14. Astrophysics

14.1 Introduction
When you study astrophysics or cosmology there are many areas within physics that have contributed to theories and experiments, such as atomic physics, molecular physics, nuclear physics, particle physics etc. When we look at the night sky one sees “old” light from distant galaxies and stars, or as the physicist Richard Feynman expresses it: 
I too can see the stars on a desert night, and feel them. But do I see less or more? The vastness of the heavens stretched my imagination—stuck on this carousel my little eye can catch one-million-year-old light.

14.2 Olbers’ Paradox
In observations of the night sky it shows that if you look in any direction, there is a star. The mean incoming radiation from our own Sun is 1.38 kW/m². Olber wanted to investigate how bright the night sky should be with that assumption. Let us suppose that the density of stars is \( \rho \) and that they are equally spread over the sky. We calculate the total amount of the light reaching the Earth from all directions.

The number of stars between radius \( R \) and \( R + \Delta R \) will be: 
\[ \Delta n = 4\pi R^2 \rho \Delta r \]

The light intensity that reaches the Earth from one distant star delivering the power \( P \) at a distance of \( R \) will be: 
\[ I_{\text{Earth}} = \frac{P}{4\pi R^2} \]

The total intensity of the light from all stars reaching Earth will then be:
This means that the night sky should be completely bright, which is not the case. This is the so-called Olber’s paradox. This assumption is based upon that the night sky and the Universe are static.

Hubble studied the velocity of astronomical objects and in 1929 he presented a linear relation between the red shift (Doppler effect) of certain spectral lines emitted from very distant objects and stars and the distance to them. This has already been discussed in the section of Special relativity in Chapter 1, which we also will discuss further in Chapter 14.7.

**Example**

Suppose that the expansion of the Universe has stopped and that the radius of the Universe is equal to the distance of the galaxy Hydra. Calculate the distance to Earth. (Use the figure in 14.7)

From the figure we get \( R = 4.0 \times 10^9 \) light years = \( 4.0 \times 10^9 \times 3.00 \times 10^8 \times 365 \times 24 \times 3600 \) m = \( 3.8 \times 10^{25} \) m

If one can estimate the density \( \rho \) the total intensity can be estimated.

### 14.3 Cosmology of the 1900-ies

One believed in the beginning of 1900 that the Universe was static and did not change. Investigations had been made on the Solar system and one knew about other stars and nebulous, and interstellar clouds. People did not know about other galaxies, which were called spiral nebulous.
14.4 Einstein’s cosmological constant $\Lambda$.

The Russian researcher Alexander Friedmann developed a model for the expansion of the Universe around 1920 that included the Hubble parameter $H$, the constant of gravity $G$, the density $\rho$, a dimensionless time-dependent scaling parameter $R$ and a parameter of curvature $k$, indicating if the Universe was open or closed.

$$H^2 = \frac{8\pi G \rho}{3c^2} - \frac{k}{R^2}$$

Einstein and several others were simultaneously working on models of the Universe and Einstein suggested a model giving a static Universe. He modified the so-called field equations by introducing a distance term $\Lambda c^2/3$:

$$H^2 = \frac{8\pi G \rho}{3c^2} - \frac{k}{R^2} + \frac{\Lambda c^2}{3}$$

The constant $\Lambda$ is called the cosmological constant. However, Einstein’s model did not work since the static Universe became instable and the observations made by Hubble ten years later, showed that the Universe was expanding. The constant $\Lambda$ was abandoned and Einstein called it the biggest blunder he ever made.

14.5 Intensity measurements of stellar objects

There are stellar objects with known output power $P$ that are called standard light sources. Since the light intensity decreases quadratically with the distance ($1/r^2$) we have:

$$I = \frac{P}{4\pi r^2}$$

We can measure the intensity reaching Earth from a standard light source and determine the distance to the object, $r$. Standard light sources include the so-called Cepheid Variable stars and the Type Ia Super novae, that has shown to have a constant output power ($\pm 18\%$).

Cepheid Variables

Already in 1912, Henrietta Leavitt studied the Cepheid variables, which are stars that had consumed their supply of hydrogen and pulsate in intensity. One needed a measurement of the distance with some other method for at least one Cepheid. The first Cepheid, Delta Cephei, was close enough to be able to be measured with the parallax method.
Nowadays one uses the Small Magellanic Cloud, which is one of our closest galaxies. Below is shown still another galaxy, the Sombrero galaxy:

14.6 Novae and super novae
Novae and super novae are stars shining brightly and then fades away. Explosions in the star give temperatures and pressures needed to produce heavier elements as uranium. The Crab nebulae’s super nova took place already in 1054 and was observed in both China and Japan. It could even be seen in daylight.

Super novae
Super novae are classified using their spectra:
Type I: super novae WITHOUT hydrogen absorption lines, Balmer $H_\alpha$, $H_\beta$, ..., Lyman $L_\alpha$, $L_\beta$ etc.
Type II: super novae WITH hydrogen absorption lines.
Type I gives large supernovae products in the form of gas clouds that are rich of iron.
Type II super novae give large products in the form of gas clouds that are rich of elements heavier than iron. Besides, there is a compressed kernel in the form of a neutron star or a black hole. The Crab nebula is what remains of a Type II super nova that also has a neutron star in its center. Tycho’s supernova remaining stems from a Type I supernova rich on iron, but it does not have a dense star body in the center.

SN 1604 (Kepler’s supernova) (Chandra) and atomic spectrum between 300-500 nm
The picture, taken in the X-ray region shows what remains of the Kepler Supernova SN 1604 (Chandra X-ray Observatory).

14.7 Hubble’s measurements
Hubble measured the speed of astronomical objects moving away from Earth and found the linear dependence between the red shift and the escape velocity, by looking at selected spectral lines emitted from the distant objects and comparing with Earth bound light sources.
Hubble’s law:

\[ v = HR \]

where \( H \) is called the Hubble constant.

The Hubble constant relates to a scaling factor \( \alpha \) proportional to the distance between the galaxies:

\[ H = \frac{1}{\alpha} \frac{d\alpha}{dt} \]

In Chapter 1, relativity, the Hubble law was also discussed. In the next picture one can see the linear dependence in making distance measurements at large distances. In the diagram we use the unit Mpc (Mparsec) [1 light year = 0.306 601 394 parsec].

**Example**

We can note that galaxies at large distances from us move with relative and very high speed. *Hydra* at a distance of \( 4 \times 10^9 \) light years from us move at a velocity of \( 0.2 \beta = 0.2 \) \( v/c \). Its relative velocity is thus \( v = 0.2 \times 3.00 \times 10^5 \) m/s = \( 6.0 \times 10^7 \) m/s!
14.8 Quasars

Quasars ("QUAsi-StellAR-Objects") seemed to be week shining stars but had large red shifts indicating that they were very far from us. Their light intensity varies weekly or monthly indicating they are small objects. Their sizes are of the order of light weeks. However, they are more intense than our galaxy that has the size of 100 000 light-years in diameter. The red shift is described by the parameter $z$, defined as

$$ z = \frac{\Delta \lambda}{\lambda} = \sqrt{\frac{1 + \beta}{1 - \beta}} - 1 $$

Here, $\beta = v/c$ as usual.

The variation in $z$ for more than 100 quasars is $z = 0.16$ to $3.53$.

We rewrite the equation and determines $\beta = v/c$ and get:

$$ \beta = \frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1} $$

**Example**

Determine the velocity of a galaxy where the measured red shift gives the parameter $z = 3.0$.

We put $z$ into the formulae above and obtains $v = c \times \frac{(2+1)^2 - 1}{(2+1)^2 + 1} = c \times \frac{8}{10} = 0.8c = 2.4 \times 10^8$ m/s

14.9 Black holes

**The escape velocity of Black holes**

In the same way as one calculates the escape velocity of rockets and gases from Earth, one can calculate the escape velocity of matter from Black holes. Objects can be released from the Black hole if its kinetic energy exceeds the gravitational energy of the Hole.
If we put $v = c$ as an upper limit we get: (We should have used relativistic calculations)
This can only be possible if the mass $M$ is within the radius:

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

$$r \leq \frac{2GM}{c^2}$$

This is called the Schwarzschild radius.

**14.10 Detection of Black holes**

Since light cannot escape a Black hole, one has to detect Black holes by indirect methods. One possibility is to study X-ray radiation appearing when masses fall into the Black hole, as can be seen in the figure below. An accompanying star’s material is falls into the Black hole generating X-ray radiation.

The energy of the radiation is as usual $\Delta E = hf = hc/\lambda$.

By observations in different wavelength regions one has seen several candidates as Black holes (Cygnus X-1, M87, NGC4261...). All objects have masses that exceeds the Suns by large.

**14.11 Black holes and the Event Horizon**

When the thermo-nuclear material of a star vanishes, there is no longer any heat resisting the gravity. Therefore, the gravitational force becomes the completely dominating force. The matter of the star will collapse into an extremely dense sphere with such a high density that no light can escape from the created Black hole. The point in the middle is called a *singularity*. 
A star collapsing in this way has a mass larger than 3 solar masses and distorts the space-time and creates a Black hole.

Karl Schwarzschild calculated the radius of a Black hole, also called the event horizon:

\[ r_s = \frac{2GM}{c^2} \]

**Example**

Suppose we have a Black hole with three solar masses. Calculate the Schwarzschild radius.

\[ r_s = \frac{2GM}{c^2} = \frac{2G(3M_s)}{c^2} = \frac{2 \times 6.66 \times 10^{-11} \times 1.989 \times 10^{30}}{(3.00 \times 10^8)^2} \approx 8.8 \text{ km} \]

We can compare the result with the solar radius 7.0x10^5 km!

**Black hole – Applet:**

http://www.astro.ubc.ca/~scharein/a311/Sim.html

Describes the development of a Black hole.

### 14.12 Hawking radiation

Depending on quantum fluctuations, particle-anti particle pairs are created near the event horizon. One particle falls into the singularity and at the same time as others can escape. Anti-particles that escape emit energy when they are being annihilated with matter.
Hawking determined the Black body radiation of the Black hole to be:

\[ T = \frac{\hbar c^3}{8\pi kGM} \]

Here, \( k \) is the Boltzmann constant.

Hawking also calculated the total emitted power of the Black hole:

\[ P(T) = 4\pi\sigma T^4 \left( \frac{\hbar c^3}{8\pi kGM} \right)^4 \]

Here, \( \sigma \) is the Stefan-Boltzmann constant.

**Example**

A Black hole has the mass equal to three solar masses. Calculate the temperature \( T \) of the Black body radiation.

\[ T = \frac{\hbar c^3}{8\pi kGM} = \frac{6.63 \times 10^{-34} \times (3.00 \times 10^8)^3}{8\pi \times 2\pi \times 1.38 \times 10^{-23} \times 6.67 \times 10^{-11} \times 1.989 \times 10^{30}} \approx 6.2 \times 10^{-8} \text{K} \]

This temperature is very close to Absolute zero.

**14.13 Evaporation of Black holes**

Very small Black holes with masses of the order of \( 10^{-19} \) solar masses can be detected by their Hawking radiation, which is negligible for large Black holes. The energy transfers to pair production at the Event horizon and the mass of the Black hole successively will diminish with a lifetime of:

\[ T = \frac{M^3}{3\nu\hbar} = 8.3 \times 10^{-26} M^3 \text{ s} / \text{ g}^3 \]

where \( \nu \) is a constant \( \nu = 1 / 15.360\pi \)

One finds that the smaller the mass, the shorter the lifetime. Black holes with masses of the order of the solar mass can live longer than the age of the Universe. Little Black holes with masses <\( 10^{11} \) kg (of the size of mountains) would explode.

**Example**

Determine the lifetime of a "small" Black hole with mass \( 10^{10} \) kg.

\[ T = \frac{(10^{10})^3}{3\nu\hbar} = 8.3 \times 10^{-26} (10^{10})^3 \text{ s} \approx 8.3 \times 10^{-4} \text{ s} = 83 \text{ ms} \]
14.14 Big Bang. The cosmical background radiation

The theory of the Big Bang leans on two fundamental principles, the general theory of relativity and the cosmological theory, where one supposes that the Universe is uniform. In any direction you look, it should be the same, which means that the Universe is isotropic and homogeneous. We will discuss several issues enlighten these questions.

Penzias and Wilson studied the cosmological background radiation in the microwave region that comes from all parts of the Universe.

A. Penzias and R. Wilson at the microwave-antenna, Crawford Hill, NJ

The blackbody radiation is billions of years old and has a Doppler shift towards the temperature 3 K today. Applying satellite measurements one sees an almost evenly spread 3 K background.

Planck curve showing the 3 K background of the Universe

14.15 The geometry and future of the Universe

Friedmann gave a theory for the Universe and described it by using density of mass, curvature and vacuum energy:
\[ H^2 = \frac{8\pi G \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3} \]

Here we recognize \( H \), the Hubble constant, \( G \), the constant of gravity and \( k \) is the curvature parameter.

In the formula we can identify three parts:

Density of mass \( \Omega_m = \frac{8\pi G \rho_m}{3H^2} \)

Curvature \( \Omega_k = -\frac{k}{a^2H^2} \)

Vacuum energy density \( \Omega_\Lambda = \frac{\Lambda}{3H^2} \)

We will return to these parameters but first we will try to decide about the future of the Universe. One puts the question, whether the Universe will expand for eternal times. It shows, that it depends on how large amount of matter the Universe contains and how fast it expands. There shows to be three possibilities.

One can show that there is a certain critical density \( \rho_c \) for which the Universe will be flat, closed or open. If the relative density is \( \Omega_0 \), i.e. if we divide \( \rho \) by \( \rho_c \) and it is equal to 1, then we have a flat Universe.

\[ \Omega_0 = \frac{\rho}{\rho_c} \]

The critical density is \( \rho_c = \frac{3H^2}{8\pi G} \)

Here \( H \) is the Hubble parameter and \( G \) the gravitational constant. We can try to deduce this expression by doing just about the same calculations as when one determines the escape velocity of gases from Earth. In that case one assumes that the change in total energy of an object with \( m \) is zero:

\[ E = K + U = \frac{1}{2}mv^2 - \frac{GMm}{R} \]

The escape velocity from Earth becomes \( v = 11 \text{ km/s} \)

Let us do the same calculation for the Universe and letting it be a sphere with radius \( R \) and density \( \rho \). The total mass within the sphere becomes \( M = 4\pi R^3 \rho /3 \). Applying the Hubble equation, we get a value of the expansion velocity \( v = HR \), where \( H \) is the Hubble parameter. Letting the mass of a galaxy be \( m \) we get an expression for the limit of return, which is the same as its escape velocity:

\[ \frac{1}{2}mv^2 = \frac{GMm}{R} \]

and with the expression above we get

\[ \frac{1}{2}m(HR)^2 = \frac{GM}{R} \left( \frac{4}{3} \pi R^3 \rho_c \right) \]
This leads to the critical density $\rho_c$:

$$\rho_c = \frac{3H^2}{8\pi G}$$

**Example**

Determine the critical density for which the Universe is flat. The Hubble parameter is $H = 21$ km/s per million light years. The critical density will be:

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 \times \left( 21 \times 10^3 / 3 \times 10^8 \times 10^6 \times 365 \times 24 \times 3600 \right)^2}{8\pi \times 6.66 \times 10^{-11}} \text{ kg/m}^3 \approx 8.83 \times 10^{-27} \text{ kg/m}^3$$

This density corresponds to roughly 5 hydrogen atoms per m$^3$ since the mass of hydrogen is $1.67 \times 10^{-27}$ kg.

Let us look at the geometry of the Universe as flat, open or closed for different values of $\Omega_0$:

There are several methods to determine the geometry of the Universe. One can study the incoming microwave radiation that reaches Earth. The picture below shows the result:

The microwave radiation from the Universe where the colors stand for intensity.
Depending on whether the Universe is flat, closed or open on can see it in the fluctuations of the pictures below. Careful measurements at different institutes added together has given the following directions:

14.16 Fundamental constants

The constant of gravity $G$ and the speed of light $c$ are two fundamental constants within the general theory of relativity. Looking at quantum physics, naturally the Planck’s constant $\hbar$ is one of the fundamental constants as well as and $\frac{\pi^2}{\hbar} = \hbar$. By combining these constants we can derive a new constant called the Planck length $\lambda_P$:

$$\lambda_P = \sqrt{\frac{G\hbar}{c^3}}$$

Putting the values of the constants above into the expression we get:

$$\lambda_P = 4.0 \times 10^{-35} \text{ m}$$

It shows, for lengths smaller than the Planck length, neither of the general theory of relativity nor quantum physics can describe situations occurring on these scales. Due to the principle of uncertainty, quantum fluctuations are permitted at dimensions larger than the Planck length.

Let us imagine something moving at the speed of light. Then it takes the time $t_P$ to move a Planck length, and we get:

$$t_P = \frac{\lambda_P}{c} = \sqrt{\frac{G\hbar}{c^5}} = 1.4 \times 10^{-43} \text{ s}$$

It shows that it is difficult to describe what happened in the Universe at times shorter than the Planck time, which we can see from the figure below:
14.17 The cosmological constants

Measurements and observations have shown that the visible matter only is 4% of the total mass of the Universe. Dark matter stands for 23% and 73% is so-called mysterious dark energy.

* Data from star clusters and galaxies is in consistence with a Universe of low density.
* The cosmic background radiation is as we have seen consistent with a Flat Universe.
* Distance determinations based on the Type I supernova data is in turn consistent with an, in space, accelerating Universe.

All these data gives conditions on the Universe constant parameter in Friedmann’s equation:

\[ \Omega_k = 0 \quad \text{where} \quad \Omega_k = -\frac{k}{a^2H^2} \quad \text{has to do with the curvature} \]

\[ \Omega_m = 0.3 \quad \text{where} \quad \Omega_m = \frac{8\pi G\rho_m}{3H^2} \quad \text{has to do with the density of mass} \]

\[ \Omega_\Lambda = 0.7 \quad \text{where} \quad \Omega_\Lambda = \frac{\Lambda}{3H^2} \quad \text{has to do with the vacuum energy}. \]

Applying modeling, the following diagram can be constructed:
The cosmological constants seem to play a most important role in the development of the Universe. One puts the question, why, and what is the meaning?

14.18 The lacking mass and dark matter
The visible, luminous mass stands for up to 4% of the critical density for the Universe. What can be said about the dark matter we cannot see?

According to the discussion above, the dark matter of a galaxy would be of the order of 70%.

With the equation below (where the parentheses stand for mean values):

\[
\frac{\langle mv^2 \rangle}{r} = \frac{\langle G M m \rangle}{r^2}
\]

We can estimate the total mass \(M\) of the galaxy. The dark halo of the galaxy below covers the region between \(10^3\) and \(3 \times 10^4\) light years from the galaxy center. Around 70% of the galaxy thus consist of dark matter.
14.19 En mysterious entity is the vacuum energy, which we do not know much about...

When you try to determine the vacuum energy concept one can start in the same way as when studying diatomic molecules, i.e. with the zero point energy concept. We start in the same way as we did in Chapter 8, Molecular physics. There we discussed the quantum mechanics behind the harmonic oscillator. Let us begin with the zero point energy of the harmonic oscillator and let it be a model for photons and other particles in vacuum.

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega \]

Here \( n \) is the number of particles and \( \frac{1}{2} \hbar \omega \) stands for the energy for each frequency \( \omega \), even without particles, that is in vacuum. We thus have an infinite energy density overall in space! Can this be the vacuum energy that is responsible for the cosmological constant? With calculations one has derived to a finite value giving an energy density of \( \rho_{\text{vac}} = 10^{119} \text{J/m}^3 \) to be compared to the cosmological observations (\( \Omega_\Lambda = 0.7 \)) giving \( \rho_{\text{vac}} = 10^{-7} \text{J/m}^3 \). The difference is 120 orders of magnitude. Why we have this discrepancy is not known!

14.20 The age of the Universe

- Calculations with astronomical data give the age of the Universe of 13.7 ± 0.2 billion years.
- By studying the radioactive decay where one has examined certain meteorites that have hit the Earth, it shows to be 4.5 billion years. Other techniques give estimates of 8 to 17.5 billion years.
- By using looking at stars one have estimated their creation to be around 200 000 years after Big Bang.
- By examining the intensity of spectral lines of old stars one has studied the intensity ratio of thorium/europium and uranium/thorium indicating an age of the Universe to be 14 billion years.
THE THORIUM CHRONOMETER IN CS 22892—052: ESTIMATES OF THE AGE OF THE GALAXY

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Received 1996 August 12; accepted 1996 November 18

ABSTRACT

We analyze the recent thorium detection in the metal-poor halo star CS 22892—052. Sneden et al. have demonstrated that all of the stable elements with Z ≥ 56, including those near the thorium nuclear region, are consistent with the solar r-process abundances. This result strongly suggests that thorium (formed in the r-process) was also produced in solar proportions in the progenitor of CS 22892—052. Theoretical calculations, presented here, that reproduce the observed stable solar system r-process abundances predict a thorium/europium (Th/Eu) ratio in close agreement with the extrapolated, corrected r-process-only ratio (0.465) at the time of the formation of the solar system. Sneden et al. found that Th/Eu = 0.219 in CS 22892—052, substantially below the current solar ratio, indicating a much greater age for this star. Ignoring additional production of thorium over time, and thus any chemical evolution effects, comparison between the observed and the solar system corrected Th/Eu ratios gives a simple radioactive-decay age for CS 22892—052 of 15.2 ± 3.7 Gyr.

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Learning goals
Describe the Olber’s Paradox
Describe the background to the Hubble law
Discuss the Einstein cosmological constant
Describe intensity measurements for distance calculations
Be able to classify supernovae
Thoroughly discuss Hubble’s law and results
Discuss the red shift of Quasars
Discuss the parameter z and perform calculations on distances and velocities
Discuss the Schawrzschild radius
Discuss Hawking radiation
Calculate the temperature of Black holes
Calculate the lifetimes of the Black holes
Roughly describe the background radiation and its temperature
Roughly describe the geometry of the Universe
Calculate the critical density
Roughly try to describe the missing mass and Dark matter
Roughly try to describe the vacuum energy

**Advices for reading**

Think about the methods used in Astrophysics often deal with the detection of starlight
Think of how Hubble’s law reformed the distance measurements to the stars
Reflect about how intensity measurements were made using the Cepheids
Think of the fact that the Schwarzschild radius is important for Black holes
Think about the difficulties to detect radiation from Black holes
Think about the critical density and its importance for the future of the Universe

**Readings**

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**WEB-readings**

- [http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html](http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html)
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