KTH Electrical Engineering

Exam in EG2050 System Planning, 20 May 2014, 14:00–19:00, Q24, Q26

Allowed aids

In this tentamen you are allowed to use the following aids:

- Calculator without information relevant to the course.
- One **handwritten**, **single-sided** A4-page with **your own** notes (original, not a copy), which should be handed in together with the answer sheet.

PART I (MANDATORY)

Write all answers on the answer sheet provided. Motivations and calculations do not have to be presented.

Part I can yield 40 points in total. The examinee is guaranteed to pass if the score is at least 33 points. If the result in part I is at least 31 points, then there will be a possibility to complement for passing the exam with the grade E.

Problem 1 (4 p)

Answer the following theoretical questions by choosing *one* alternative, which you find correct.

a) (1 p) Which of the following statements is the best description of the function of a balance responsible player in an electricity market?

1. A balance responsible player is responsible for safe operation of the power system.
2. A balance responsible player builds and maintains a regional or local grid.
3. A balance responsible player is economically responsible that the system during each trading period (for example one hour) is supplied as much energy as consumed by the customers of the player.

b) (2 p) The consumers in a bilateral electricity market has the following choices: I) They can choose which system operator they want, II) They can choose which retailer they want, III) They can choose which player should manage their balance responsibility.

1. Only I is true.
2. Only III is true.
3. I and II are true but not III.
4. II and III are true but not I.
5. All the statements are true.

c) (1 p) What does a take-and-pay contract mean?

1. The customer buys the same amount of energy in each trading period as long as the contract is valid.
2. During the time the contract is valid, the customer is allowed to consume as much energy they want each trading period, provided that the maximal power is not exceeded.
3. The customer buys energy at the power exchange. If the price of the power exchange exceeds a specified maximal price, the customer is receives a compensation corresponding to the difference between the power exchange price and the contracted maximal price.

Problem 2 (6 p)

Consider the common electricity market of the two countries Rike and Maa. Assume that there is perfect competition, that all players have perfect information, and that there are neither reservoir nor capacity limitations in the power plants. The power systems of Rike and Maa are interconnected by an HVDC line, which can transfer a certain 10 TWh per year. Assume that the electricity price in Rike is 370 ¤/MWh. Data for the power plants in Rike and Maa are shown in table 1. The variable production costs are assumed to be linear in the intervals; the production is zero if the price is on the lower price level and the production is maximal at the higher price level.

Table 1 Data for the power plants in Rike and Maa.

Power source	Production capability [TWh/year]		Variable cost [¤/MWh]
	Rike	Maa	
Hydro power	50	10	30–60
Nuclear power	50	20	100–120
Coal condensing	15	15	300–450
Gas turbines	5	5	800–1 000

a) (1 p) How much are the hydro power plants in Rike producing?

b) (1 p) How much are the gas turbines in Rike producing?

c) (2 p) How much are the coal condensing power plants in Rike producing?

d) (2 p) Assume that the elctricity consumption in Maa is 50 TWh/year. What will the electricity price be in Maa?

Problem 3 (6 p)

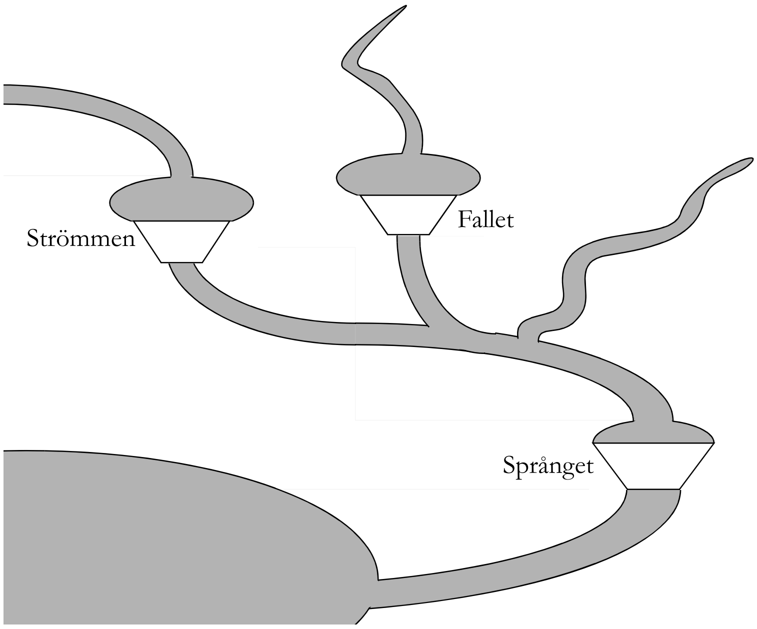
The hydro power plant Forsen has an installed capacity of 70 MW. To avoid damage on the turbines of the power plant, it is never allowed to generate less than 26 MW when it is committed. The power plant is part of the primary control of Land and has a gain of 200 MW/Hz. Forsen is producing 42 MW at 2 pm. The other power plants in Land that participate in the primary control has a total gain of 7 800 MW/Hz. The gain of the other power plants is available in the frequency range 50±0.5 Hz.

The frequency of the system is 50.190 Hz at 2 pm. Shortly thereafter a nuclear power plant in Land is started again after an earlier outage, which means that the power system is supplied another 1 030 MW.

a) (3 p) How much will Forsen produce when the primary control has restored the balance between generation and consumption in the system?

b) (3 p) What will the frequency be in Land? Answer with three decimals!

Problem 4 (12 p)



AB Vattenkraft owns three hydro power plant located as in the figure above. The following symbols have been introduced in a short-term planning problem for these hydro power plants:

Indices for the power plants: Strömmen 1, Fallet 2, Språnget 3.

γ_i = expected future production equivalent for water stored in reservoir i, i = 1, 2, 3,
λ_t = expected electricity price at ElKräng hour t, t = 1, …, 24,
λ_{25} = expected electricity price at ElKräng after the end of the planning period,
$\overline{M}_i$ = maximal contents of reservoir i, i = 1, 2, 3,
$M_{i,0}$ = contents of reservoir i at the beginning of the planning period, i = 1, 2, 3,
$M_{i,t}$ = contents of reservoir i at the end of hour t, i = 1, 2, 3, t = 1, …, 24,
$\mu_{i,j}$ = marginal production equivalent in power plant i, segment j, i = 1, 2, 3, j = 1, 2,
$\overline{Q}_{i,j}$ = maximal discharge in power plant i, segment j, i = 1, 2, 3, j = 1, 2,
$Q_{i,j,t}$ = discharge in power plant i, segment j, during hour t, i = 1, 2, 3, j = 1, 2, t = 1, …, 24,
$S_{i,t}$ = spillage from reservoir i during hour t, i = 1, 2, 3, t = 1, …, 24,
$V_{i,t}$ = local inflow to reservoir i during hour t, i = 1, 2, 3, t = 1, …, 24.

a) (4 p) All electricity generated by AB Vattenkraft is sold to the local power exchange, ElKräng. Formulate the objective function if the purpose of the planning is to maximise the income of generated hydro power plus the value of stored water. Use the symbols defined above.

b) (3 p) Formulate the limits for the optimisation variables in the short-term planning problem of AB Vattenkraft as defined above. To receive full score for this problem, you also have to state the possible values for each index value and limit!

c) (4 p) The best efficiency in the hydro power plant Språnget is obtained at the discharge 250 m^3/s and the production equivalent is then 1,00 MWh/HE. The maximal discharge is 375 m^3/s and then the relative efficiency is 96%. Assume that we need a piecewise linear model of electricity generation as function of the discharge in Språnget. The model should have two segments and the breakpoint between them should be located at the best efficiency. Calculate the following parameters:

$\mu_{3,j}$ = marginal production equivalent in Språnget, segment j,
$\overline{Q}_{3,j}$ = maximal discharge in Språnget, segment j.

d) (1 p) The following variables and parameters have been introduced in a short-term planning problem for a a thermal power plant:

C^* = start-up cost of the power plant after one hour down-time,
C^{**} = start-up cost of the power plant after at least two hours down-time,
G_t = generation in the power plant during hour t,
s_t^* = start-up of the power plant at the beginning of hour t after one hour down-time (1 if the power plant is started after one hour down-time, otherwise 0),
s_t^{**} = start-up of the power plant at the beginning of hour t after at least two hours down-time (1 if the power plant is started after at least two hours down-time, otherwise 0),
u_t = unit commitment of the power plant in hour t (1 if the power plant is committed, otherwise 0),
β = variable operation cost.

The following objective function is used in the planning problem:

$$\text{minimise} \quad \sum_{t \in \mathcal{T}} (\beta G_t + C^* s_t^* + C^{**} s_t^{**}).$$

Two linear constraints are needed in order to get correct values of s_t^* and s_t^{**} The first constraint forces s_t^{**} to be equal to one if the power plant is committed in hour t, while it has not been committed during hours $t - 1$ and $t - 2$ respectively:

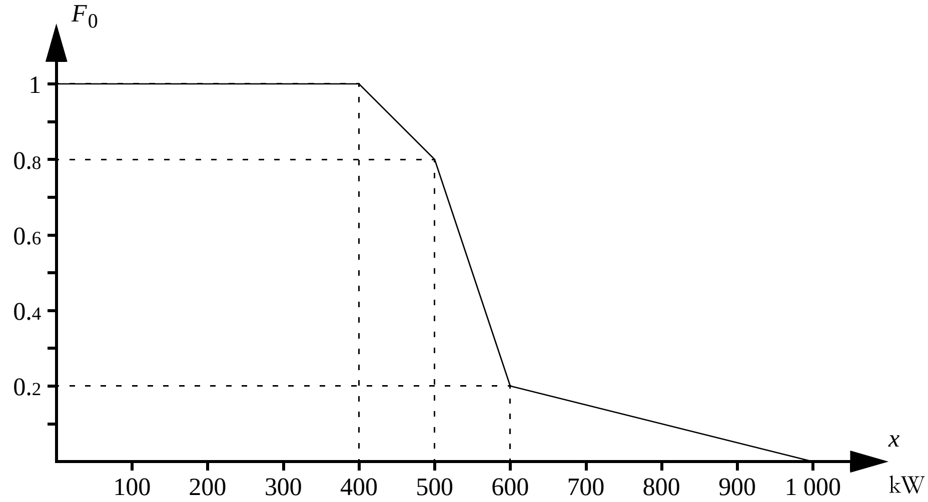
$$s_t^{**} \geq u_t - u_{t-1} - u_{t-2}.$$

The second constraint should force s_t^* to be equal to one if the power plant has been committed in hours t and $t - 2$ respectively, but has not been committed during hour $t - 1$. Which of the following relations should be used for the second constraint?

1. $s_t^* \geq u_t - u_{t-1} - s_t^{**}$.
2. $s_t^* \geq u_t - u_{t-1} - u_{t-2}$.
3. $s_t^* - u_{t-2} \geq u_t - u_{t-1}$.

Problem 5 (12 p)

Ekibuga is a town in East Africa. The town is not connected to a national grid, but has a local system of its own, which is supplied by a hydro power plant at nearby Omugga. The local grid also includes some smaller villages along the road between Omugga and Ekibuga. The hydro power plant does not have a reservoir, but the water flow is always sufficient to generate the installed capacity (800 kW) and the risk for outages in the power plant is negligible. The duration curve of the total load of the system is shown below.



a) (2 p) How large is the expected energy not served per hour for the system with only the hydro power plant?

b) (2 p) To improve the security of supply in the system, a diesel generator set is considered in Ekibuga. The planned diesel generator set would have 100 kW installed capacity, 80% availability and the operation cost 2 ¤/kWh. The expected energy not served when including both the hydro power plant and the diesel generator set is 4 kWh/h. Calculate the expected total operation cost per hour for the system with a hydro power plant and a diesel generator set.

c) (3 p) Another option to improve the security of supply is to build a wind power plant in Kasozi. The wind power plant would have 200 kW installed capacity and the available generation capacity is modelled according to table 2. What is the *LOLP* of the system with a hydro power plant and a wind power plant?

Hint: The convolution formula for a multi-state model reads

$$\tilde{F}_g(x) = \sum_{i=1}^{N_g} p_{g,i}\tilde{F}_{g-1}(x - x_{g,i}).$$

Table 2 Model of the wind power plant in Kasozi.

Available generation capacity [kW]	Probability [%]
0	20
50	30
100	20
150	20
200	10

d) (4 p) Assume that a combination of complementary random numbers and control variates is used to simulate the system with a hydro power plant and a diesel generator set. The simulation comprises 1 000 original scenarios, y_i, i = 1, …, 1 000. The corresponding complementary scenarios, y_i*, i = 1, …, 1 000, have also been generated. The simplified model $\tilde{g}(Y)$, corresponds to the model used in probabilistic production cost simulation, whereas the detailed model, $g(Y)$, considers factors such as the losses being dependent on which power plants that are operated and that the load is varying in different parts of the system. The results are shown in table 3. Which estimate of *ETOC* is obtained for the detailed model?

Table 3 Results from the Monte Carlo simulationin problem 5d.

Detailed model		Simplified model	
Total operation cost in the original scenarios, $\sum_{i=1}^{1000} g(y_i)$ [¤/h]	Total operation cost in the complementary scenarios, $\sum_{i=1}^{1000} g(y_i^*)$ [¤/h]	Total operation cost in the original scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i)$ [¤/h]	Total operation cost in the complementary scenarios, $\sum_{i=1}^{1000} \tilde{g}(y_i^*)$ [¤/h]
45 120	43 550	12 120	11 950

e) (1 p) Assume that stratified sampling is used instead of complementary random numbers and control variates in a simulation of the same system. Results for each stratum are given in table 4. Which estimate of *LOLP* is obtained for this simulation?

Table 4 Results from the Monte Carlo simulation in problem 5e.

Stratum, h	1	2	3	4	5
Stratum weight, ω_h	0.85	0.05	0.02	0.02	0.06
Estimated risk of power deficit, m_{LOLOh}	0	0	0	0.25	1

PART II (FOR HIGHER GRADES)

All introduced symbols must be defined. Solutions should include sufficient detail that the argument and calculations can be easily followed.

The answer to each problem must begin on a new sheet, but answers to different parts of the same problem (a, b, c, etc.) can be written on the same sheet. The fields *Namn* (Name), *Blad nr* (Sheet number) and *Uppgift nr* (Problem number) must be filled out on every sheet.

Part II gives a total of 60 points, but this part will only be marked if the candidate has obtained at least 33 points in part I. Then the results of parts I and II and the bonus points will be added together to determine the examination grade (A, B, C, D, E).

Problem 6 (10 p)

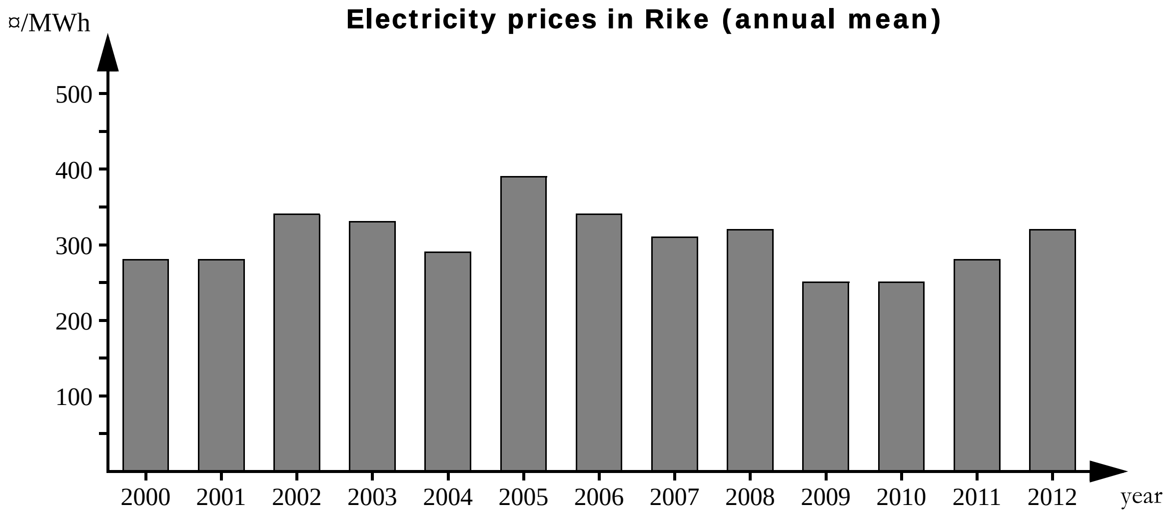
Today the wind power generation in Rike is about 1 TWh, but there are considerations for a large-scale expansion. To support this venture, it has been proposed that Rike should introduce a system with green certificates. The owner will receive a green certificate for each MWh generated in the new wind power plants. The consumers are then obliged to buy certificates corresponding to 10% of their electricity consumption, which means that a consumer with an annual consumption of 100 MWh will have to buy 10 green certificates per year. The Rikish Energy Agency estimates that this would result in an expansion of 4 000 MW wind power, having an average annual generation of 10 TWh vindkraft. The price of certificates is estimated to 250 ¤/certificate.

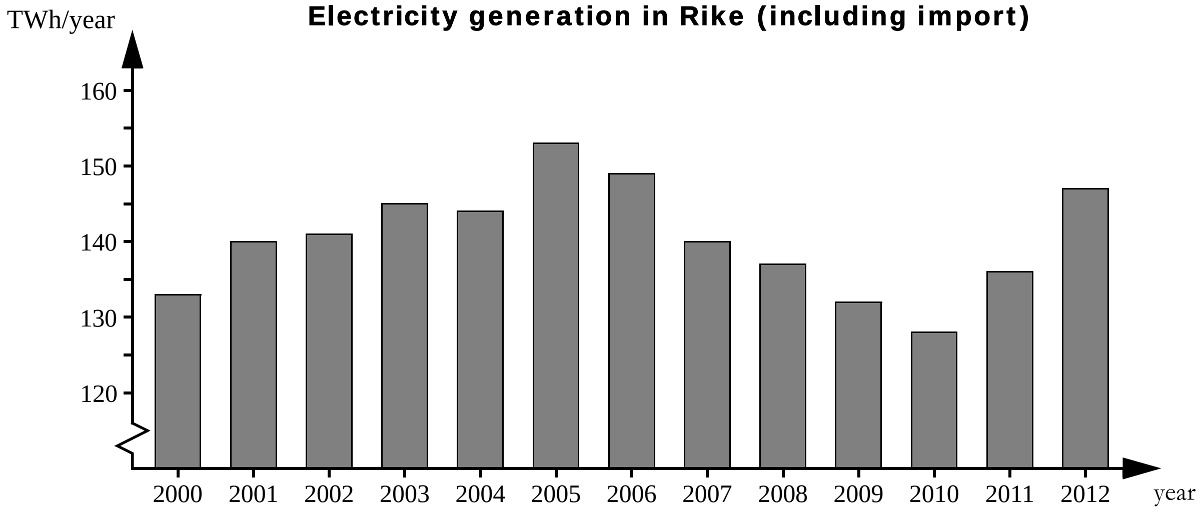
Due to these plans The Rike Association for Protection of Views (an organisation that is well-known opponents of wind power) has written a polemical article in the largest paper in Rike. The article is concluded as follows: "Wind power is of no use whatsoever to the electricity supply in Rike and it is very hard to understand why the people in Rike should use at least 2.5 billions ¤ to subsidise destroyed forests, blast devastated rocks, trailer truck roads in the mountains and destroyed world heritage sites!"

Use the statistics on the next page and make your own analysis of the consequences of a wind power expansion for the consumers in Rike.

NOTICE! The answers to this problem will be evaluated not on the conclusion of whether wind power in Rike is profitable or not, but on how well you can reason around and evaluate the statements above.

Hint: Are there any other factors increasing or decreasing the costs due to wind power except those mentioned above?





Problem 7 (10 p)

Consider a power system comprised by the three countries Aland, Beland and Celand, which are interconnected by AC transmission lines. Each transmission line is equipped with a protection system which after a short time delay disconnects the line if the power flow exceeds the maximal capacity of the line. Further data about the system are given in table 5.

The primary control in the three countries is divided in a normal operation reserve and a disturbance reserve. The normal operation reserve, which is used to manage normal variations in for example load and wind power generation, is available in the frequency range 49.9–50.1 Hz, whereas the disturbance reserve is used to manage outages in larger power plants and is available in the frequency reange 49.5–49.9 Hz. Data for the primary control is also supplied in table 5.

a) (4 p) At a certain occasion there is balance between production and consumption and the frequency in the system is 49.96 Hz. Due to a leakage in the coolant system, a nuclear power plant in area A must be stopped immediately, which means that 900 MW of generation is lost. What will the new frequency become in each area?

Table 5 Data for the power system.

Country	Normal operation reserve [MW/Hz]	Disturbance reserve [MW/Hz]	Transmission line			
			To	Type	Capacity [MW]	Current power flow [MW]
Aland	800	700	Beland	AC	1 000	Import 200
Beland	1 500	700	Aland Celand	AC AC	1 000 800	Export 200 Export 400
Celand	700	600	Beland	AC	800	Import 400

b) (6 p) The three countries have common real-time balancing market, and the three system operators shares the costs of activating bids from this market. Consequently, they always choose to activate the bids that give the least total cost, independent on in which country the bid is activated.

To relieve the primary control after the situation described in part a, it is necessary to activate some up-regulation. The available up-regulation bids are listed in table 6. The bids do not have to be accepted as a whole, but the system operators may choose how many MW that should be activated in each bid. In total, the system operators wishes to activate 900 MW up-regulation. Which bids should be activated if the up-regulation should be carried out minimising the total costs without overloading any transmission lines?

Table 6 Available up-regulation bids.

Bid	Up-regulation		
	Maximal volume [MW]	Price [¤/MWh]	Country
1	250	410	Celand
2	250	415	Celand
3	200	420	Beland
4	150	430	Celand
5	200	450	Aland
6	200	480	Beland

Problem 8 (20 p)

The electricity company Stads energi AB owns the oil condensing power plant Röksta as well as a wind farm. The oil condensing power plants has two blocks. The company has decided that it is not profitable to shut down a block for just an hour; therefore, they have decided that a block should not be restarted within less than two hours. More data about Röksta are given in table 7.

The company has a firm power contract of selling 100 MWh/h to AB Elleverantören. Stads energi AB supplies this amount either by the power plants owned by the company or by trading at the local power pool ElKräng. In order to submit the bids for ElKräng in time, Stads energi AB must determine the operation plan for Monday already during Sunday morning. The operation of Röksta during Sunday is shown in table 8. When planning the operation for Monday, the company uses the forecasts of electricity prices and wind power generation described in table 9. It is also assumed that it is possible to purchase and sell unlimited quantities at the forecasted prices.

a) (10 p) Formulate the planning problem of Stads energi AB as a MILP problem. Use the notation in table 10 for the parameters (it is however permitted to add further symbols if you consider it necessary).

NOTICE! The following is required to get full score for this problem:

- The symbols for the optimisation variables must be clearly defined.
- The optimisation problem should be formulated so that it is easy to determine what the objective function is, which constraints there are and which limits there are.
- The possible values for all indices should be clearly stated for each equation.

b) (10 p) The figure on the next page swhows the efficiency as a function of the electricity generation in Röksta block I. Assume that the oil costs 1 860 SEK/ton and that the heat contents of the oil is 11.9 kWh/kg. Assume that the generation cost as function of the generation should be modelled using a piecewise linear model. The model should have two segments, and it should re-

Table 7 Data for the thermal power plant Röksta.

	Block I	Block II
Installed capacity[MW]	120	96
Minimal generation when committed [MW]	40	30
Generation cost[SEK/MWh]	460	490
Start-up cost [SEK/start]	36 000	31 000
Minimal down time [h]	2	2

Table 8 Operation plans for Röksta during Sunday.

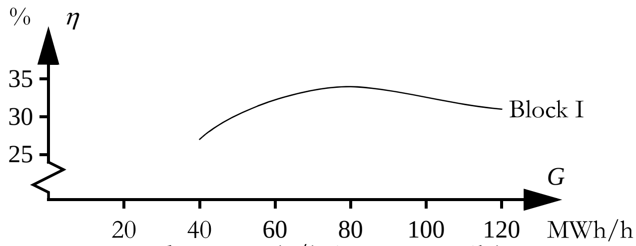
Time	Generation [MWh]	
	Block I	Block II
0-5	0	0
5-22	120	0
22-24	0	0

Table 9 Forecasts for Monday.

Hour	0-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12
Price at ElKräng [SEK/MWh]	448	446	444	436	442	448	471	523	566	558	529	489
Wind power generation [MWh]	11	11	9	9	9	8	8	7	6	6	5	5
Hour	12-13	13-14	14-15	15-16	16-17	17-18	18-19	19-20	20-21	21-22	22-23	23-24
Price at ElKräng [SEK/MWh]	470	464	457	453	452	468	513	483	460	458	452	448
Wind power generation [MWh]	4	4	5	6	8	7	5	5	7	10	15	19

Table 10 Notation for the planning problem of Stads energi AB.

Symbol	Explanation	Value
$\overline{G}_g$	Installed capacity in block *g*	See table 7
$\underline{G}_g$	Minimal generation when block *g* is committed	See table 7
β_{Gg}	Variable generation cost in block *g*	See table 7
C_g^+	Start-up cost in block *g*	See table 7
$u_{g,0}$	Unit commitment in block *g* before the start of the planning period	0 (cf. table 8)
λ_t	Expected electricity price at ElKräng hour *t*	See table 9
W_t	Expected wind power generation hour *t*	See table 9
D_t	Contracted load	100



flect that generation between 0 and 40 MWh/h is not possible. How must the planning problem from part a be reformulated in order to consider this model? Do not forget to define all new variables and parameters that you introduce, and to motivate the choice of breakpoint between the two segments!

Hint: The generation cost for a certain hour can be expressed as

$$\beta_{u1}u_{1,t} + \beta_{G1,1}G_{1,1,t} + \beta_{G1,2}G_{1,2,t},$$

where β_{u1}, $\beta_{G1,1}$ and $\beta_{G1,2}$ are parameters and $u_{1,t}$, $G_{1,1,t}$ and $G_{1,2,t}$ are optimisation variables.

Problem 9 (20 p)

Akabuga is a town in East Africa. Despite many promises from the electricity company in Eggwanga, Akabuga is still not connected to the national grid. A number of entrepreneurs and citizens in Mji are therefore considering to start a cooperative, Akabuga Electricity Consumers Cooperative Limited (AECCO), which will build and operate a local power system in Akabuga, The idea is that members will be allowed to connect to the local grid and that all members should pay a uniform tariff per consumed kWh. This tariff, which will be the only income of AECCO, should be set to cover all costs (fixed and variable) in an annual basis. Moreover, the tariffs should generate a surplus around 5%, which can be used for future investments in the power system.

AECCO considers two options to supply the local grid. The first option is to procure three diesel generator sets, each having a capacity of 250 kW and 90% availability. The diesel generator sets would represent an investment cost of in total 600 000 ¤/year and would have the variable cost 10 ¤/kWh. The other option is to build an 800 kW hydro power plant in Ekiyira, which is located at some distance from Akabuga. The variable costs and risk of outages are assumed to be negligible for the hydro power plant. On the other hand, there would be some losses on the line between Ekiyira and Akabuga. These losses can be computed according to

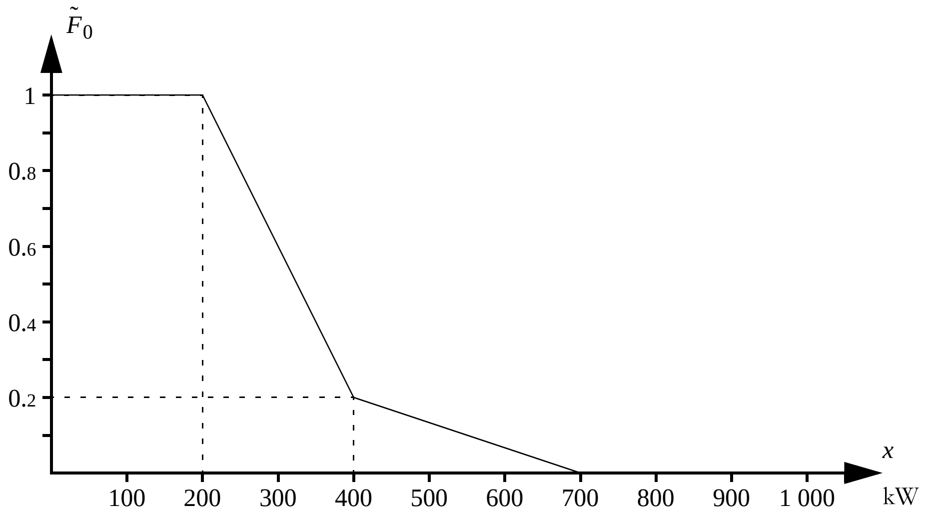
$$L = \beta_L \cdot P^2,$$

where

L = losses on the line [kW],
β_L = loss coefficient [kW^{-1}] = 0.00015,
P = injected power on the line [kW].

The annual cost of the hydro power plant (investment cost for the plant itself and the line, salaries for staff and maintenance costs, etc.) would be 25 M¤.

Based on experience from other parts of Eggwanga, the following load duration curve has been estiamted for Akabuga:



a) (8 p) Suggest a method to compute the expected operation cost of the power system in Akabuga. Describe which assumptions you make and what you can do to obtain a result that is as accurate as possible considering the limited data given above and the limited time you have at disposal.

b) (12 p) Should AECCO procure diesel generator sets or build the hydro power plant? Assume that the fixed costs for the copperative (investments in the local distribution system, salaries for the staff, etc.) are 23 M¤/year. The losses in the distribution grid in Akabuga can be considered negligible.

Table 11 Random numbers from a $U(0, 1)$ distribution.

0.81	0.63	0.96	0.96	0.42	0.66	0.68	0.66	0.28	0.69
0.91	0.10	0.97	0.49	0.92	0.04	0.76	0.17	0.05	0.32
0.13	0.28	0.16	0.80	0.79	0.85	0.74	0.71	0.10	0.95
0.91	0.54	0.97	0.14	0.96	0.93	0.39	0.03	0.82	0.03

KTH Electrical Engineering

Answer sheet for part I

Name: ....

Personal number: ....

Problem 1

a) Alternative .... is correct.

b) Alternative .... is correct.

c) Alternative .... is correct.

Problem 2

a) .... TWh/year **b)** .... TWh/year

c) .... TWh/year **d)** .... ¤/MWh

Problem 3

a) .... MW **b)** .... Hz

Problem 4

a) ....

b) ....

....

....

c) $\mu_{3,1}$ .... MWh/HE $\mu_{3,2}$ .... MWh/HE

$\bar{Q}_{3,1}$ .... HE $\bar{Q}_{3,2}$ .... HE

d) Alternative .... is correct.

Problem 5

a) .... kWh/h **b)** .... ¤/h

c) .... % **d)** .... ¤/h

e) .... %

Problem 1

a) 3, **b)** 4, **c)** 2.

Problem 2

a) As the electricity price is higher than the variable operation cost in the most expensive hydro power plant, all hydro power in Rike will be utilised, i.e., 50 TWh/year.

b) As the electricity price is lower than the variable operation cost in the least expensive gas turbine, no gas turbines in Rike will be used.

c) The part of the coal condensing which has a lower variable operation cost than the electricity price 370 ¤/MWh will be used, i.e., (370 – 300)/(450 – 300) · 15 = 7 TWh/year.

d) Assume that we have the same electricity price in Maa as in Rike, i.e., 370 ¤/MWh. At this price there will be 10 TWh hydro generation, 20 TWh nuclear and 7 TWh coal condensing, which results in a total generation of 37 TWh. The remaining 13 TWh must then be imported from Rike. However, this is not possible, as the transmission capacity between the countries is only 10 TWh/year. Hence, we can conclude that the electricity price in Maa must be higher than the electricity price in Rike. We will therefore have in total 40 TWh from hydro, nuclear and import, which means that we need 10 TWh kolkondens i Maa. Thus, we use 10/15 of the price interval for coal condensing, i.e., the electricity price in Maa must be 300 + 10/15 · 150 = 400 ¤/MWh.

Problem 3

a) The power plants that are part of the primary control compensates the start of the nuclear power plant by decreasing the generation by in total 1 030 MW. The contribution of each individual power plant is determined by its share of the total gain. Forsen corresponds to 2.5% of the gain in the system; hence, it should reduce the generation by 25.75 MW. However, the power plant only has the possibility to reduce by 16 MW. Consequently, the power plant will generate 26 MW (least possible power when the unit is committed).

b) The other power plants in the system must reduce the generation by 1 014 MW, which requires a that the frequency is increasing by 1 014 MW/7 800 MW/Hz = 0.13 Hz. The new frequency in the system is then 50.190 + 0.130 = 50.320 Hz.

Problem 4

a) maximise $\sum_{t=1}^{24} \lambda_t \sum_{i=1}^{3} \sum_{j=1}^{2} \mu_{i,j} Q_{i,j,t} + \lambda_{25}((\gamma_1 + \gamma_3)M_{1,24} + (\gamma_2 + \gamma_3)M_{2,24} + \gamma_3 M_{3,24})$.

b) The optimisation variables involved in the problem are reservoir contents, discharge and spillage, which yields the following limits:

$0 \le M_{i,t} \le \overline{M}_i, \qquad i = 1, 2, 3, t = 1, \ldots, 24,$

$0 \le Q_{i,j,t} \le \overline{Q}_{i,j} \qquad i = 1, 2, 3, j = 1, 2, t = 1, \ldots, 24,$

$0 \le S_{i,t}, \qquad i = 1, 2, 3, t = 1, \ldots, 24.$

c) The following data are given in the problem text:

$\overline{Q}$ = maximal discharge in Språnget = 375,
$\hat{Q}$ = discharge in Språnget at best efficiency = 250,
γ_{max} = maximal production equivalent in Språnget = 1,
$\eta(\overline{Q})$ = relative efficiency at maximal discharge in Språnget = 0.96.

To calculate the marginal production equivalents, we need the electricity generation at the corresponding discharges. These are calculated using the formula $H = \gamma_{max} \cdot \eta(V) \cdot V$:

$\hat{H}$ = electricity generation in Språnget at best efficiency = 1 · 1 · 250 = 250,
$\overline{H}_i$ = maximal electricity generation in Språnget = 1 · 0.96 · 375 = 360.

The marginal production equivalents are now calculated according to

$$\mu_1 = \frac{\hat{H}}{\hat{Q}}$$

and

$$\mu_2 = \frac{\overline{H} - \hat{H}}{\overline{Q} - \hat{Q}},$$

which results in the following linear model of the power plant:

$$\mu_j = \text{marginal production equivalent in Språnget, segment } j = \begin{cases} 1.00 & j = 1, \\ 0.88 & j = 2, \end{cases}$$

$$\overline{Q}_j = \text{maximal discharge in Språnget, segment } j = \begin{cases} 250 & j = 1, \\ 125 & j = 2. \end{cases}$$

d) 1.

Problem 5

a) The unserved energy during an hour is given by

$$EENS_1 = 1 \cdot \int_{800}^{\infty} \tilde{F}_1(x)dx = 0.1 \cdot 200/2 = 10 \text{ kWh/h}.$$

b) The expected generation in the diesel generator set is $EG_2 = EENS_1 - EENS_2 = 6$ kWh/h. Hence, the expected operation cost is $ETOC = 2EG_2 = 12$ ¤/h.

c) $LOLP = \tilde{F}_2(1\,000) = 0.1\tilde{F}_1(1\,000) + 0.2\tilde{F}_1(950) + 0.2\tilde{F}_1(900) + 0.3\tilde{F}_1(850) + 0.15\tilde{F}_1(800) = 0.1 \cdot 0 + 0.2 \cdot 0.025 + 0.2 \cdot 0.05 + 0.3 \cdot 0.075 + 0.2 \cdot 0.1 = 5.75\%$.

d) In practice there is no need to differentiate between observations based on the original scenarios and the complementary scenarios; the expected difference between the detailed and simplified models is given by

$$m_{(TOC-\tilde{TOC})} = \frac{1}{2000}\left(\sum_{i=1}^{1000}(g(y_i) - \tilde{g}(y_i)) + \sum_{i=1}^{1000}(g(y_i^*) - \tilde{g}(y_i^*))\right) = 32.3 \text{ ¤/h}.$$

The expectation value of the simplified model (which corresponds to a PPC model) was calculated to 12 ¤/h in part b. The estimate of *ETOC* for the detailed model is thus

$$m_{TOC} = m_{(TOC-\tilde{TOC})} + \mu_{\tilde{TOC}} = 32.3 + 12 = 44.3 \text{ ¤/h}.$$

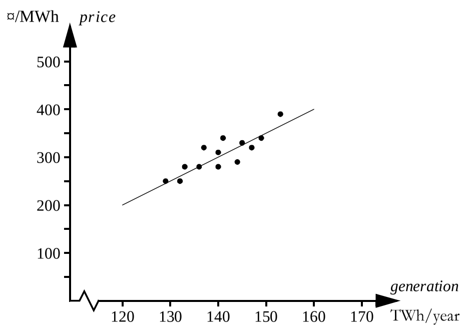
e) $m_{LOLO} = \sum_{h=1}^{5} \omega_h m_{LOLOh} = 0 + 0 + 0 + 0.02 \cdot 0.25 + 0.06 \cdot 1 = 6.5\%$.

Problem 6

Considering that we do not have much information about the power system in Rike, we will have to assume that the data presented are correct; apparently, the Rike Association for Protection of Views do not seem to question the numbers of the Energy Agency, as the cost that the association is referring to corresponds to the cost of 10 TWh green certificates at the price 250 ¤/certificate. Obviously, this represents a cost increase for the consumers in Rike, namely 25 ¤/MWh to be precise.

As wind power generation is varying continuously, there will also be an increased need for frequency control, which of course also represents a cost. We are however not able to comment on the size of these costs based on the data provided in the problem text.

Wind power is however not only causing increased costs, but will also decrease the electricity price in Rike. How much the price will decrease depends on the profile of the variable costs in the power plants that can be replaced by wind power, as well as the price sensitivity of the demand. Based on the statistics provided, we can make a rough estimate of how the electricity price is affected for each TWh of wind added to the system. We do this by assuming that there is a linear relation between the electricity price and demand. This relation can be estimated by drawing the price as a function of the generation:



Based on this estimate, it seems as if the generation in the other power plants is reduced by 10 TWh/year then the electricity price decreases by 50 ¤/MWh.

Thus, we can conclude that the costs for the consumers will increase due to the green certificates, but the consumers will also receive lower electricity prices in the same size of order or even larger than the costs of certificates. Besides, there will be increased costs for frequency control and grid expansion (which at the end of the day will be paid by the consumers). In addition to the economical perspective, the value of wind power also depends on the priority of different objectives for the environment (lower carbon dioxide emissions, value of untouched nature, etc..). Here we lack way too much information in order to draw any conclusions whether wind power is profitable or not for Rike.

Problem 7

a) When the nuclear power plant is disconnected, the normal operation reserve will be activated first. The total gain in the normal operation reserve is 3 000 MW/Hz and the frequency can drop by 0.06 Hz before the normal operation reserve is fully utilised; this means that the normal operation reserve will compensate 3 000 · 0.06 = 180 MW of the outage. The remaining 720 MW of the outage will then be covered by the disturbance reserve. As the total gain in the disturbance reserve is 2 000 MW/Hz the frequency will drop by another 720/2 000 = 0.36 Hz, which means that the frequency will be stabilised at 49.54 Hz.

We must now check that the interconnections can manage the resulting power flows. Each country will increase its generation according to its share of the total gain of the normal operation reserve and disturbance reserve respectively:

$$\Delta G_A = \frac{800}{3\,000} \cdot 180 + \frac{700}{2\,000} \cdot 720 = 300 \text{ MW},$$

$$\Delta G_B = \frac{1\,500}{3\,000} \cdot 180 + \frac{700}{2\,000} \cdot 720 = 342 \text{ MW},$$

$$\Delta G_C = \frac{700}{3\,000} \cdot 180 + \frac{600}{2\,000} \cdot 720 = 258 \text{ MW}.$$

The new power flows are then

$$P_{B\to C} = 400 - \Delta G_C = 142 \text{ MW},$$

$$P_{B\to A} = 200 + \Delta G_B + \Delta G_C = 800 \text{ MW}.$$

These flows are within the capacity limits of each line and thus the system will be in balance at the frequency 49.54 Hz.

b) We supply the system as much new generation as the earlier outage; hence, we will have the same situation as before the error, except that some generation in Aland has been shifted to Beland and Celand. If we activate the least expensive bids (i.e., bids 1–4 as well as 50 MW from bid 5) then we will add in total 650 MW i Celand, which results in the power flow $P_{B\to C} = 400 - 650 = -250$ MW, which does not exceed the capacity of the line. In Beland we will add 200 MW, which results in the flow $P_{B\to A} = 200 + 200 + 650 = 1\,050$ MW—this flow exceed the capacity of the line by 50 MW. To stay within the limits, another 50 MW must be supplied in Aland, while we decrease the up-regulation by 50 MW in Celand (as bid 4 is the most expensive of the bids outside Aland). Thus, we should activate all from bids 1–3, 100 MW from bid 4 and 100 MW from bid 5.

Problem 8

a) The problem we want to solve is

maximise *value of sold electricity – cost of purchased electricity*

+ value of stored water
– generation cost in Röksta – start-up cost in Röksta,

subject to *hydrologiscal balance for Forsen,*
limitations in generation capacity,
relation between unit committment, start and stop in Röksta,
requirement of minimal down time in Röksta,
load balance.

Indices for thermal power plants

Röksta block I - 1, Röksta block II - 2.

Parameters

The parameters are defined in table 10 of the exam.

Optimisation variables

$G_{g,t}$ = generation in block g during hour t, t = 1, …, 24,
$s^+_{g,t}$ = start-up of block g in the beginning of hour t, t = 1, …, 24,
$s^-_{g,t}$ = stop of block g in the beginning of hour t, t = 1, …, 24,
$u_{g,t}$ = unit commitment in block g during hour t, t = 1, …, 24,
p_t = purchase from ElKräng during hour t, t = 1, …, 24,
r_t = sales to ElKräng during hour t, t = 1, …, 24.

Objective function

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t(r_t - p_t) - \sum_{t=1}^{24}\sum_{g=1}^{2} (\beta_{Gg} G_g + C_g^+ s^+_{g,t}).$$

Constraints

Maximal generation in the thermal power plants:

$$G_{g,t} \le u_{g,t}\overline{G}_g, \qquad g = 1, 2, t = 1, …, 24.$$

Minimal generation in the thermal power plants:

$$u_{g,t}\underline{G}_g \le G_{g,t}, \qquad g = 1, 2, t = 1, …, 24.$$

Relation between unit commitment, start-up and stop in the thermal power plants:

$$u_{g,t} - u_{g,t-1} = s^+_{g,t} - s^-_{g,t}, \qquad g = 1, 2, t = 1, …, 24.$$

Requirement on minimal down time:

$$s^-_{g,t} + s^+_{g,t+1} \le 1, \qquad g = 1, 2, t = 1, …, 23.$$

Load balance:

$$G_{1,t} + G_{2,t} + p_t - r_t = D_t - W_t, \qquad t = 1, …, 24.$$

Variable limits

$s^+_{g,t} \in \{0, 1\}$, $g = 1, 2, t = 1, …, 24,$
$s^-_{g,t} \in \{0, 1\}$, $g = 1, 2, t = 1, …, 24,$
$u_{g,t} \in \{0, 1\}$, $g = 1, 2, t = 1, …, 24,$
$0 \le p_t$, $t = 1, …, 24,$
$0 \le r_t$, $t = 1, …, 24.$

b) The generation in block I must be divided in two segments. The breakpoint between the two segments should be lactated at best efficiency, as the optimal solution of LP as well as MILP problems favours the maximal and minimal values of the optimisation variables.

Introduce the following optimisation variables:

$G_{1,j,t}$ = generation in I, segment j, during hour t, j = 1, 2, t = 1, …, 24.

Notice that the lower limit of the generation in each segment is 0.

To calculate the generation cost we must determine the marginal cost for electricity generation in each segment, i.e.,

$\beta_{G1,j}$ = marginal generation cost in block I, segment j, j = 1, 2.

To determine the marginal costs we must know the generation cost for three levels of the generation: minimal generation, best efficiency and maximal generation. Using the formula

$$C(G) = \phi\frac{G}{h \cdot \eta(G)},$$

where the following parameters are given in the problem:

Φ = fuel price = 1 860,
h = heat contents of the fuel = 11.9,
$$\eta(G) = \text{efficiency at the generation } G = \{\text{cf. the figure}\} = \begin{cases} 0.27 & g = 40, \\ 0.34 & g = 80, \\ 0.31 & G = 120, \end{cases}$$

we can calculate

$C(40) \approx 23\,156$ SEK, $C(80) \approx 36\,777$ SEK, $C(80) \approx 60\,504$ SEK.

Hence, we get the marginal generation costs

$$\beta_{G1,1} = \frac{C(80) - C(40)}{80 - 40} \approx 340.53, \beta_{G1,2} = \frac{C(120) - C(80)}{120 - 80} \approx 593.18.$$

If both $G_{1,1,t}$ and $G_{1,2,t}$ are 0 then the power plant is generating 40 MW if it is committed, i.e., if $u_{1,t} = 1$, and 0 otherwise. As $C(40) \approx 23\,156$ we introduce a parameter $\beta_{u1} = 23\,156$, representing the cost to generate 40 MW. This cost is added to the generation cost of each segment. This gives us the following new objective function:

$$\text{maximise} \quad \sum_{t=1}^{24} \lambda_t(r_t - p_t) - \sum_{t=1}^{24}\left(\sum_{g=1}^{2} C_g^+ s^+_{g,t} + \beta_{u1}u_{1,t} + \sum_{j=1}^{2} \beta_{G1,j}G_{1,j,t} + \beta_{G2}G_{2,t}\right).$$

It is not possible to generate anything in the two segments if the power plant is not committed, this is represented by the following constraint:

$$G_{1,j,t} - u_{1,t}\overline{G}_{1,j} \le 0, \qquad j = 1, 2, t = 1, …, 24.$$

where

$\overline{G}_{1,j}$ = maximal generation in block I, segment j = 40, j = 1, 2.

We must also state the lower limits of the new optimisation variables:

$0 \le G_{1,j,t}$, $j = 1, 2, t = 1, …, 24.$

Finally, it should be noted that generation in segment 1 is more profitable than generation in segment 2; hence, it is not necessary to introduce an extra integer variable to guarantee that $G_{1,2,t} = 0$ although $G_{1,1,t} < \overline{G}_{1,1}$.

Problem 9

a) The tariff if AECCO would choose the hydro power plant can easily be determined directly from the data provided. This is because the maximal losses are $0.00015 \cdot 800^2 = 96$ kW, which means that even if losses are included, the hydro power plant will always have sufficient capacity to cover the load in Akabuga and consequently will the variable costs always be negligible.

The alterantive using diesel generator sets is well suited for probabilistic production cost simulation. The challenge to carry out such a simulation in this context is that quite a lot of computations are required to compute the equivalent load duration curve. This can either be solved by using a suitable discrete approximation of the continuous load duration curve provided in the assignment. However, by ignoring computing partial results that are of no importance for the final result, we can manage also a simulation using the continuous duration curve. To compute the expected generation in the diesel generator sets we need *EENS* with and without the three diesel generator sets, i.e.,

$$T\int_{0}^{\infty}\tilde{F}_0(x)dx$$

and

$$T\int_{750}^{\infty}\tilde{F}_3(x)dx.$$

The first expression is given, whereas the other requires that we compute the equivalent load duration curve including outages in all three units for the interval $x \geq 750$. To save some more work, we can consider the three diesel generators as one single power plant with four possible states (they have the same variable costs; thus, there is no need to differentiate them in this case).

b) To simulate the diesel generator set option, we choose to consider the three diesel generators as one power plant with four states. The probability of each state can be compiled from the following table:

Diesel generator 1	0	250	0	250	0	250	0	250
Diesel generator 2	0	0	250	250	0	0	250	250
Diesel generator 3	0	0	0	0	250	250	250	250
Total capacity	0	250	250	500	250	500	500	750
Probability	$0.1^3 = 0.001$	$0.1^2 \cdot 0.9 = 0.009$	$0.1^2 \cdot 0.9 = 0.009$	$0.9^2 \cdot 0.1 = 0.081$	$0.1^2 \cdot 0.9 = 0.009$	$0.9^2 \cdot 0.1 = 0.081$	$0.9^2 \cdot 0.1 = 0.081$	$0.9^3 = 0.729$

The equivalent load duration curve including outages in all three diesel generators is then given by

$$\tilde{F}_3(x) = 0.729\tilde{F}_0(x) + 0.243\tilde{F}_0(x-250) + 0.027\tilde{F}_0(x-500) + 0.001\tilde{F}_0(x-750).$$

Hence, we can compute

$$EENS_0 = 8\,760\int_{0}^{\infty}\tilde{F}_0(x)dx = 8\,760\cdot(1\cdot200 + (1+0.2)/2\cdot200 + 0.2/2\cdot300) = 8\,760\cdot350 =$$

$$= 3.066 \text{ GWh/year.}$$

$$EENS_3 = 8\,760\int_{750}^{\infty}\tilde{F}_3(x)dx =$$

$$= 8\,760\int_{750}^{\infty}(0.729\tilde{F}_0(x) + (0.243)\tilde{F}_0(x-250) + 0.027\tilde{F}_0(x-500) + 0.001\tilde{F}_0(x-750)) =$$

$$= 8\,760\left(\int_{750}^{\infty}0.729\tilde{F}_0(x) + \int_{500}^{\infty}0.243\tilde{F}_0(x) + \int_{250}^{\infty}0.027\tilde{F}_0(x) + \int_{0}^{\infty}0.001\tilde{F}_0(x)\right) =$$

$$= 8\,760(0.729\cdot0 + 0.243\cdot(0.1667/2\cdot200) + 0.027\cdot((0.8+0.2)/2\cdot150 + 0.2/2\cdot300)$$
$$+ 0.001\cdot(1\cdot200 + (1+0.2)/2\cdot200 + 0.2/2\cdot300) = 0.056 \text{ GWh/year,}$$

which results in the expected generation $EG_3 = EENS_3 - EENS_2 = 3{,}010$ GWh/year. The variable costs are then 30.10 M¤/year. If they instead choose to build the hydro power plant, there will be no variable costs, but an additional fixed cost of 25 M¤/year. The total costs will thus be less with the hydro power plant and will also be distributed on a larger volume (since the unserved energy is negligible for the hydro power plant, whereas it amount to about 56 MWh/year for the diesel generator sets). Therefore, we can conclude that the hydro power plant is the better option without even looking at the other costs.