

Modern physics Chapter 9-10. Solutions to exercises.

9.1.1 The resistivity is given by $\rho = \rho_0(1 + \alpha\Delta T)$. For copper $\rho_0 = 1.7 \times 10^{-8} \Omega\text{m}$ and the constant $\alpha = 4.0 \times 10^{-3} \text{ T}^{-1}$. The temperature $T_0 = 273 \text{ K}$. Equation:

$$\rho_{373} = 1.7 \times 10^{-8} (1 + 4.0 \times 10^{-3} (373 - 273)) = 2.38 \times 10^{-8} \Omega\text{m} \approx 2.4 \times 10^{-8} \Omega\text{m}$$

9.1.2 The relative resistivity ρ/ρ_0 where $\alpha = -7.0 \times 10^{-2} \text{ T}^{-1}$. $\Delta T = (25 - 20) \text{ K}$

Equation $\frac{\rho}{\rho_0} = 1 - 7.0 \times 10^{-2} (25 - 20) = 1 - 0.35$, i.e. a reduction by 35%.

9.2.1 Bragg's formula $2d \sin \theta = m\lambda$ where $\theta = 30^\circ$ is the angle for grazing incidence. First order with $m = 1$ gives

$$d = \frac{m\lambda}{2 \sin \theta} = \frac{1 \times 1.2}{2 \sin 30^\circ} \text{ \AA} = 1.2 \text{ \AA}$$

9.3.1 The potential energy is given by $V = -6 \frac{e^2}{4\pi\epsilon_0 a}$ where $a = 0.281 \text{ nm}$. Equation:

$$V = -6 \frac{e^2}{4\pi\epsilon_0 a} = -9.00 \times 10^9 \frac{6 \times 1.602 \times 10^{-19}}{0.281 \times 10^{-9}} \text{ eV} = 30.74 \text{ eV} \approx 31 \text{ eV}$$

9.3.2 A linear crystal model gives $V = \alpha \times \frac{e^2}{4\pi\epsilon_0 a}$ where $V = 7.2 \text{ eV}$ and $a = 0.281 \text{ nm}$.

We determine α : $7.2 \text{ eV} = \alpha \times \frac{9.00 \times 10^9 \times 1.6 \times 10^{-19} \text{ e}}{0.281 \times 10^{-9}}$ giving $\alpha = 1.4$ (no unit)

9.6.1 The energies can be written using the quantum numbers: $E_{n_1 n_2 n_3} = \frac{h^2}{8ma^2} (n_1^2 + n_2^2 + n_3^2)$

With $n_1 = n_2 = n_3 = 1$ and $a = 1.0 \text{ nm}$ we have:

$$E_{111} = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (1.0 \times 10^{-3})^2} \times \frac{1}{1.6 \times 10^{-19}} \text{ eV} = 1.3 \times 10^{-13} \text{ eV}$$

9.7.1 The Fermi energy $u_F = \frac{h^2}{2m} \left(\frac{3}{8\pi} \times \frac{N}{V} \right)^{2/3}$ where N/V is searched for. We get:

$$(u_F)^{3/2} = \left(\frac{h^2}{2m} \right)^{3/2} \frac{3}{8\pi} \times \frac{N}{V} \quad \text{Ekvation:} \quad \frac{N}{V} = \frac{8\pi}{3} \frac{(u_F \times 2m)^{3/2}}{h^3}$$

$$\frac{N}{V} = \frac{8\pi (1.5 \times 1.6 \times 10^{-19} \times 2 \times 9.1 \times 10^{-31})^{3/2}}{3 \times (6.63 \times 10^{-34})^3} \text{ electrons/m}^3$$

$$\approx 3.3 \times 10^{26} \text{ electrons/m}^3$$

9.7.2 The Fermi function $f(u) = \frac{1}{e^{(u-u_F)/kT}}$ At $T = 500 \text{ K}$ and $u = u_F - 0.1 \text{ eV}$, we get

$$f(u) = \frac{1}{e^{(u-u_F)/kT}} = \frac{1}{e^{-0.1 \times 1.6 \times 10^{-19} / 1.38 \times 10^{-23} \times 500}} = 0.09838 \approx 0.0984$$

9.9.1 The probability is $P = N/N_0$ where $N = N_0 e^{E_g/kT}$ and $E_g = 1.5 \text{ eV}$.
 $- E_g/kT = - 1.5 \times 1.6 \times 10^{-19} / (1.38 \times 10^{-23} \times 293) = - 59.356$

The probability is $P = e^{-59.356} \approx 1.66 \times 10^{-26}$ vanishingly small.

9.10.1 At equilibrium the magnetic and electric forces are equal $F_E = F_B$,
i.e. $eE = evB$, why $B = E/v = 10 \times 10^{-3} / 0.42 \text{ T} = 24 \text{ mT}$