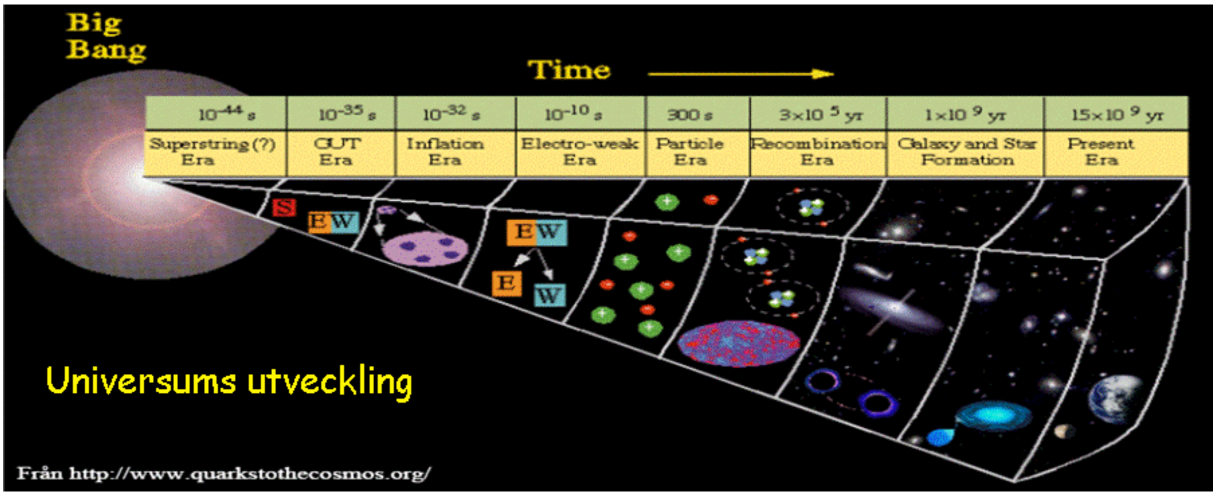
# Modern Physics 9p ECTS



Från http://www.quarkstothecosmos.org/

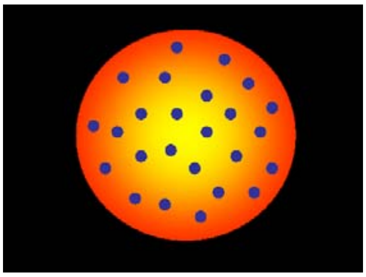
## *Contents*

1. Introduction
2. The special relativity
3. The original quantum theory
4. The photon
5. Statistical physics
6. The Schrödinger equation
7. Atoms
8. Molecules
9. Solid state physics
10. Nanophysics
**11. Nuclear physics**
**12. Nuclear energy**
13. Particle physics
14. Astrophysics

# 11. Nuclear physics

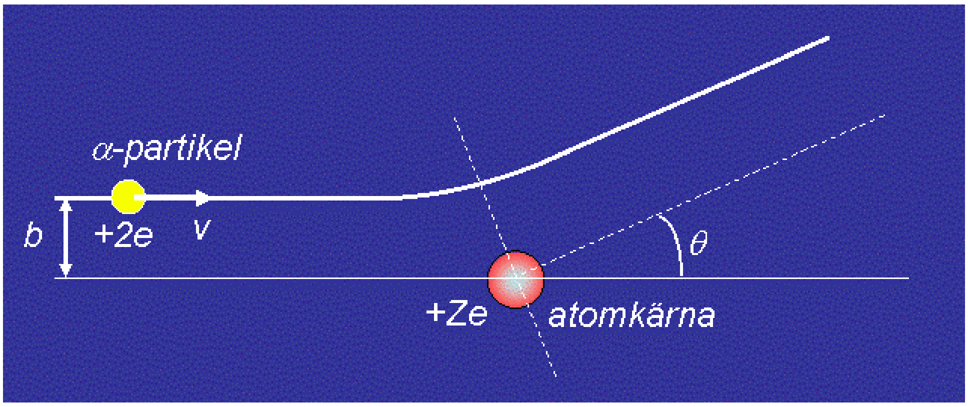
### 11.1 Introduction

Thomson discovered the electron in 1897 (but its mass was still unknown) and presented a model for how the atom was built



Thomsons ”plumpudding”-modell av atomen

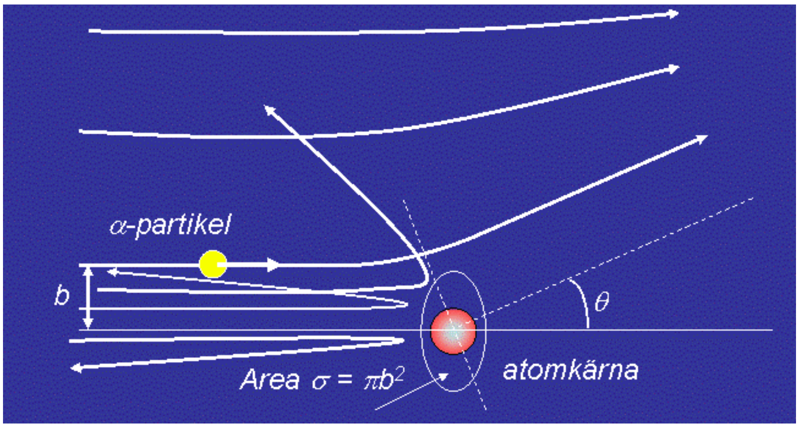
Rutherford (Geiger, Marsden) investigated his ”plum-pudding-model” and found that the positive charge of the atom (and also its mass) is concentrated to a very small region, a nucleus, with a width of around 1/10000 of the atom.



Since one expected very small scattering angles (”A bullet through a sack of snowballs”), the result of the experiment was extremely astonishing. With Rutherford’s own words:

*”It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”*

Rutherford, Geiger and Marsden investigated thin gold foils that were irradiated by α-particles and examined the angle dependence.



They studied different values on the parameter $\theta$ and derived the so called Rutherford’s scattering formula:

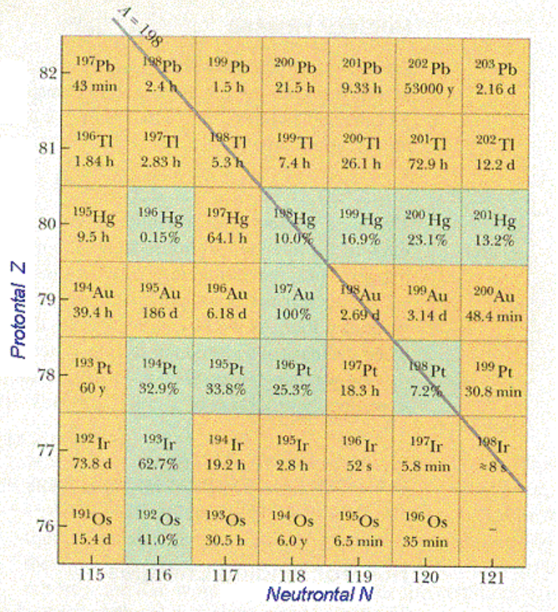
$$N(\theta) = \frac{N_{tot} n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2 \sin^4(\theta/2)}$$

Here $N_{tot}$ is the number of α-particles per area unit that reached the folium, having the thickness $t$. The distance from the α-source to the folium is $r$. $K$ is the α-particle.

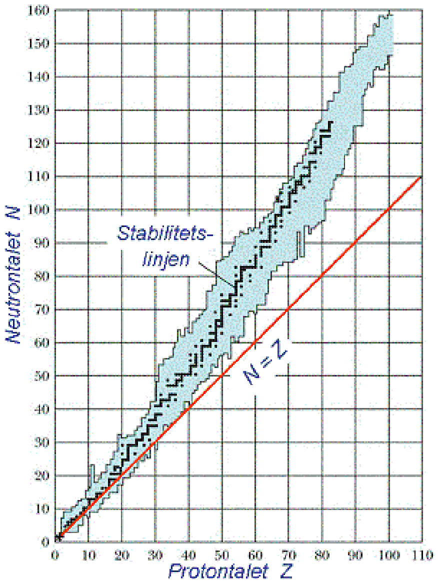
Rutherford suggested the following (1920) reaction p+e$^-$ to produce a neutral particle in the nucleus, needed since the mass of the number of protons is less than the total mass/2. But this solution is not good enough (according to the Heisenberg uncertainty relation). The *Neutron* was thus postulated (1928) and found in 1932 by Chadwick

The nucleus consists of protons and neutrons (called *nucleons*) and the total *mass number* is *A*.

The number of protons in the nucleus is given by the *atomic Z*
The number of neutrons in the nucleus is given by *N*
$A = Z+N$



| Protontal Z \ Neutrontal N | 115 | 116 | 117 | 118 | 119 | 120 | 121 |
|---|---|---|---|---|---|---|---|
| 82 | $^{197}$Pb<br>43 min | $^{198}$Pb<br>2.4 h | $^{199}$Pb<br>1.5 h | $^{200}$Pb<br>21.5 h | $^{201}$Pb<br>9.33 h | $^{202}$Pb<br>53000 y | $^{203}$Pb<br>2.16 d |
| 81 | $^{196}$Tl<br>1.84 h | $^{197}$Tl<br>2.83 h | $^{198}$Tl<br>5.3 h | $^{199}$Tl<br>7.4 h | $^{200}$Tl<br>26.1 h | $^{201}$Tl<br>72.9 h | $^{202}$Tl<br>12.2 d |
| 80 | $^{195}$Hg<br>9.5 h | $^{196}$Hg<br>0.15% | $^{197}$Hg<br>64.1 h | $^{198}$Hg<br>10.0% | $^{199}$Hg<br>16.9% | $^{200}$Hg<br>23.1% | $^{201}$Hg<br>13.2% |
| 79 | $^{194}$Au<br>39.4 h | $^{195}$Au<br>186 d | $^{196}$Au<br>6.18 d | $^{197}$Au<br>100% | $^{198}$Au<br>2.69 d | $^{199}$Au<br>3.14 d | $^{200}$Au<br>48.4 min |
| 78 | $^{193}$Pt<br>60 y | $^{194}$Pt<br>32.9% | $^{195}$Pt<br>33.8% | $^{196}$Pt<br>25.3% | $^{197}$Pt<br>18.3 h | $^{198}$Pt<br>7.2% | $^{199}$Pt<br>30.8 min |
| 77 | $^{192}$Ir<br>73.8 d | $^{193}$Ir<br>62.7% | $^{194}$Ir<br>19.2 h | $^{195}$Ir<br>2.8 h | $^{196}$Ir<br>52 s | $^{197}$Ir<br>5.8 min | $^{198}$Ir<br>≈8 s |
| 76 | $^{191}$Os<br>15.4 d | $^{192}$Os<br>41.0% | $^{193}$Os<br>30.5 h | $^{194}$Os<br>6.0 y | $^{195}$Os<br>6.5 min | $^{196}$Os<br>35 min | - |



Chemical identical substances, but with different masses are called *isotopes.* They have different *atomic mass* (different A) but the same *atomic number* (same number of protons Z). Nuclides with the same number of neutrons (same N), but different Z are called *isotones*, while nuclei with the same A are called *isobars*.
Of the around 2000 known nuclei only 280 are stabile, the rest are *radioactive*, that is they decay spontaneously. In Nature, there are a few natural radioactive elements, the rest have been produced. The first are called *natural radioactive,* while the rest are called *induced radioactive*.

### 11.2 Natural radioactivity

Radiactive radiation was detected by Becquerel (1896) and can have three components,

- $\alpha$-particles (that are helium nuclei)
- $\beta$- particles (that are electrons or positrons)
- $\gamma$- particles (that are photons)

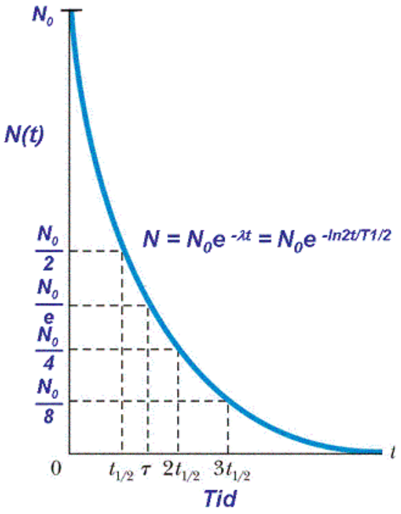
In rare cases, also neutrons and big nuclear fragments can be present.
The number of decays from a radioactive source decreases exponentially. There is a probability that a nucleus shall decay during the time *dt* (dt is supposed to be << 1). If there are *N* radioactive nuclei at time *t*, the number of nuclei will decay (the reason for the minus sign)

$$-dN/dt = N$$

with the solution

$$N = N_0\, e^{-\lambda t}$$

$N_0$ is the number of nuclei at time t=0 and $\lambda$ is the *decay constant* that varies for different nuclei. It depends on the interaction between the nucleus and the *inner* electrons of the atom and is thus independent of the atom if it is a part of a gas, fluid or a solid.



The activity (decay rate) $R$, is defined as the number of decays per time unit and can be written

$$R = - dN/dt = N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

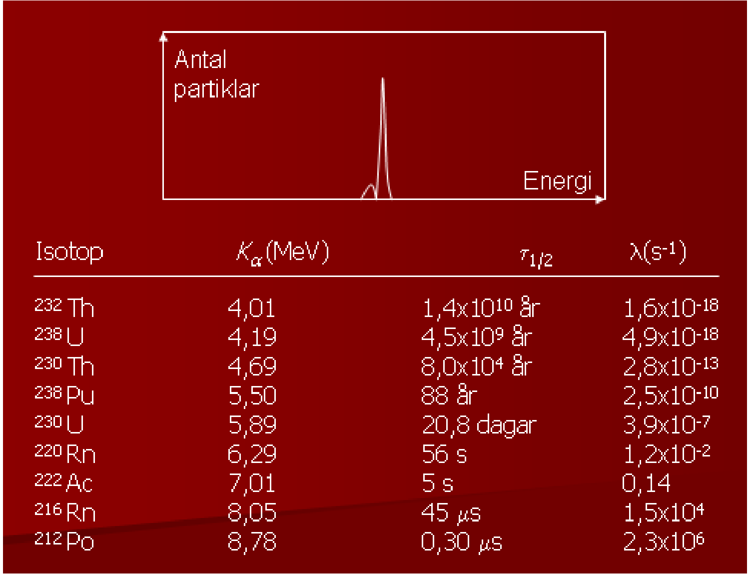
that also can be written

$$R = \lambda N$$

The activity is measured in the unit1 Bq (Becquerel) = 1 decay/s. Earlier the unit 1 Cu (Curie) = 3.7 $10^{10}$ decay/s was used

## 11.3 α-decay

- α- particles are helium nuclei
- α- particles are only emitted of nuclei with A>200
- α- particles kinetic energy has a specified value (or perhaps a few determined values if the daughter-nucleus can exist in different energy levels) and is normally within the region 4-10 MeV, as can be seen in the figure below



| Isotop | $K_\alpha$(MeV) | $\tau_{1/2}$ | $\lambda$($s^{-1}$) |
|---|---|---|---|
| $^{232}$Th | 4,01 | 1,4x$10^{10}$ år | 1,6x$10^{-18}$ |
| $^{238}$U | 4,19 | 4,5x$10^{9}$ år | 4,9x$10^{-18}$ |
| $^{230}$Th | 4,69 | 8,0x$10^{4}$ år | 2,8x$10^{-13}$ |
| $^{238}$Pu | 5,50 | 88 år | 2,5x$10^{-10}$ |
| $^{230}$U | 5,89 | 20,8 dagar | 3,9x$10^{-7}$ |
| $^{220}$Rn | 6,29 | 56 s | 1,2x$10^{-2}$ |
| $^{222}$Ac | 7,01 | 5 s | 0,14 |
| $^{216}$Rn | 8,05 | 45 $\mu$s | 1,5x$10^{4}$ |
| $^{212}$Po | 8,78 | 0,30 $\mu$s | 2,3x$10^{6}$ |

The reaction formula can generally be written

$${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$$

as the uranium isotope ${}^{238}U$ decay can be written

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

**Example**

${}^{238}U$ decays via α-decay according to

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$$

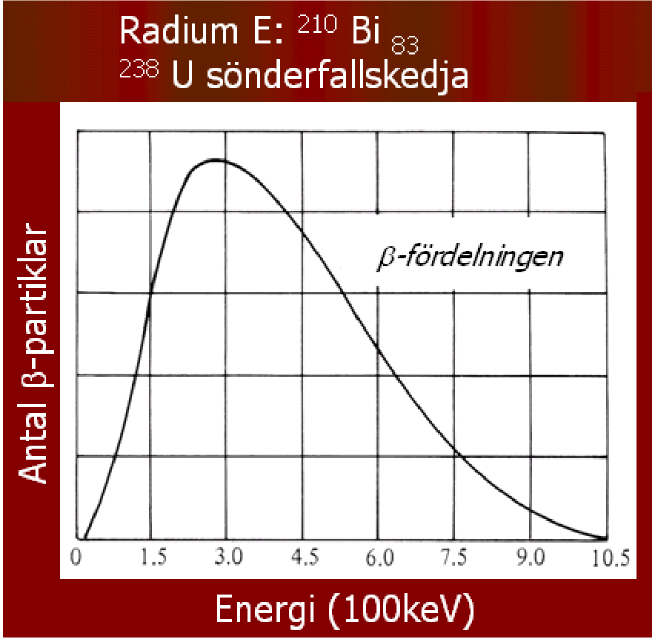
Calculate the liberated energy (Q-value) for the reaction if the atomic masses are the following (in atomic units, u)

| | | |
|---|---|---|
| 238,05079 | for | ${}^{238}U$ |
| 234,04363 | for | ${}^{234}Th$ |
| 4,00260 | for | ${}^{4}He$ |

$Q = \Delta mc^2 = \{238,05079 - (234,04363 + 4,00260)\}$ u x 931,5 MeV/u = 0,00456 u x 931,5 MeV/u = 4,2849 MeV

### 11.4 **β-decay**

In the β-decay an electron (or a positron) is released. If you measure its kinetic energy, one obtains a continuous spectrum although one would expect just *one single value.*



In order to fulfil the laws about the conservation of energy and momentum, one has to make one more assumption, namely that there must be still one more particle emitted together with the electron, the so called the *neutrino* (electron neutrino) $\nu_e$. Pauli suggested the existence of the neutrino around 1930 to explain the β-decay. It is extremely difficult, but not impossible to detect. In a famous experiment from 1953 Reines and Cowan succeeded in doing this by using the neutrino flow from a reactor. The probability, *cross section*, for a neutrino reaction to take place is extremely small, of the order of $10^{-20}$ barn, giving a free mean distance of many thousands of light years in water. 1 *barn* = $10^{-28}$ m$^2$.

The reaction formula for the β-decay can generally be written

$^A{}_Z X \rightarrow {}^A{}_{Z+1}Y + e^- +$ anti-$n_e$ for the $\beta^-$-decay

$^A{}_Z X' \rightarrow {}^A{}_{Z-1}Y' + e^+ + n_e$ for the $\beta^+$-decay

For certain nuclei there is another process present, so called *electron capture* that is a kind of β-decay. The nucleus captures a K-electron (inner shell). In this case just only one neutrino is ejected, but it is followed by X-ray radiation from the electronic transition when the temporary free place of the K-shell is filled.

## 11.5 γ-decay

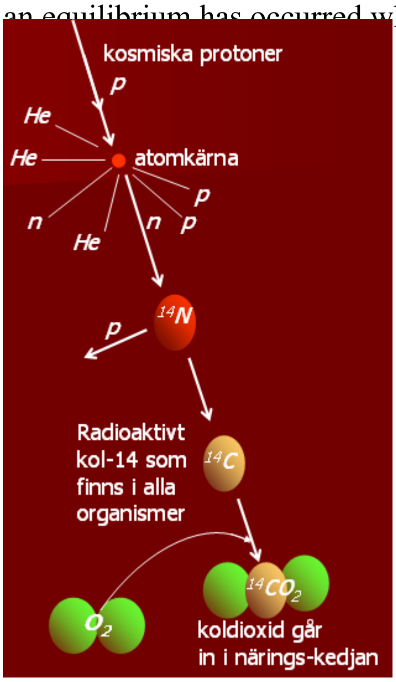
At γ-decay electromagnetic radiation is emitted. The number of nucleons does not change in this process, why the reaction formula generally can be written

$A_Z X^* \rightarrow A_Z X + \gamma$

The nucleus goes from an excited state (denoted *) to a state of lower energy. The spectrum is discrete why the emitted radiation only has just one energy (or a few energies), or one can say that the nucleus energy is ruled by quantum physics. A competing process is the so-called *inner conversion*. In this case, the nucleus transfers its excess energy to an electron (in the K-shell) and is ejected from the atom, thus giving X-ray radiation. Within archaeology, the isotope $^{14}$C is used, having a suitable half-life of 5730 years. This isotope is produced in the atmosphere where $^{14}$N-nuclei are hit by thermal neutrons and then the following reaction occurs

$^{14}N + n \rightarrow {}^{14}C + H$

The production of this isotope is supposed to have been constant during a long time and that an equilibrium has occurred where the number of created $^{14}$C-atoms = number decays.



All living organisms contain carbon and are thus to some extent radioactive. When the organism is alive the activity is constant, but when it dies, it decays according to

$$R = R_0 \cdot e^{-(\ln 2)t/5730}$$

By measuring $R$ one can determine the time of the organisms death.

# 12 Nuclear energy

## 12.1 Introduction

In α-particle scattering towards nuclei, one gets a deviation from Rutherford's theory for high energies of the α-particle (and light nuclei). This gives us a possibility to determine the nuclear radius. Corresponding experiments have been done with electrons, but this was done muck later since a rather high energy is required to give the electron a deBroglie-wavelength of the same order as the diameter of the nucleus. These result showed that the nucleus at a first approximation can be regarded as spherical with a radius $R$, depending on its mass number $A$ according to

$$R = R_0 A^{1/3}$$

Here $R_0 = (1{,}1\text{-}1{,}4)\ 10^{-15}$ m and the lower value is taken from electron scattering and the higher from $\alpha$-particle scattering.

The nuclear mass, that can be measured with high accuracy is given in **atomic** mass units defined by the mass of 1/12 of the $^{12}$C-atom and has the value

1 u = 1,66056 $10^{-27}$ kg

Perhaps it can be worth noticing that $uc^2$ = 931,494 MeV and in the same way we get

$m_pc^2$= 938,3 MeV $m_nc^2$= 939,6 MeV $m_ec^2$= 0,511 MeV

The nuclear mass, M, is less than the sum of the masses of the protons and the neutrons. This *massdefect* $\Delta m$ corresponds to an energy $\Delta E_{be}$ (>0), the nuclear *binding energy* $\Delta E_{be} = \Sigma(mc^2) - Mc^2$ $(= \Delta mc^2)$

If one adds energy to the nucleus, one can split it into its parts. If the electrons binding energies of the atom can be neglected, the binding energy $E_{be}$ for a nucleus with $Z$ protons and $(A-Z)$ neutrons be written

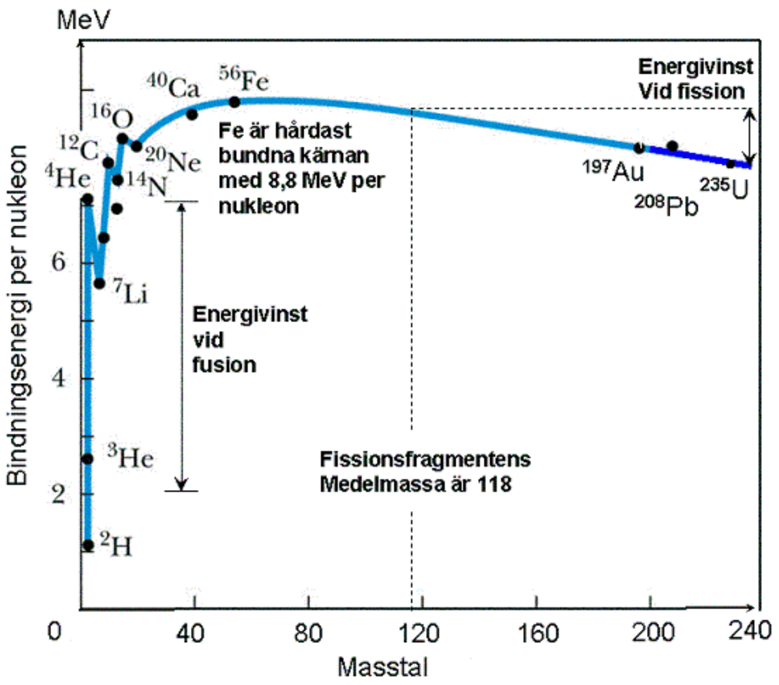
$$E_{be} = Z(M_H - m_e)c^2 + (A-Z)m_nc^2 - (M-Zm_e)c^2$$

$M$ is the ATOMIC MASS and $m_e$ the mass of the electron. The electron masses vanish in the expression, why

$$E_{be} = ZM_Hc^2 + (A-Z)m_nc^2 - Mc^2$$

The binding energy/nucleon then becomes $\varepsilon = E_{be}/A$

and $\varepsilon$ depends on $A$ according to the diagram below.



We observe from the diagram that one can obtain *kinetic energy* from *rest energy* if
1) two light nuclei add to form a heavier nucleus, in a process called ***fusion***
or
2) a heavy nucleus is split into two lighter nuclei, a process called ***fission***

## 12.2 Fission

If a nucleus with mass number $A$ and atomic number $Z$ decays into two equal parts (a possible decay, but not the most common) the sum of the rest energies before the reaction is:

$$ZM_{H}c^2 + (A-Z)m_nc^2 - A\varepsilon_A$$

and after:

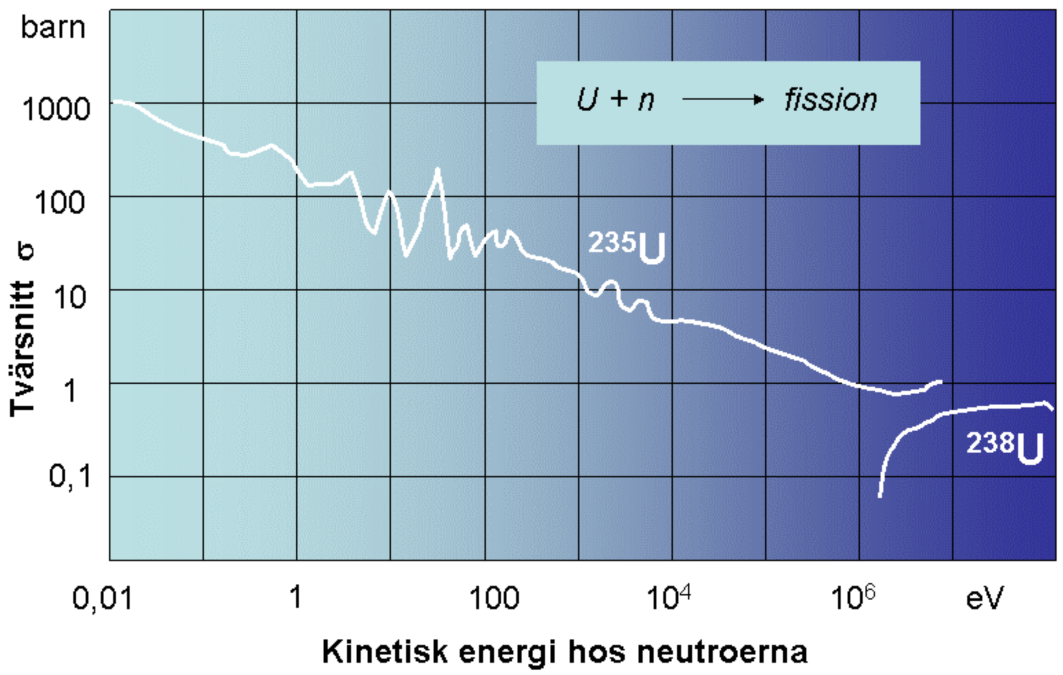
$$2(ZM_{H}c^2/2 + (A-Z)m_nc^2/2 - A\varepsilon_{A/2}/2)$$

The difference in rest energies (fore-after) becomes

$$A(\varepsilon_{A/2} - \varepsilon_A) > 0$$

With A = 238 and $\varepsilon_{A/2}$ = 8,5 and $\varepsilon_A$ = 7,6 MeV/nucleon respectively, we get
238(8,5-7,6) MeV/nucleus = more than 200 MeV/nucleus!

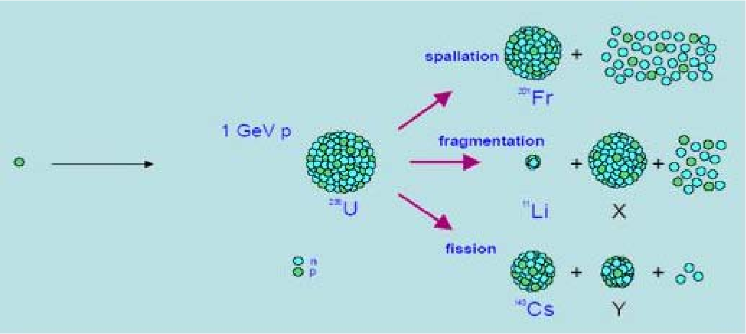
In order to achieve fission, by making the nucleus to become unstable in some way, which can be done, by sending neutrons to hit the nucleus. If one studies normal fission elements like $^{235}$U and $^{238}$U, we can look at the diagram below, showing the probability or the cross section, for fission to occur. From the diagram we see that it for $^{235}$U is enough with slow (thermal) neutrons for the reaction to start. For $^{238}$U on the other hand, neutrons with kinetic energies in the MeV-region are needed.



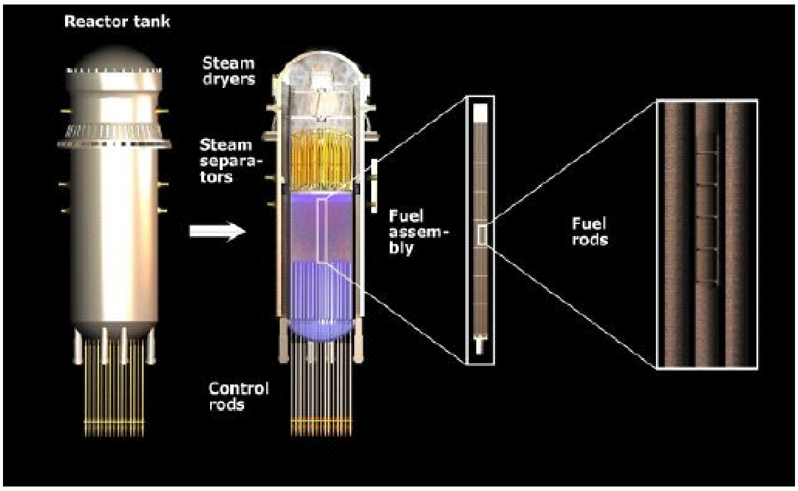
As we have mentioned, normally two equally large fission fragments are not created at nuclear reactions. An example is shown below where $^{235}$U captures a neutron:

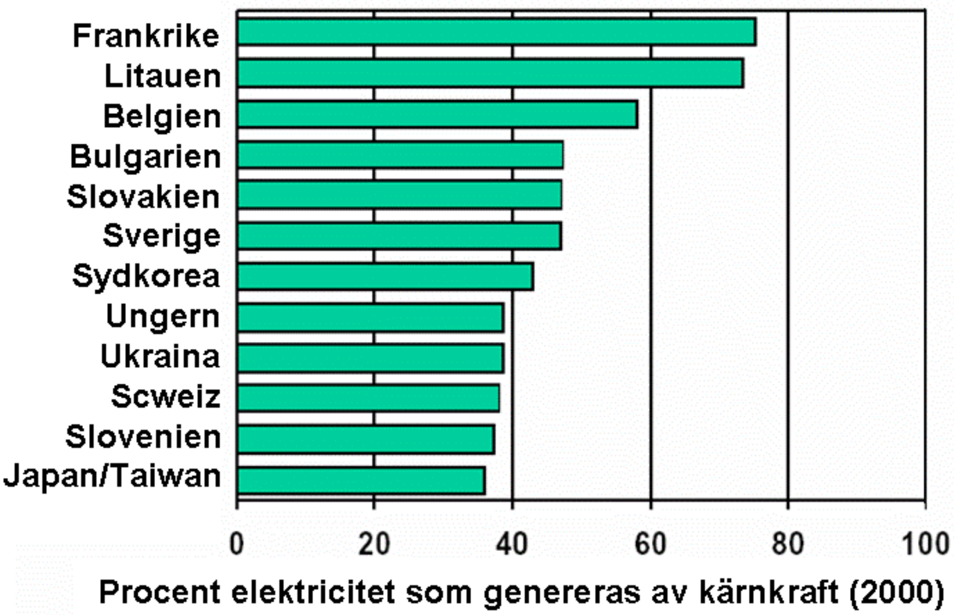
$$ {}^{1}n+{}^{235}U \rightarrow {}^{236}U^{*} \rightarrow {}^{144}Ba+{}^{89}+{}^{89}Kr+3\,{}^{1}n \text{ or } {}^{1}n+{}^{235}U \rightarrow {}^{236}U^{*} \rightarrow {}^{140}Xe+{}^{89}+{}^{94}Sr+2\,{}^{1}n $$

In the figure below, we see different alternatives for fission when a high-energy proton (1 GeV) falls upon a uranium nucleus:



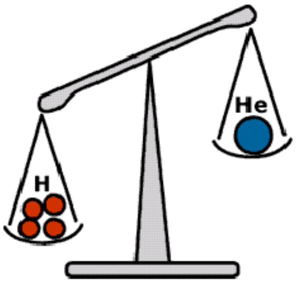
Since there are more neutrons created in the nuclear reaction, all these secondary neutrons can react with another nucleus, thus creating still more neutrons, why a *nuclear chain reaction* can follow. The energy release is due to the kinetic energy of the participating daughter nuclei.





## 12.3 Fusion

We have already realized that one can extract energy by letting two lighter nuclei form a heavier nucleus thus gaining kinetic energy due to the mass defect. The fusion reaction of the Sun is based on the figure below:



The repulsive Coulomb barrier has to be overcome in order to make the two positive nuclei come close enough to each other, making the distance very short to reach the region where the strong attractive force acts. One can make a simple estimation of how large the Coulomb barrier is for two colliding hydrogen nuclei.

**Example:**
What average kinetic energy must each proton have if they are to overcome the repulsive Coulomb barrier making two protons cause a fusion?

The Sun radiates isotropically the energy 3.9 $10^{26}$ J/s. The fusion process in the inner part of the Sun is a process containing several steps, where hydrogen is converted to helium. In the first step, two hydrogen nuclei (protons) collide and create deuterium under the emittance of one positron and one (electron)-neutrino according to (Q-value is liberated energy)

$^1H^+ + {}^1H^+ \rightarrow {}^2H^+ + e^+ + \nu_e$ (Q = 0.42 MeV)

(After this reaction, the positron is of course annihilated by an electron $e^+ + e^- \rightarrow \gamma + \gamma$ (Q = 1.02 MeV))
This is a rare process. Normally the protons touch each other elastically, but once in $10^{26}$ collisions, deuterium is created. It is this "bottleneck" making the Sun burn "slowly" (but still there are some $10^{12}$ kg/s deuterium created depending on the high number of available protons).
When a deuterium nucleus has been created, instantly a $^3$He-nucleus is created according to
$^2H^+ + {}^1H^+ \rightarrow {}^3He^{2+} + \gamma$ (Q = 5,49 MeV)
During $10^5$ years in average (What makes the process take so long?) there are two $^3$He-nuclei created and one $^4$He-nucleus:

$^3He^{2+} + {}^3He^{2+} \rightarrow {}^4He^{2+} + {}^1H^+ + {}^1H^+$ (Q = 12.86 MeV)

In this way one can summarize the Suns total *p-p*-cycle to

$2({}^1H^+ + {}^1H^+) \rightarrow {}^4He^{2+} + 2(e^+ + n_e) + 2\gamma$

and if one adds four electron masses on each side, one realizes that one can use atomic masses and can calculate the Q-value of the process:

$Q = \Delta mc^2 = 4(1.007825)\text{uc}^2 - 4.002603\ \text{uc}^2 = 0.028697\ \text{uc}^2 = 26.7\ \text{MeV}$

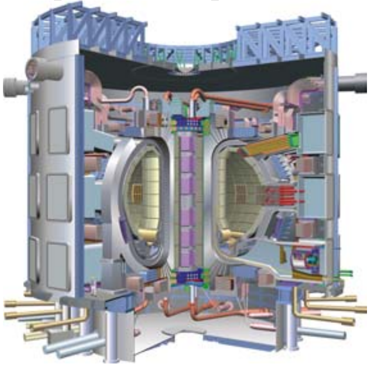
Both neutrino particles carry around 0.5 MeV of this energy from the Sun. When all the hydrogen of the Sun has burnt (in about $5 \times 10^9$ years), the Sun's mass will not be large enough to start a helium cycle, but the Sun will end its phase as a star to become a Red Giant. Other massive stars can continue the fusion reactions. However, they cannot create fusion elements heavier than $^{56}$Fe.
Element with masses larger than 56 are believed to be created in Supernovae explosions. One has tried to copy the elegant fusion reaction of the Sun, but it has failed since the reaction is too "slow". Tempting alternative fusion reacions that would possibly work are:

$^2H^+ + {}^2H^+ \rightarrow {}^3He^{2+} + n$ (Q = 3.27 MeV)
$^2H^+ + {}^2H^+ \rightarrow {}^3H^+ + {}^1H^+$ (Q = 4.03 MeV)
$^2H^+ + {}^3H^+ \rightarrow {}^4He^{2+} + n$ (Q = 17.59 MeV)

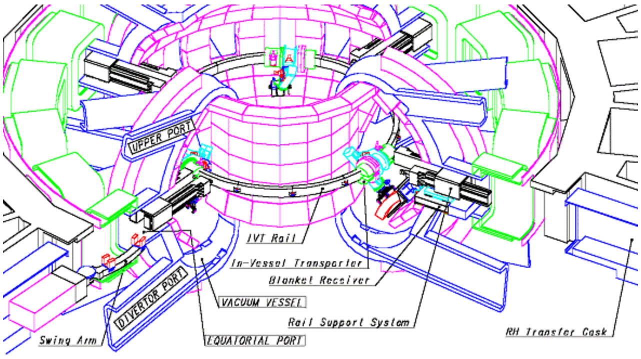
There are three criteria that have to be fulfilled in order to obtain a successful (Earth based) fusion reactor.

- High particle density (a neutral plasma) $n$
- High plasma temperature $T$
- Long enough confinement time ($\tau$) of the plasma. One can show that a fusion reactor using the Deuterium-Tritium reaction must obey the Lawson criterion: $n\tau > 10^{20}$ s/m$^3$

Magnetic confinement is shown below in a so-called Tokamak. ITER is the name of the last research plant that is planned to be constructed and in operation 2016.



ITER

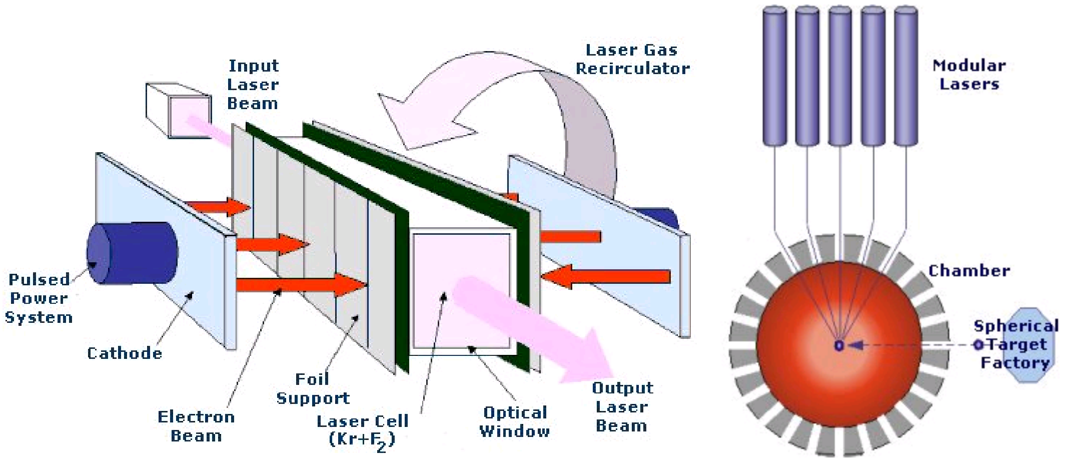


Tvärsnitt genom ITER

**Parameters for ITER:**

| | |
|---|---|
| **Total power** | **500 MW (700MW)** |
| **Q = fusion power/auxiliary heating power** | **≥10 (inductive)** |
| **Plasma burning time** | **≥ 300 s** |
| **Plasma max radius** | **6.2 m** |
| **Plasma min radius** | **2.0 m** |
| **Plasma current** | **15 MA (17.4 MA)** |
| **Magnetic field “Toroidal” @ 6.2 m radius** | **5.3 T** |
| **Plasma volyme** | **840 m$^3$** |
| **Plasma surface** | **680 m$^2$** |
| **Heating/Driving power** | **73 MW (100 MW)** |

There are different ways in trying to fulfill the Lawson criterion. Another method is to shoot powerful laser beams on to pellets of Tritium (http://www.llnl.gov/str/Petawatt.html) in order to obtain fusion.



A krypton fluoride laser is based on the reaction: (Electron energy) + Kr + $F_2$ => KrF* + F => Kr + $F_2$ +light

There are a number of Laser Parameters needed to achieve a fusion reaction in a pellet. The Laser energy has to be around 1.5 Mega joule. The laser pulse time has to be $4\text{x}10^{-9}$ s. This gives an output power of $P$ = (Energy/burning time) = $4 \text{ x } 10^{14}$ Watt.

**Contents Chapter 11-12**

11.1 Introduction. Nuclear physics
11.2 Natural radioactivity
11.3 $\alpha$-decay
11.4 $\beta$-decay
11.5 $\gamma$-decay
12.1 Nuclear energy
12.2 Fission
12.3 Fusion

**Learning goals**

Be able to discuss basics of nuclear physics, proton number, mass, neutron number
Be able to discuss the introduction of the decay constant
Discuss the law of decay and activity
Calculate the activity and the half-life
Be able to discuss about the $\alpha$-decay
Write reaction formulas for the $\alpha$-decay
Calculate the Q-value
Be able to discuss for the $\beta$-decay
Discuss the introduction of the neutrino
Be able to discuss electron capture
Be able to discuss inner conversion
Be able to discuss for the $\gamma$-decay
Be able to discuss for the C-14 method
Be able to discuss the size of the nucleus and nuclear models

Be able to discuss about the nuclear bonding energy
Be able to roughly discuss the bonding energy per nucleon
Describe fission and nuclear reactors
Describe fusion
Give a view of ITER
Give a view of Laser fusion

**Advices for reading**

Think of the methods used in physics are often based on scattering
Rutherford's contribution was important in the study of the atom
Also think of the neutron beeing difficult to detect, one had first postulated its existence
Make some thoughts over the nuclear decay. It was at first not easy to understand since there were several mechanisms
Think about the difference between α-and β-decay.
Study the decay spectrum at α-and β-decay. Notice the difference and explain it
Discuss the neutrino's role in the decay
Think of the importance of the Q-value for reactions. Study it closely
Think of how the γ-decay differs from α-and β-decay
Think of how the γ-decay can be used for practical age determinations
Try to describe the nucleus dimensions with a simple model
Think of the curve for the binding energy per nucleon decides how to obtain fission and fusion
Think of that the Q-value plays an important role and the need for new neutrons in the fission process

**Readings**

- Thornton, Rex, Modern Physics, Saunders
- Krane, Modern Physics, Wiley
- Beiser, Concepts of Modern Physics, McGraw-Hill
- Serway, Moses, Moger, Modern Physics, Saunders
- Eisberg, Resnick, Quantum Physics of Atoms, Molecules, Solids and Particles, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Halliday and Resnick, Fundamentals of Physics, Wiley
- Benson, University physics, Wiley

**WEB-readings**

- http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html
- http://nobelprize.org/nobel_prizes/physics/articles/lecuyer/index.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1935/index.html
- http://www.llnl.gov/str/Petawatt.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1967/index.html