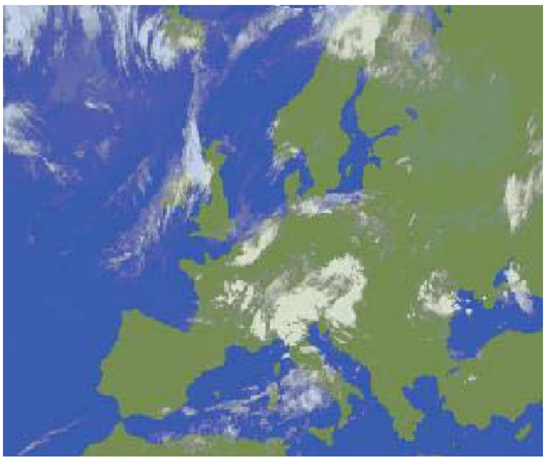
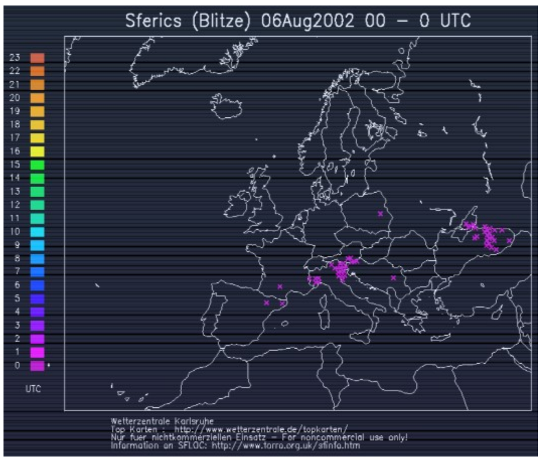
Environmental Science. Physics and Applications

Chapter 5. Weather and Climate

5. Weather and Climate

Here we will discuss weather and climate, which are subjects that are not too easy to discuss and to perform calculations on. People are interested in temperature, high and low pressures, sunshine, rain, snow, wind and so on. We are also interested in looking at climate changes over short and over longer periods and try to study if we can deduce any trends in the climate changes.





Data representing what we call *the weather* is collected by a large amount of measuring units both on ground but also via satellites, balloons, lasers etc.
While the *weather* for us means sunshine, rain, snow, temperature, clouds as we see and hear from daily forecasts, while the word *climate* stands for average of measurable entities over longer periods as years.
One way of trying to perform these studies is to use physical concepts regarding energy transport phenomena and to study both vertical and horizontal movements of the atmosphere.

5.1 Energy transport

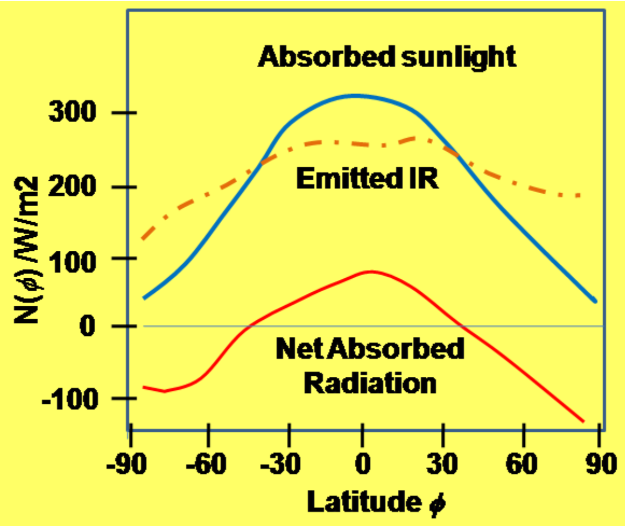
In Chapter 2, we discussed the incoming sunlight through the atmosphere and the corresponding outgoing infrared radiation from the Earth. We found that

The blackbody radiation from Earth has a wavelength maximum at
$\lambda_{\max Earth} = 2898 \times 10^3 \frac{1}{T}$ nm $\lambda_{\max} = 2898 \times 10^3 \frac{1}{288}$ nm $\approx$ 10060 nm $\approx$ 10 μm that we compared with the wavelength maximum from the solar radiation at
$\lambda_{\max Sun} = 2898 \times 10^3 \frac{1}{5800}$ nm $\approx$ 499.7 nm $\approx$ 500 nm
The emitted radiation from the Earth has an intensity maximum around 25 W/(m^2μm) using the Planck formula and the corresponding intensity of the Sun is 0.85×10^8 W/(m^2μm). We also evaluated the total emission by using the Stefan-Bolzmann law where

$e = \sigma T^4$ $\sigma = 5.67 \times 10^{-8}$ W/(m^2 K^4)

We obtained the following intensities of the emission: $I_{Earth} = 390$ W/m^2 and $I_{Sun} = 64$ MW/m^2, respectively.

However, all these values are only mean values where we ignored the variations over the latitudes ϕ and longitudes λ. Data have been collected at the top of the atmosphere for different latitudes ϕ. In the picture below we have sketched their data showing the annual mean of the absorbed sunlight and the corresponding emitted infrared light as a function of the latitudes ϕ. The intensity is described by the function $N(\phi)$ in the unit W/m^2.



Incoming solar radiation and emitted infrared radiation

In this picture the blue curve represents the absorbed incoming solar radiation at the different latitudes from ϕ = -90º to ϕ = 90º, whereas the dotted red curve is the emitted infrared radiation from the Earth. We observe that the net radiation that is been absorbed at the poles (ϕ = -90º, ϕ = 90º) is negative.
The red solid curve shoes the net absorbed radiation that has been derived as the difference between the two other corves. We have a situation where net absorption near the Equator is positive and at higher latitudes the absorption is slightly positive or negative. This implies that there will be a net heat transport from the Equator towards both poles in order to achieve thermal equilibrium.

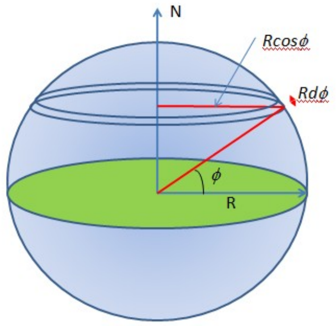
One interesting feature of the picture is that the difference in emission from the Earth does not differ too much between the Equator and the poles. How can that be? The Stefan-Bolzmann law that was discussed in Chapter 2 can explain this fact. The law gives the emitted radiation:

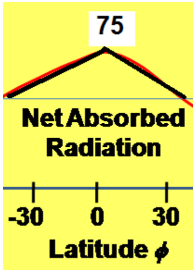
$$e = \sigma T^4 \text{, with } \sigma = 5.67\text{x}10^{-8}\ \text{W/(m}^2\ \text{K}^4\text{)}$$

The temperature near the poles lies around –10 ^{o}C or 263 K and at the Equator around 30 ^{o}C or 303 K. The ratio is 263/308 = 0.854 and the power of 4 gives 0.53. The emission should be almost 50 % lower at the poles, and from the figure we have the intensities 250 W/m^2 and 150 W/m^2 respectively, which gives 150/250 = 0.6 that fits with the Stefan-Bolzmann equation.

Energy transport from the latitude 35^{o}

If we look at the energy difference at the latitude ϕ = 35^{o}, the net radiation absorbed shows to be zero. The peak value at the Equator from the figure is 75 W/m^2. The curve between ϕ = -35^{o} and ϕ = 35^{o} is shown below:





*Let us assume that the net positive radiation south of the Equator goes to the South and vice versa for the northern radiation. This means that we just have to investigate the energy going from $\phi = 0^o$ to $\phi = 35^o$. We have to study a strip having the width $Rd\phi$ we can see in the left figure above. The radius to the strip is $Rcos\phi$ why the area of the strip becomes

$$dA = 2\pi Rcos\phi \ Rd\phi = 2\pi R^2 cos\phi \, d\phi$$

Let us calculate the power dP[W/m^2] streaming through the strip with area dA:

$$dP = N(\phi) \, dA = N(\phi) \, 2\pi R^2 cos\phi \, d\phi$$

In order to calculate the total power streaming in between $\phi = 0^O$ to $\phi = 35^O$, we have to make an integration:

$$P = \int dP = \int_0^{35^o} dP = \int_0^{35^o} N(\phi) 2\pi R^2 \cos\phi \, d\phi$$

However, we don't have any analytical expression yet for $N(\phi)$ so we have to make an approximation. If we see in the right figure, it looks like a triangle with height 75 W/m^2 and with a base that goes from $\phi = -35^O$ to $\phi = 35^O$. In order to make the integrating we have to use radians instead of degrees, i.e. $\phi = -35^O x(\pi/180^O)$ to $\phi = 35^O x(\pi/180^O)$ or from $\phi = -0.611$ to $\phi = 0.611$. Half the triangle goes from 75 W/m^2 to zero when the angle goes from $\phi = 0$ to $\phi = 0.611$, and if we can describe the function N with:

$N(\phi) = 75 - k \, \phi$ *(just as* $y = kx$*).*

We can get the slope k in the following way: $0 = 75 - k \times 0.611 \Rightarrow k = 75/0.611 = 122.7$ W/ m^{2}rad.

So, at last, we can perform the integration:

$$P = \int_0^{0.61} N(\phi) 2\pi R^2 \cos\phi \, d\phi = \int_0^{0.61} (75 - 122.7\phi) 2\pi R^2 \cos\phi \, d\phi$$

With the radius of Earth $R = 6.36 \times 10^6$ m we can evaluate the integral $\Rightarrow$ $P = 5.64 \times 10^{15}$ W. This is a huge power so we can divide it by the circumference of the latitude circle and then we end up with 170 MW/m, which is the size of one power plant per meter!

5.2 The Atmospheres vertical structure and motion

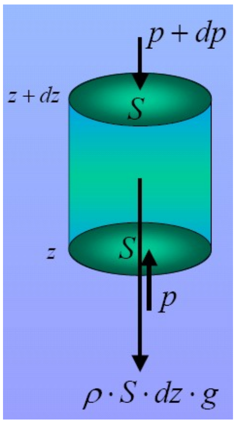
Let us study the atmosphere and look at the pressure variations vertically. We use the ideal gas law:

$$pV = RnT$$

Here the pressure is p, the volume V and the temperature is T (K). The number of moles is n and the constant R = 8.314 J/K mol

We can determine the pressure as a function of height above the ground and start with the ideal gas law [http://en.wikipedia.org/wiki/Ideal_gas_law]:

$$p = \frac{RnT}{V} = R\frac{m}{M}\frac{T}{V} = \left[\rho = \frac{m}{V}\right] = \frac{\rho RT}{M}$$



Let us look in the z-direction study a cylinder with height dz and bottom area S. If the pressure is p on the lower part of the gas column and $p + dp$ on the higher part, the following relation holds at hydrostatic equilibrium:

If the column has the weight $m = \rho S dz$ we get:

$$mg + Sdp = 0 \Rightarrow \rho S \cdot dz \cdot g + Sdp = 0$$

$$\frac{dp}{dz} = -g\rho = -g\frac{pM}{RT}$$

This equation gives the expression for how the pressure changes with distance.

If we have a situation where the temperature T is constant we introduce a constant H, where

$H = \frac{RT}{Mg}$, why we obtain $\frac{dp}{dz} = -\frac{1}{H}p$

We rearrange the equation and get $\frac{dp}{p} = -\frac{dz}{H}$

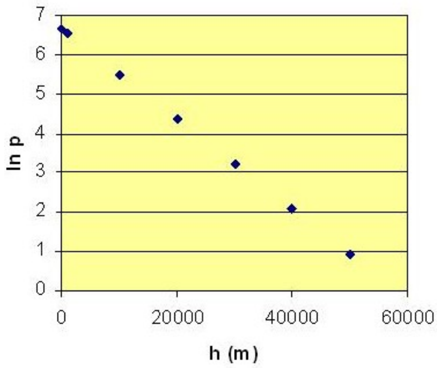
We integrate this expression and obtain $\int \frac{dp}{p} = -\frac{1}{H}\int dz$

The integration gives $[\ln]_{p_0}^{p} = -\frac{z}{H} \Rightarrow \ln\frac{p}{p_0} = -\frac{z}{H}$

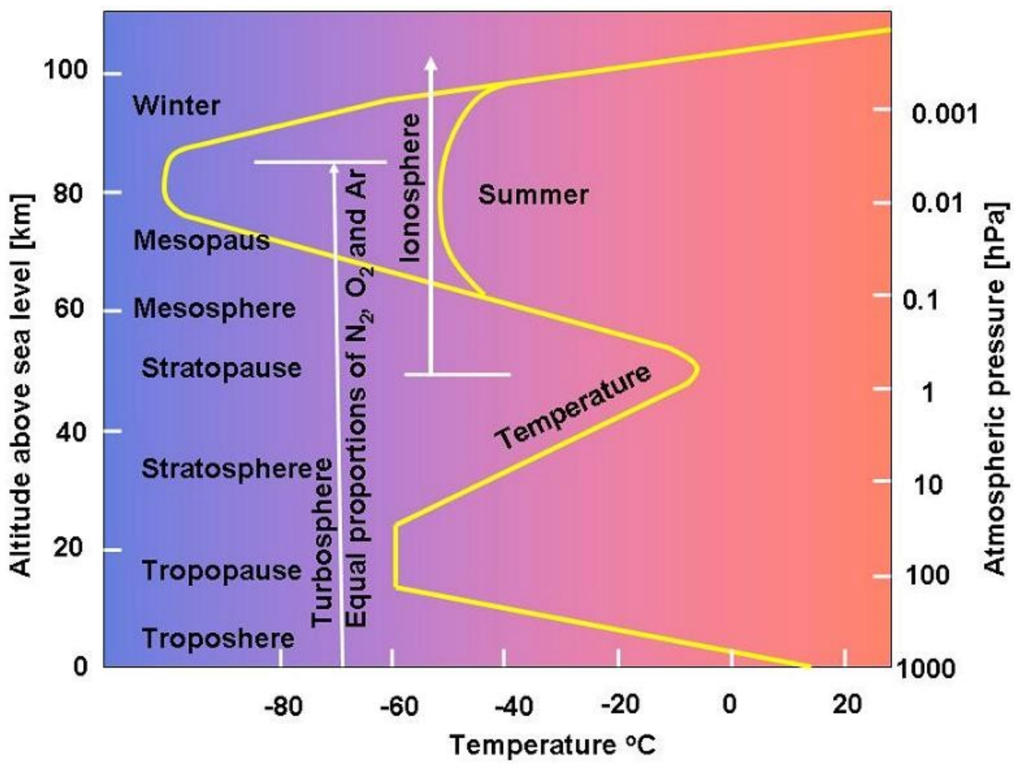
With $e^{\ln x} = x$, we get

$$p = p_0 e^{-z/H}$$

If we make a graph of the logarithmic expression we will obtain a straight line, if we put height *h* above ground as a function of *ln p (*Torr*)*:



We can also derive the pressure above ground with the following link: http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html (go to barometric formula)



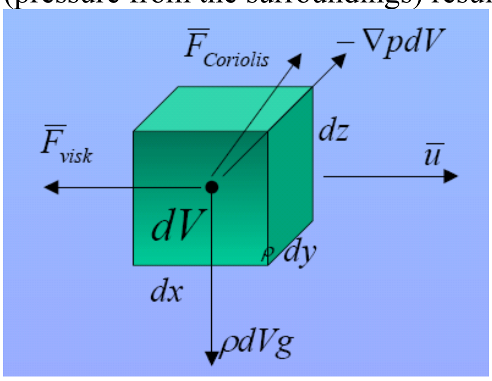
However, we have regarded the temperature as being constant, and this is really not the case as we see in the next figure, where the temperature drops almost linearly from 18 ºC at the surface of the Earth to around – 60 ^{O}C at 18 km height. In the Stratosphere the temperature on the other hand increases linearly from -60 ^{O}C to near zero from 25 km to 50 km height. As you can see, there are more regions where the temperature versus height dependence is linear.

Example
Calculate the temperature drop from sea level to 18 km above sea level in terms of ^{O}C/km.
Solution

$$k = \frac{-60-(18)}{18-0}\frac{^{o}C}{km} = -4.3^{o}\,C/km$$

5.3 The Atmosphere's horizontal motion

If we study the forces ($\vec{F}_{tot}$) acting on a volume (dV) of air, there are several forces influencing it, the gravitational force ($mg = \rho dVg$), the viscosity force ($\vec{F}_{visc}$), wind force (pressure from the surroundings) resulting in:

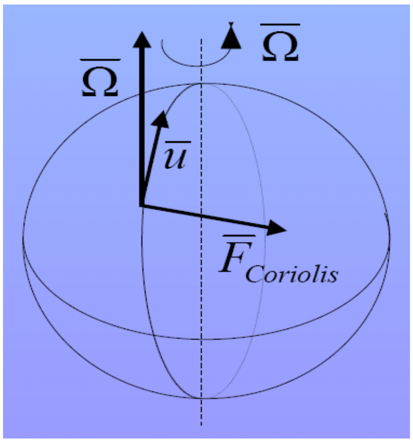


There is also a force called the Coriolis force influencing the system.

This gives the following equation $F = m\vec{a} = m\frac{d\vec{u}}{dt} = \rho\,dV\frac{d\vec{u}}{dt}$ or written with the forces above:

$$\vec{F}_{pressure} + \vec{F}_{gravity} + \vec{F}_{viscocity} + \vec{F}_{Coriolis} = \rho\,dV\frac{d\vec{u}}{dt}$$

The Coriolis force is a inertial force present in rotating systems, i.e. a train or a ship moving in South-North direction in the Northern hemisphere is affected by this force acting to the right relative to the direction of the movement. This is due to the rotational motion of the Earth. In the Southern hemisphere, the force is acting in the opposite direction.

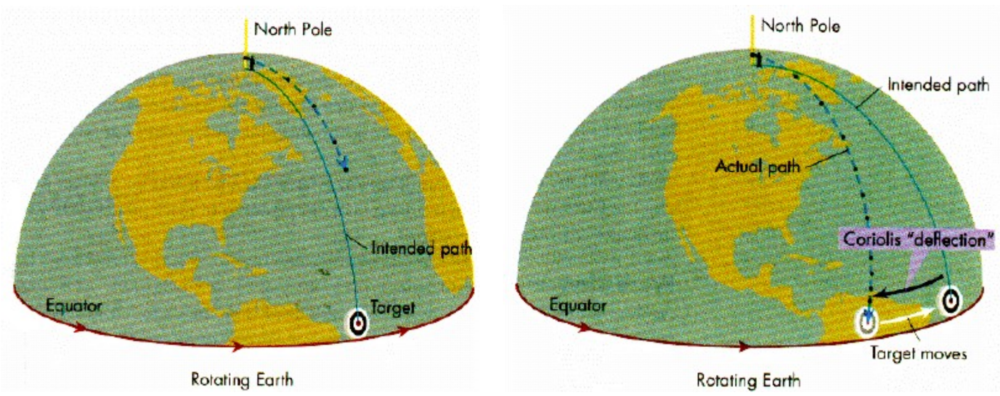


Omega ($\vec{\Omega}$) is the rotation vector, $\vec{u}$, the velocity of the system and the Coriolis force is given by:

$$\overline{F}_{Coriolis} = 2m\overline{u} \times \overline{\Omega} = -2m\overline{\Omega} \times \overline{u}$$

A particle on the surface of the Earth is moving an angle of 2π rad during 24 h

$$\left|\vec{\Omega}\right| = \frac{2\pi}{24 \times 60 \times 60}\ \text{rad/s} = 7.27 \times 10^{-5}\ \text{rad/s}\ .$$



Above is shown a situation where there is an object moving from the North towards the South and is influenced by the Coriolis force.

Example
A TGV train is moving in the North-South direction from Paris to Lyon at a speed of $u = 350$ km/h at a latitude θ of 47^{o}. The mass of one wagon is 28 tons. Calculate the Coriolis force on the wagon and compare with the weight of the wagon.

Solution
The Coriolis force is given by $F = 2mu\Omega\sin\theta$ where $\Omega = 7.27\text{x}10^{-5}$ rad/s. We get

$F = 2 \times 28 \times 10^{4} \times 350 \times \frac{10}{36} \times 7.27 \times 10^{-5} \times \sin\theta\ \ \text{N} = 2895\ \text{N} \approx 2.9\ \text{kN}$ to be compared with the weight of the wagon that has the weight $mg = 28 \times 10^4 \times 9.8\,\mathrm{N} \approx 2.74 \times 10^6\,\mathrm{N}$. The relative force is $\frac{F_{Coriolis}}{mg} = \frac{2895}{2.744 \times 10^6} = 0.00106 \approx 0.11\%$

The Easterly direction of the Gulf Stream towards Northern Europe, in the Northern hemisphere, is due to the Coriolis force. The Southern winds blow to the East on the Northern hemisphere but to the West in the Southern hemisphere. Other examples where the Coriolis force influences the environment is water moving for example in a great stream up North leading to a force on the East side of the shore in the Northern hemisphere, why it affects the western shore when moving in the Southern hemisphere. There will be more erosion on the shores on the side affected by the Coriolis force.

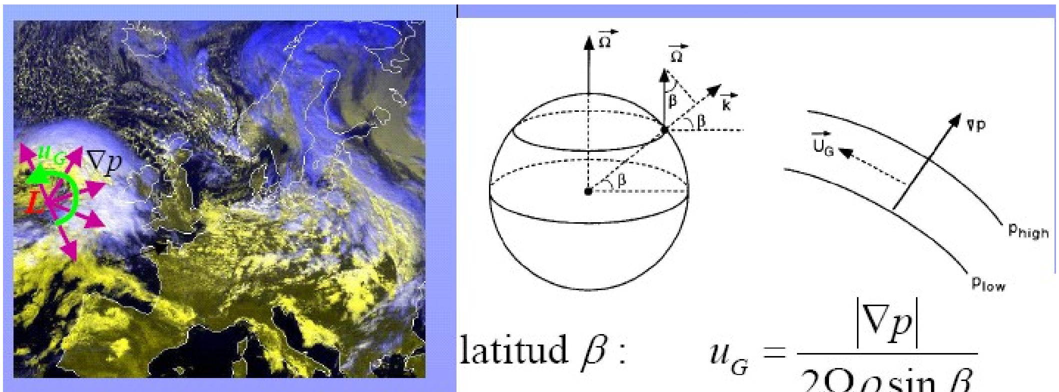
Simulations of these and other weather phenomena can be found on:
http://www.atmos.uiuc.edu/courses/atmos100/all_programs.html

Geostrophic airflow
Earlier we derived the complex equation for airflow, but higher up in the atmosphere one can neglect the viscosity forces due to friction, and the pressure forces due to the low pressure. If we also neglect the vertical components (as *mg*), we end up with:

$$\vec{F}_{resultant} = -m\frac{d\vec{u}}{dt} - 2\rho\,\vec{\Omega} \times \vec{u} = 0$$

We can introduce a pressure force acting perpendicular to isobars (lines with equal pressure) when we have isobars with a pressure difference. We thus also introduce a vector called gradient $\frac{\partial p}{\partial s} = |grad p| = |\nabla p|$ describing how the pressure changes with the distance *s*.

Here we also have assumed vertical equilibrium.



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latitud β : $u_G = \frac{|\nabla p|}{2\Omega\rho\sin\beta}$

If we look at airflow with mass *m* moving at latitude β and observe that we have a pressure gradient of ∇p resulting in a velocity u_G around the Low pressure, counterclockwise, and of course clockwise for a High pressure. This wind will blow parallel to the isobars.

Example

Calculate the wind velocity around a Low pressure at latitude 60^{o} in a region where the pressures are 945 hPa and 950 hPa between the centre part and the outer parts of the Low pressure at a difference of 10 km. At this height the air density is 0.5 kg/m^3.

Solution

With the expression above we get

$$u_G = \frac{\nabla p}{2\Omega\,\rho\,\sin\beta} = \frac{(950 - 945)\times 10^2/10\times 10^3}{2\times 0.5\times 7.27\times 10^{-5}\sin 60^o}\ \text{m/s} \approx 750\ \text{m/s}$$

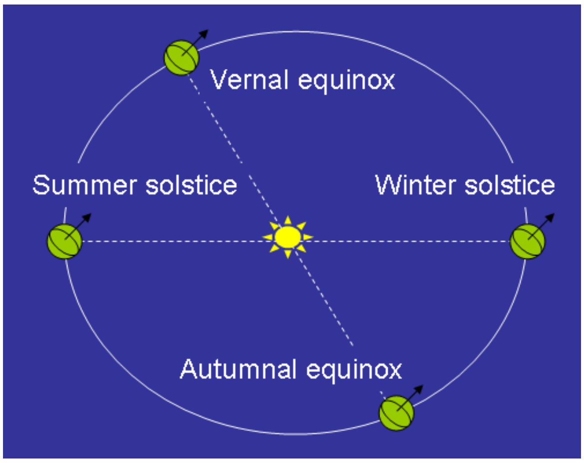
Here we have discussed how pressure differences can generate wind circulating in a Low pressure. The origin of these pressure differences is of course due to the incoming solar radiation. Like in Hadley circulation, the heated air moves upwards, is cooled down and moves downwards. This also holds far away from the Equator where the Hadley circulation occurs. Both in the North and South the solar influence produces the pressure differences. Many effects can be mentioned about temperature differences as well. At the end of the summer, the oceans reach their highest temperatures, and at the same time the land is cooling down. This means that there will be a temperature gradient between the land and sea, both on a larger timescale, but also du to the day-night changes.

We see that there are many parameters to keep in mind when performing weather studies and it is difficult to make physical models to perform accurate description of the weather. There are many parameters that have to be used if we start from scratch in our weather description. The famous physicist and Nobel laureate Richard Feynman said: “Give me seven parameters and I can simulate an Elephant”.

5.4 Natural Climate changes

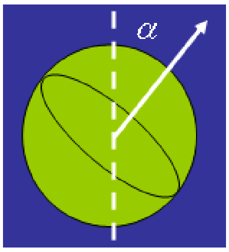
Orientation of the Earth. Milankovitch effect.

The orbit of the Earth around the Sun is elliptical, with an eccentricity of e = 0.0167. The ellipse can be described by its long and short axis, a respectively b, where $b = a\sqrt{1 - e^2}$. Here we get $b = a\sqrt{1 - e^2} = 0.99986a$. We can deduce that the orbit is almost a circle. However, when looking at longer periods one has shown that the eccentricity can vary between 0.002 and 0.055 with a period of about 100,000 years.



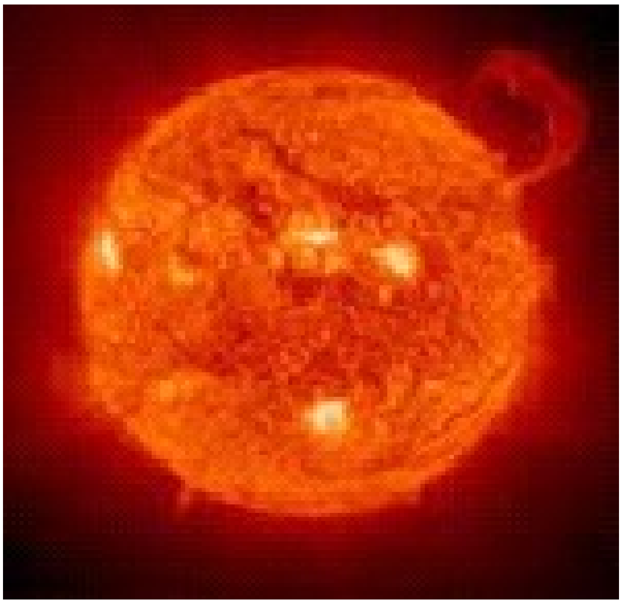
The Earth's orbit around the Sun is an ellipse with the Sun in one of the two foci.

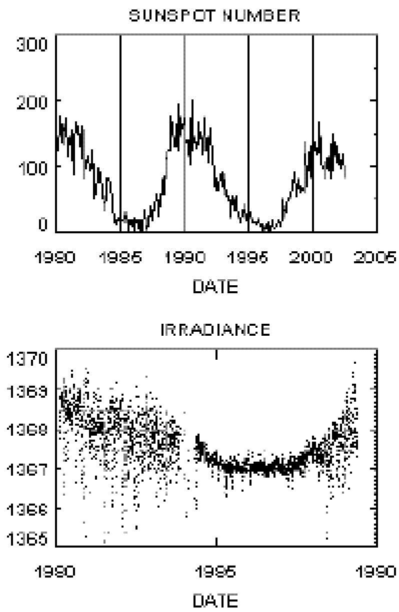
The Earth is tilted an angle $\alpha = 23.5^{o}$ and is rotating around its own axis. The angle is slowly varying with time between 22.5^{o} and 24.5^{o} with a periodicity of 41,000 years.



Still, the axis of the Earth also performs a kind of vibration, a so-called precession around the normal of the orbit with a period of 23,000 years. Because of this the vernal point (when night and day are equally long) moves along the eclipse of the zodiac in 23,000 years.
Thus, we have three different periods to take into account; the 100,000-year period, the 23,000-year period and the 41,000-year period.
The results of these periods show up as:
100,000 period: Variations in solar influx on Earth
41,000 and 23000 periods: Variation in Intensity at the surface of the Earth.

Changes in the solar radiation

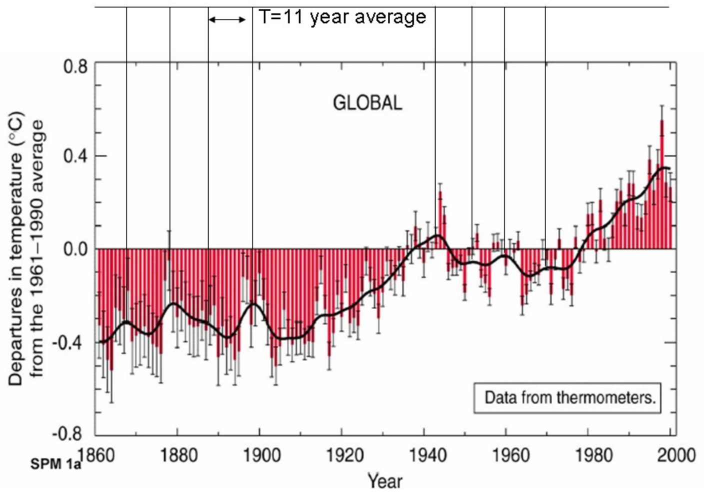




As we have discussed in earlier chapters, the solar constant is around 1380 kW/m^2. One can wonder if this value is constant or if it varies in time. Research has been going on looking at the sunspots and that radiation reaching the Earth. Above is shown a diagram showing such variations. The solar radiation also has small fast periodical variations correlated to the number of sunspots. One has found a periodicity of around 11 years. However, the effect on the solar luminosity is rather small, just around 1%.

Being a star, the Sun has a continuous development scheme and lifecycle. Looking back some three billion years, the luminosity of the Sun was around 30% less than today.

The 11-year period can bee seen as variations in the surface temperature below:



Catastrophes

One of the toughest predictions to make is when there is going to be a catastrophe. For instance volcanoes, and their influence in the amount of emitted SO_2 in the atmosphere as well as their general disturbance of the atmosphere is a difficult task to predict.

"A snapshot of a meteorite taken 65 million years go"

An extreme example is meteorites and their sudden and long-lasting effect of the atmosphere. Some 65 million years go a large meteorite landed in the Mexican Gulf and created a large dust cloud in the atmosphere that lasted for several years, blocking the sunlight to reach the surface of the Earth. The life of plants decreased and the animals at that time (dinosaurs) were in shortage of food and they died out.