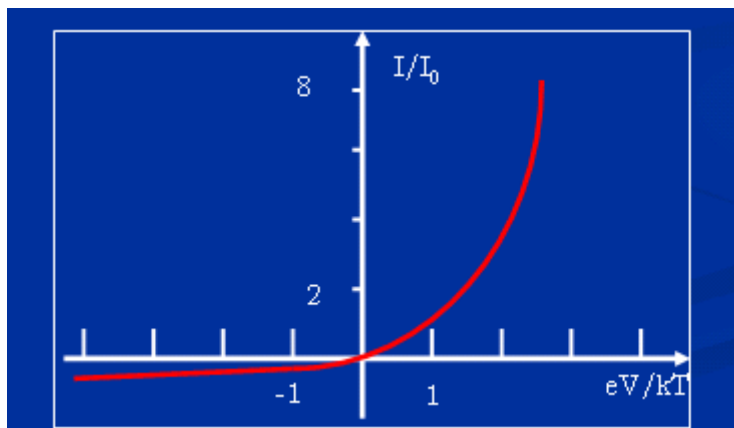
Since *kT* = 0.025 eV at room temperature one will get a rather high increase in current if one changes the voltage by just a few tens of a volt.

Example
We connect two identical diodes in a series to a voltage of 1.000 V. One is connected in the forward direction and the other in the backward direction.

Determine the current in the circuit and the voltage across each diode; diode 1 and diode 2.

It is the same current that flows through the two diodes: $I_1 = I_2$
We get the following equation for the current (the diodes characteristics):
$$I_0(e^{eV/kT} - 1) = -I_0(e^{-e(1,000-V/kT)} - 1)$$
We see tat the exponential of diode 2 is <<1, which gives $e^{eV/kT} - 1 \approx 1$
We take the logrithm and gets
$V_2 = kT\,ln2 = 8.862 \times 10^{-5} \times 300 \times ln2 = 0.0179$ volt
The voltage over diode 1 is then $V_1 = V - 0.0179$ volt $= 1.000 - 0.0179$ volt $= 0.982$ volt.
The current through both diodes is $I_1 = I_2 = I_0$.

Since the thin layer, d_F, between the *p*-and *n*-layers normally contains few charge carriers, this region must have a high resistance. When you put on a voltage making the current go in the *p-n* direction, the layer is decreased in size considerably but also the resistance. The diode will make it easier for the current to pass in this direction. That is why I_F gets a high value. The so-called drift current I_{drift} has the same size all the time. If you then connect the diode in the opposite direction, namely in the *n-p* direction, the middle zone will expand considerably and also its resistance. The width d_F will become bigger, which means that the diode has difficulties in conducting the current in the backward direction. This can easily be seen in the figure below:
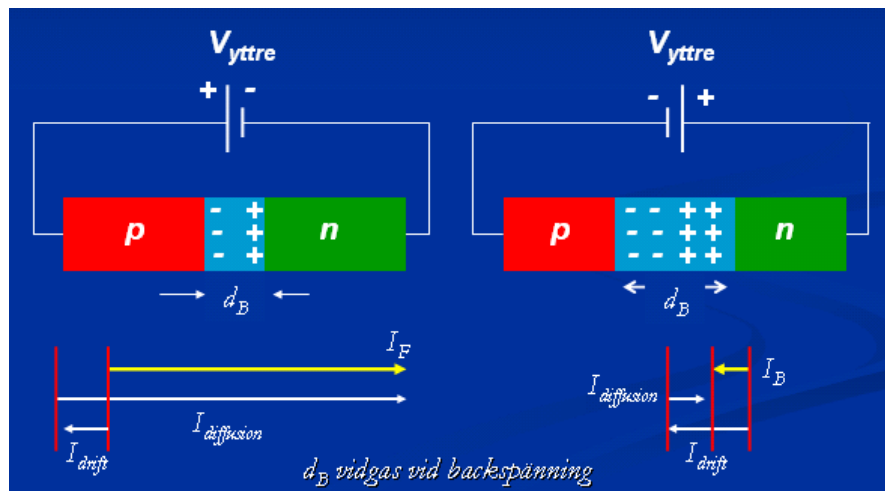
If we connect the diode to a circuit an puts on an alternating $V_{in} = V_0 \sin 2\pi ft$, where the voltage source has the frequency *f* or the period $T = 1/f$. We study the situation graphically and study the outgoing voltage V_{ut}. We will obtain a pulsation direct current.
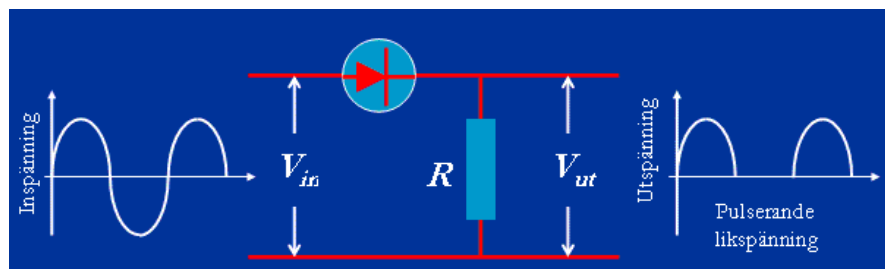
9.12 The LED-diode (Light Emitting Diode)
How can a *p-n* transition generate light? When an electron in the conduction band recombines with a hole in the valence band, energy corresponding to the band gap E_g is

transmitted. In many semi-conductors this energy is transferred to create thermal energy in the vibrating crystal. In other semi-conductors, containing gallium arsenide (GaAs), a photon can be emitted with the wavelength

$$\lambda = c/f = c/(E_g/h) = hc/E_g$$

The material has, in order to be able to emit much light, in order to function as a LED, have enough number of electron-hole recombinations. However, in intrinsic, pure, semi-conductors, the number of electron-hole pairs are few. A doped semi-conductor does not work either. Surely the number of electrons raise, but not the number of holes. What is needed is a semi-conductor with many electrons in the conduction band and simultaneously many holes in the valence band. It is possible to make a heavily doped crystal with a voltage in the forward direction of a *p-n* transition, and at the same time have a large current passing the diode that is injecting electrons on the *n*-side and holes on the *p*-side. If the doping is large enough the middle region will be very small, perhaps a few micrometers wide. This will result in a large density of electrons and a great density of holes, only separated by the thin region. As a result we will have a lot of recombinations with the corresponding emittance of light from that small zone. Commercial LED's are designed for visible light and are often based on gallium doped with arsenide and phosphor.

9.13 The photo-diode
By letting a current pass through a *p-n* transition one can generate light. The opposite is also true. If we let light fall on a suitable *p-n* transition one can obtain a current in the circuit where the transition is a part. This is the basis of a photo-diode.

9.14 The junction laser
In a junction laser there are many electrons in the conduction band of the *n*-side and many holes of the valence band of the *p*-side. This population inversion is a requirement to obtain laser action. When an electron jumps from the conduction to the valence band it can stimulate (trig) another electron also to make a jump, and if the current is high enough it is possible to chain reaction if the ends of the crystal are flat and parallel. In this way a *p-n* transition works as a junction laser, often used in CD-players. The wavelength is most often found in the IR-region ($\lambda = 1{,}31$ and $1{,}51$ μm).

9.15 The transistor and the integrated circuit
A transistor is a semi-conductor crystal consisting of two *p-n* transitions and that can be built as an *n-p-n* or a *p-n-p* crystal. If we first study an *n-p-n* type where electrons are the charge carriers, while the *p-n-p* transistor work with a *"hole"* current. The ends of the transistor, which are called emitter and collector, are heavily doped, while the middle region is weakly doped. This region is also very thin. In the figure below, it is shown how a transistor can be connected. The *emitter-base-circuit* is put in the forward direction. One connects the voltages positive pole to the *p*-doped material.
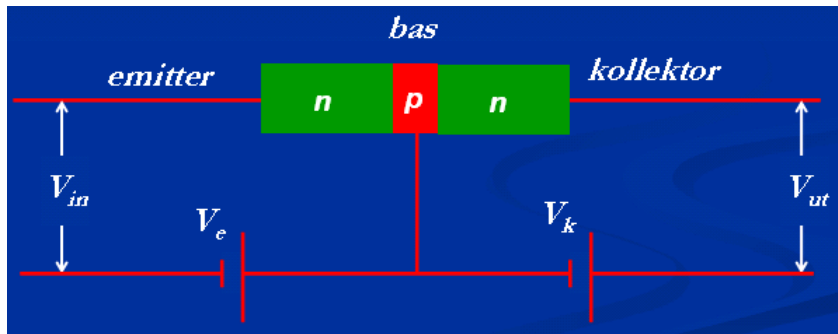
From the emitter's conduction band, many electrons reach to the base, which potential and energy are lowered by eV_e. Since the base is thin, most electrons reach the *p-n* transition between base and collector, connected in the back direction. The electrons coming from the *p*-side can in this way continue to the collector.

How does a transistor work? If you do a small change of the signal on the *n*-side will get a large change of the current from the emitter due to the steep rise of the diode curve (see the diode). The large change in the current will give rise to a large change in voltage over the resistor (large resistance) on the exit of the transistor. The transistor works as a voltage amplifier.

9.16 Solar cells
A *solar cell* is a thin layer of doped semi-conductor material that converts light to electricity. A typical material is silicon that has an efficiency of 15 – 20%, which means that an area of a square decimetre will give 1.5 W. With a voltage of 0.5 V, this means 3 A. The solar cells are then connected parallel and in series to obtain the right voltage and high enough current.


Solar panels of the ISS space programme

9.17 Amplifiers for rock and roll guitars
In the 60:ies the amplifiers were made by electronic tube devices and gave, due to the high level of amplification, a heavily distorted sound. Later, when transistors replaced the electronic tubes, the sound became more "pure", which did not make all artists happy (Jimi Hendrix).

9.18 Integrated circuits and other electronic devices, consist of thousands or even millions of transistors. There are also passive components as capacitors, solenoids and resistors. They are manufactured in one unit of semi-conductor material, one semi-conductor chip, which constitutes the integrated circuit.
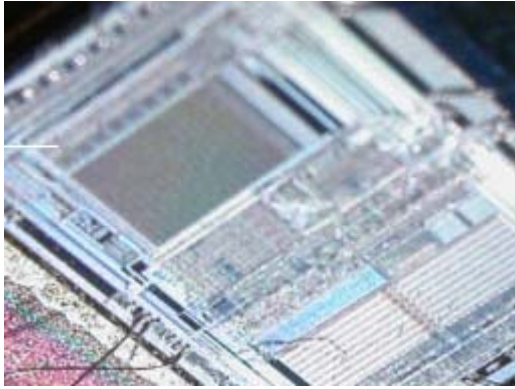
Example of an integrated circuit with thousands
of transistors.

9.19 Super conductivity
Kammerling Onnes discovered that the resistance of mercury almost vanishes when the
temperature was close to 4K. That is why the term super conductivity arouses and that has an
enormous technological potential. This means that the current in a circuit can flow without
thermal losses. One has obtained electric currents in super conducting materials that has been
flowing for years without diminishing in strength. In the figure below one can see how the
resistance changes with temperature.
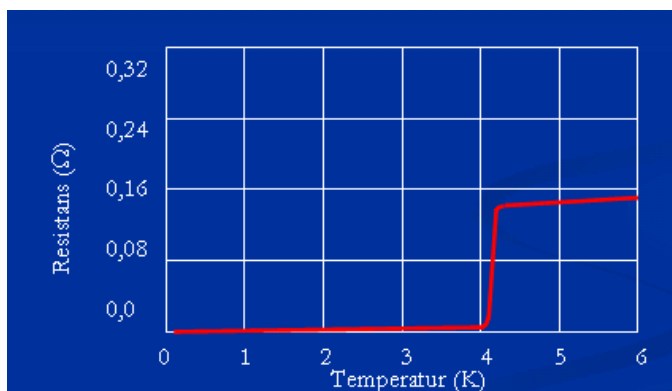
One can explain super conductivity with the fact that the electrons constituting the current
consist of coordinated pairs. One of the electrons of the pair can disturb the molecular
structure of the super conducting material when they move in the conductor. The other
electron of the pair is attracted by the positive charge of the material. One such is that the
coordination of the pairs hinders them to collide with the molecules of the material. In this
way the resistance of the material is eliminated.

10. Nanophysics

Nanophysics and nanotechnology are some of the most exciting research fields today.
The word *nano* comes from the Greek word for dwarf or small. When nano is used as a prefix
it means one part in a billion ($1:10^9$). Objects of sizes between 1 and 100 nanometres, in short
nm, are sizes within nanotechnology. Atoms are smaller than one nm, molecules and cells
vary in size from 1 to several nm. One nanometre (nm) is so small that it can not be seen in an
ordinary light microscope. Nanoscience and nanotechnology deals about study and create

material structures of sizes 1- 100 nm (as a comparison, the hydrogen atom with the electron in the inner shell has a diameter of 0.1 nm).
Researchers work with materials on atomic levels and create materials and components with new properties – to be used in micro electronics and bio medicine.
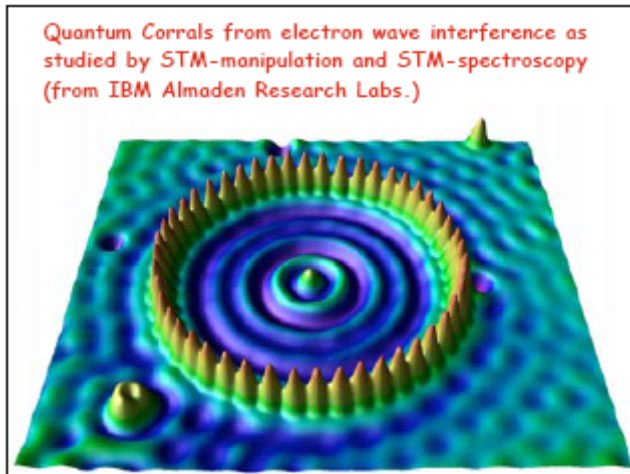
At KTH there is a modern nano laboratory for creating small structures in cooperation with many institutes like the KTH Kista electronic departments among others.

Nano technology is a term used for many scientific areas and cross-disciplinary cooperation and is often a requirement to solve nano-related scientific problems.
Nano technology is a natural development within several technological areas. Within electronics one has made the transistors and IC circuits smaller and smaller in order to make them faster and to reduce the size of the computers as well as more reliable. The instruments and apparatus one has constructed within microelectronics have made it possible to study very small parts of Nature like Gecko-feet, and other materials of nano size. Molecules and cells has always been nano technology, but now there exists possibilities to connect biology and electronics.
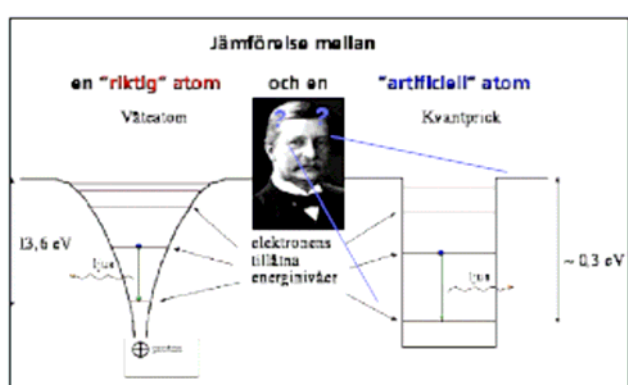
One can ask oneself if nano particles are dangerous, which depends of if things are of nano size, or if they belong to a nano structured surface. If there is an emission of nano particles, it can be dangerous, since it can hurt the lungs if they enter them. But nano particles are already

around us, for example in the exhaust from traffic, toning powder from copying machines or laser printers. A nano structured glass surface to keep it clean does no emit particles.

Devices for studying small objects:

Neutron scattering
The applications of neutron scattering spans over a wide field of great importance; material science, energy research, nano-and biotechnology as well as medicine. Today there are some 100 researchers in Sweden using neutron scattering within their research. They are represented in most fields of Natural science; chemistry, physics, material science, biochemistry, biophysics, medicine and geology.
In Sweden there is a large neutron scattering laboratory at Studsvik but there are several all over Europe and further away.

Electron microscopy
The electron microscope is a useful instrument for studies of small structures and gives information about magnetic and electric information of a sample. With different methods as electron diffraction and spectroscopy the properties of materials can be determined down to 1Å. A focused electron-or ion beam can be used to analyse or to structure materials on nano–meter level.
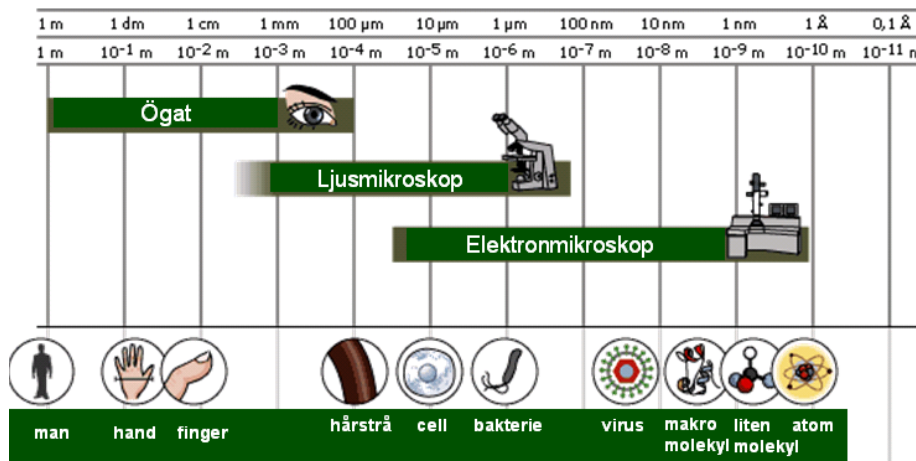
The resolving power of a microscope can be estimated with the Rayleigh criterion:

$$\theta_c = 1{,}2\frac{\lambda}{D}$$

Here θ_c is the critical angle for resolving an object. The smaller the angle, the smaller objects can be resolved. D is the diameter of the lens or the entrance of the system and λ is the wavelength of the radiation. An eye with an circular opening of 6 mm can resolve objects down to around 10^{-4} m.
Example
We look at an object from a distance of 25 (distance for clear seeing). Can one see an object of the size $b = 0,1$ mm?
Solution

We put the diameter of the iris to $D = 3$ mm and with the wavelength $\lambda = 550$ nm. We use the Rayleigh criterion and gets $\theta =$ object/radius $= b/R = b/(0.25$ m$)$

$$\theta_c = 1.22\frac{\lambda}{D} \quad \Rightarrow \quad b/0.25 = \frac{1.22\lambda}{D} \quad \Rightarrow$$

$$b = \frac{0.25 \times 1.22 \times \lambda}{D} = \frac{0.25 \times 1.22 \times 550 \times 10^{-9}}{3 \times 10^{-3}}m \approx 0.06 mm \approx 1 \times 10^{-4} m$$

b is around 0.1 mm, which fits with the figure above.

Electron microscopy contributes to the search for unique materials both within applied and basic science by analysis on atomic level. With this knowledge a materials meso-and macroscopic properties and the connection between its structure and its mechanical properties can be characterized.

An electron microscope wit a resolving strength of 100 000 times

Electron microscopy is suitable for studies of structural changes and properties of polymers, metals, ceramics, and biological materials. Nano-crystalline materials with sizes between 1 nm 10 nm show quite new magnetic, electric and optical properties. High resolving electron microscopy is a powerful technique to study the orientations of crystals, phases, and chemical composition of a electron transparent sample.

The shorter the wavelength λ the smaller objects can be observed.

Example
We can try to estimate the smallest object that can be observed in an ordinary electron microscope. Suppose that the acceleration voltage is 120 V. The deBroglie-wavelength can be determined.

We calculated the velocity in section 3.3 with deBroglie's postulate $\lambda = h/mv$ when electrons were accelerated across 120 V. We got the result $\lambda = 1.12$ Å, which is the limit for resolving a small object, which fits with the figure above describing enlargements.

The limit is around 10^{-10} m=1 Å.

The electron beam interacts with a magnetic sample and is used to study and visualize patterns of domains as well as magnetization of one-domain particles. In order to accomplish this, one can apply electron holography and perform magnetic observations with so-called Lorentz microscopy.

Spintronic applies the electrons spin to observe information. The spin is possible to measure due to its magnetic properties if one can align the spin one can use the magnetic North and South poles to represent 1 and 0. This means that one can build an electronic circuit much smaller than what is possible using ordinary electronics.

Qunatum bit, or **qubit**, from *qu*antum *bi*nary digi*t*, represents the smallest unit of quantum information. There is a fundamental difference between a qubit and a classical bit. The classical bit can have the value 0 or 1. A qubit corresponds to a quantum state and can have the value 0 or 1 but also a linear superposition of these states, a so-called quantum superposition.

Contentsl Chapter 9-10

Learning goals
Be able to describe conductivity and other electrical properites
Be able to describe the temperature dependence of conductors and semi-conductors

Calculate the conductivity of meatls and semi-conductors
Be able to describe about mobility
Be able to describe X-ray diffraction in crystal studies
Calculate the grating constant with the Bragg formula
Be able to describe ionic crystals
Calculate the Madelung constant in simple grating models
Discuss energy models of crystals
Calculate energy states at different quantum numbers
Be able to generally describe the Fermi energy derivation
Calcualte the density of free electrons in crystals with the Fermi expressions
Be able to describe the energy distribution in crystals
Perform calculations with the Fermi function
Be able to describe the band theory
Discuss the difference between conductors, semi-conductors and isolators
Be able to describe conductivity and holes
Be able to describe the Hall effect and do calculations on this effect
Describe doped semi-conductors and other solid-state instruments
Be able to describe super conductivity
Be able to generally describe nano physics and nano technology
Describe methods for studies of small objects

Advices for reading
Think of the fact that one uses general electric studies to characterize materials within solid-state physics
Study electric conductivity, Ohm's law and mobility
X-ray methods can be used when studying crystals since the wavelength is of the same size as the objects
With simple models one can describe crystal structures
Energy models are used here as well as in atomic physics
Fermi distribution made a breakthrough in the description of crystal structures
Think of that band theory makes a bridge between atomic physics and crystal physics
Think of that the doping concept is important in semi-conductor physics
Solid state-physics has lead to a development of technology and applications
Nano physics is an area leading to many new applications about electronics, new material and fast computers

Readings
- Thornton, Rex, Modern Physics, Saunders
- Krane, Modern Physics, Wiley
- Beiser, Concepts of Modern Physics, McGraw-Hill
- Serway, Moses, Moger, Modern Physics, Saunders
- Eisberg, Resnick, Quantum Physics of Atoms, Molecules, Solids and Particles, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Halliday and Resnick, Fundamentals of Physics, Wiley

- Blatt, Modern Physics, McGraw-Hill
- Benson, University physics, Wiley

WEB-addresses
- http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html
- http://nobelprize.org/nobel_prizes/physics/articles/lecuyer/index.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1956/index.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1985/index.html