

Modern physics Chapter 11-12. Solutions to Exercises.

11.1.1 After one half-life there are $\frac{1}{2}$ of the nuclei left. After still one half-life there are $\frac{1}{4}$ left, after still one half-life there are $\frac{1}{8}$ left and finally after on more there are $\frac{1}{16}$ left. We have totally 4 half-lives, which means $4 \times 0.30 \mu\text{s} = 1.2 \mu\text{s}$.

11.1.2 The number of nuclei in 1.0 g of Po is $N = \frac{1}{226} \times 6.026 \times 10^{23} = 2.26 \times 10^{21}$ nuclei.

11.1.3 The activity of a radioactive substance is given by $R = \lambda N$. The decay constant λ can be written as a function of the half-life as $\lambda = \ln 2 / T_{1/2}$.

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} N = \frac{\ln 2}{1620 \times 365 \times 24 \times 3600} 2.26 \times 10^{21} \approx 37 \times 10^{10} \text{ decays per second.}$$

This was earlier the unit for activity, i.e. 1 Ci = 1 Curie

11.1.4 Measured activity $R = 735/60$ decay/s. $R = (\ln 2 / T_{1/2}) N$ where $N = \frac{m}{M} N_0$

$$\text{The half-life: } T_{1/2} = \frac{\ln 2}{R} \times \frac{m}{M} N_0 = \frac{\ln 2}{735/60} \times \frac{1.0 \times 10^{-3}}{238} \times 6.023 \times 10^{23} \text{ s} \approx 4.5 \times 10^9 \text{ ears}$$

(We have divided by $365 \times 24 \times 60 \times 60$ s/year)

11.2.1 $M_{\text{Po210}} = 209.982876$ u. $M_{\text{Pb206}} = 205.974455$ u. $m_{\text{He4}} = 4.00260$ u.

$$\begin{aligned} \text{Q-value is given by: } Q &= M_{\text{Po210}} c^2 - (M_{\text{Pb206}} c^2 + m_{\text{He4}} c^2) - K = \\ &= (209.982876 - 205.974455 - 4.00260) \times 931.4 - 5.3 \text{ MeV} = \\ &= 0.005821 \times 931.4 \text{ MeV} - 5.3 \text{ MeV} = 5.4216794 - 5.3 \text{ MeV} = \\ &= 0.122 \text{ MeV} \end{aligned}$$

$$\begin{aligned} 11.3.1 \quad m_n &= m_p + m_e + \frac{E_{K \max}}{c^2} = M_H + \frac{E_{K \max}}{c^2} = \\ &= 1.007825 + (0.782 \text{ MeV}) / (931.5 \text{ u/MeV}) = 1.008665 \text{ u} \end{aligned}$$

$$\begin{aligned} 11.4.1 \quad \text{The power } P &= n \times hf = N \lambda hf = \\ &= \frac{1}{60} \times 6.023 \times 10^{23} \frac{\ln 2 \times (1.173 + 1.333)}{5.3 \times 365 \times 24 \times 3600} 0.99 \text{ MeV} / c \approx 16.5 \text{ W} \end{aligned}$$

11.4.2 The activity/g = S = 0.415 decay/(minutes.g). We get the equation $0.415 = 14 \times e^{-0.693t/5730}$. Take the logarithm and we gets $t = 2.91 \times 10^4$ years.

$$\begin{aligned} 11.5.1 \quad \text{The reaction becomes: } & {}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + {}^1\text{n} \\ Q &= (M_{\text{before}} - M_{\text{after}}) c^2 = \\ &= (3.016050 + 2.014102 - 4.002603 - 1.008665) \times 931.5 \text{ MeV} = \\ &= 17.6 \text{ MeV} \end{aligned}$$

$$\begin{aligned} 11.6.1 \quad \text{The reaction becomes: } & {}^{235}\text{U} + {}^1\text{n} \rightarrow {}^{236}\text{U} \\ Q &= (M_{\text{before}} - M_{\text{after}}) c^2 = (235.1170 + 1.008665 - 236.1191) \times 931.5 \text{ MeV} = \\ &= 6.1153 \text{ MeV.} \quad \text{The potential barrier is only } 5.3 \text{ MeV} \text{ why it is enough with slow} \\ & \text{thermal neutrons.} \end{aligned}$$