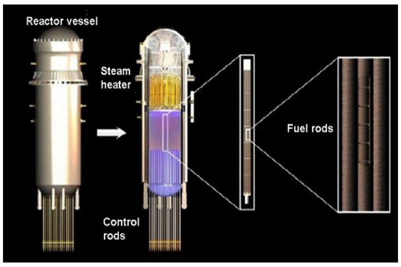
Chapter 7. Carbon free energy

The waterfalls at Trollhättan

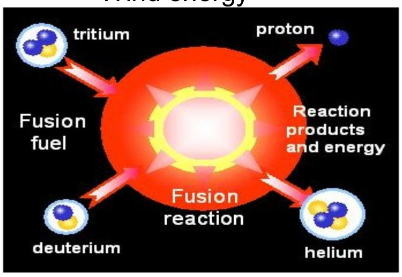
Hydro energy

Wind energy

ISS space station with solar cells



Fission reactor



Fusion processes

Chapter 7

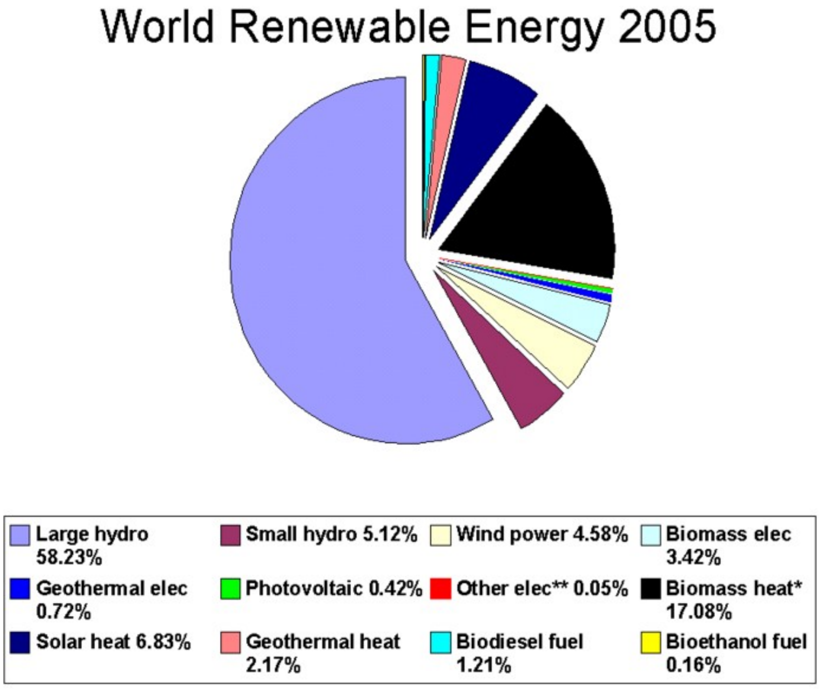
7.1 Introduction

Here, in this chapter, we will discuss power of carbon free energy sources such as solar cells, wind mills, hydro energy plants including traditional water falls, bio energy sources, but also nuclear plants based on both fission and fusion.
Let us first look at the electricity production in Sweden, where one observes the two main producers, hydropower as well as nuclear power. However, due to the closure of nuclear power plants the situation is changing, and also due to the increasing number of wind energy plants.

Electricity production in Sweden
[TWh and nettoproduction in % 2001]

Total net use:	150	
Total net production	158	
Hydro Power	78	49
Wind Power	0.5	0.3
Nuclear Power	69	44
Heat Power	6	4
Oil condense	<0.1	
Distribution losses	12	
Export – Import	7	

Let us then look at the world renewable energies (nuclear power plants excluded). The status of the energy production is shown in the picture below.



If we look worldwide we see that the large hydro plants are dominating and produce almost 60% of the energy. Biomass has also a significant sector of the figure. We will discuss most of these energy generators in this chapter and their advantages as well as drawbacks.

7.2 Solar Energy


Solar panels used for power production.

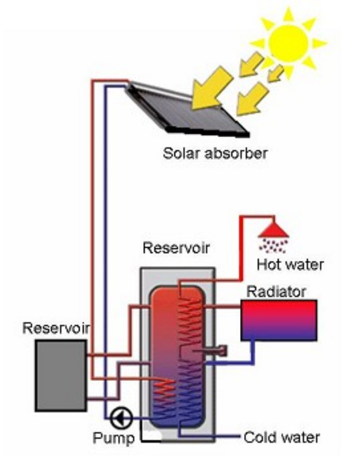

Solar tower onto which the sunlight is focused.

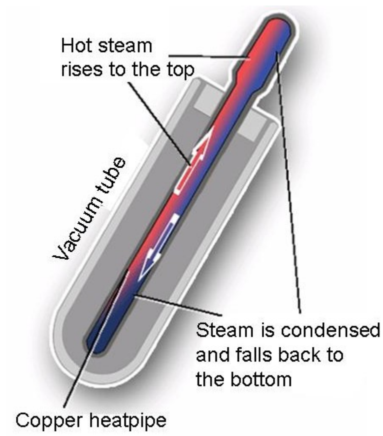
7.2.1 Introduction.

In this chapter we will discuss both the **solar absorber** and the **solar cells** based on the photo-voltaic effect. The solar absorber is a rather simple device where the solar influx is directly converted into heat normally using water as a medium.

7.2.2 Solar absorber

The incoming energy from the Sun has a mean Intensity of 1.38 kW/m^2 (the so-called solar constant). If we construct a thermal solar absorber and use water as the transport medium for the heat, we could make a construction as shown in the figures below.





Let us discuss the solar absorber by means of thermodynamics.

Example
Suppose we in the summer have an incoming solar flux Φ_{in} of 600 W/m^2 and the area of the absorber is 50 m^2. The glass temperature is 30 ^{O}C, the air temperature 25 ^{O}C and the sky temperature – 10 ^{O}C. Let the glass of the absorber transmit 90 % of the light and have an emissivity of ε = 95 %.
Let us simplify the situation and just look at how much energy is transferred to the water. Suppose also the water is circulating at a speed of 0.1 dm^3/s, and that the convection of heat from the hot to the cold side is 12 W/Km^2.
Solution
The net influx is then $Q_{in} = 0.90 \times \Phi_{in} - \varepsilon\sigma\left(T^4{}_{glass} - T^4{}_{surr}\right) - h_{out}\left(T_{glass} - T_{air}\right)$
$Q_{in} = 0.90 \times 600 - 0.95 \times 5.67 \times 10^{-8}\left(303^4 - 263^4\right) - 12(303 - 298)$
We get Q_{in} = 284 W/m^2 and $P_{in} = Q_{in} A$ = 14200 W.
In steady-state, we must have $P_{in} = P_{out}$.
From thermodynamics we get: $P_{out} = \Phi_{water}\, c_p\, \Delta T_{water}$
With these expressions we can determine the water temperature difference:

$$\Delta T_{water} = \frac{P_{in}}{\Phi_{water} c_p} = \frac{14200}{0.1 \times 4.2 \times 10^3} \approx 34\,^{o}\text{C}$$

The efficiency then becomes $\eta = \frac{Q_{in}}{\Phi_{in}} = 47\ \%$

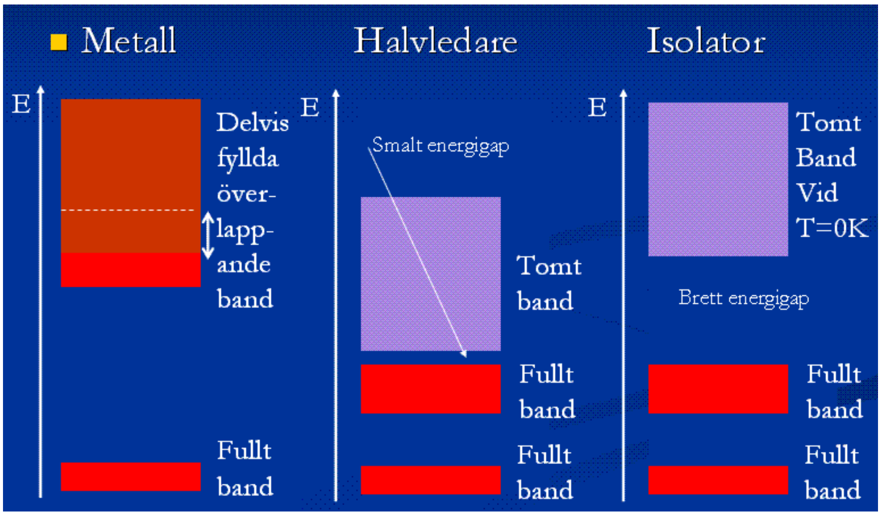
7.2.3 Solar cells. Introduction

One of the main problems with solar energy has been the low efficiency and their high cost, For quite a long time their efficiency has been around or less than 10 %. However, there seems to have been a breakthrough in this research field rather recently:
New World Record Achieved in Solar Cell Technology
New Solar Cell Breaks the "40 Percent Efficient" Sunlight-to-Electricity Barrier ***WASH-INGTON, DC*** *– U.S. Department of Energy (DOE) Assistant Secretary for Energy Efficiency and Renewable Energy Alexander Karsner today announced that with DOE funding, a concentrator solar cell produced by Boeing-Spectrolab has recently achieved a world-record*

conversion efficiency of 40.7 percent, establishing a new milestone in sunlight-to-electricity performance. This breakthrough may lead to systems with an installation cost of only $3 per watt, producing electricity at a cost of 8-10 cents per kilowatt/hour, making solar electricity a more cost-competitive and integral part of our nation's energy mix.

*7.2.4 Metals, isolators and semiconductors

Before we discuss solar cells, let us compare metals, isolators and semi-conductors with respect to the energy structure and draw conclusions about similarities and discrepancies by looking at the figure below:



Energy level diagrams of metals, semi-conductors and Isolators.

We see that metals have partly filled and overlapping bands, making the electrons move freely in the metal. This means that metals are good conductors with respect to electric conductivity and heat conductivity. Looking at semi-conductors there is only a small energy gap between the valence and conduction bands. Thus it is possible for the semi-conductors to conduct electricity by implanting atoms in the crystal, i.e. doping of the crystal. If we then look at the isolators, we see that the distance between the valence and conduction bands is large. We thus have difficulties to excite electrons from the valance band to the conduction band. That is why the isolator's properties of conduction differ from metals and semi-conductors.
We can examine how easy it is to move electrons from the valence band to the conduction band. Looking at an isolator the band gap is relatively large. For diamond the band gap 5.5 eV. Using the Boltzmann distribution function we can estimate how much thermal energy an electron can have at temperature *T*.

$$N = N_0 e^{-(E-E_0)/kT}$$

Here $k = 8{,}62\text{x}10^{-5}$ eV/K is the Boltzmann constant.

N/N_0 is the relative population of atoms at the energy E compared to the population N_0 with energy E_0.

Example

What is the relative probability that an electron can jump from the valence band to the conduction band of diamond?

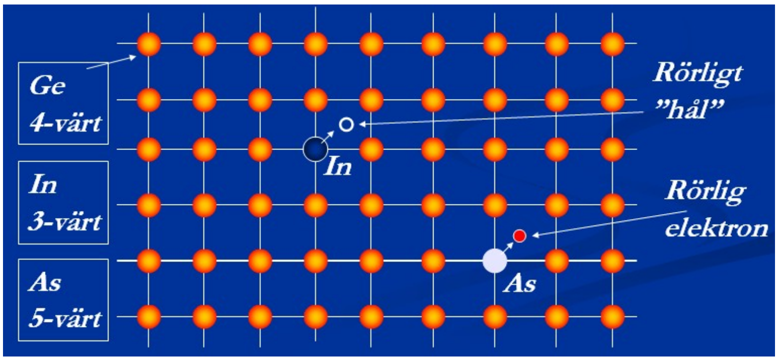
Solution

We put $(E\text{-}E_0)$ = band gap E_g = 5.5 eV for diamond and T = 293 K (room temperature) and obtains $-\frac{E_g}{kT} = 5.5/(8.62 \times 10^{-5} \times 293) \approx -218$

The probability will become $P = N/N_0 = e^{-218} = 4.6\text{x}10^{-94}$ which is an extremely small number. The probability is close to zero.

7.2.5 Intrinsic conductivity - doping

One observes a difference in conductivity where ”pure” crystals of silicon and germanium with high purity are compared with doped materials. Often “pure” materials are doped with other atoms of the order of $1{:}10^7$. By choosing special dopants the crystal can get an excess of moving electrons or the lack of electrons, which is generating ”holes”. Ordinary semi-conductor materials are silicon and germanium. Germanium has 4 electrons in its outer shell and has 4 valence electrons.



If one dopes germanium with atomic As, with 5 valence electrons in small amounts, typically 1 As-atom on 10^7 Ge-atoms, then you produce a free electron moving in the crystal. If you instead implant 1 indium atom, with 3 valence electrons, there will be a lack of electrons, or we can say that a positive ”hole” will be created.

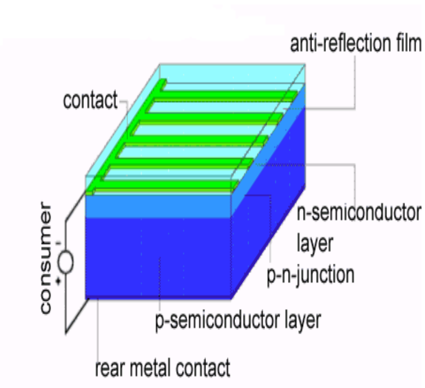
7.2.6 Doped semiconductors

By doing as in the former section we can dope a material with dopants in order to change the properties of the crystal. All modern materials are based on doping. In the figure above we see the different types, **n-doped** respectively **p-doped** materials.
The introduction of dopants in germanium and silicon crystals can be controlled in getting a material of the *n-type* and the opposite material *p-type*, with only a small area in between. The simplest way to produce such crystals is to pick out a growing crystal, for instance melted germanium containing a donor, and rapidly introduce an acceptor in the melt. The first part of the crystal created is of *n-type* and the rest of *p-type*.

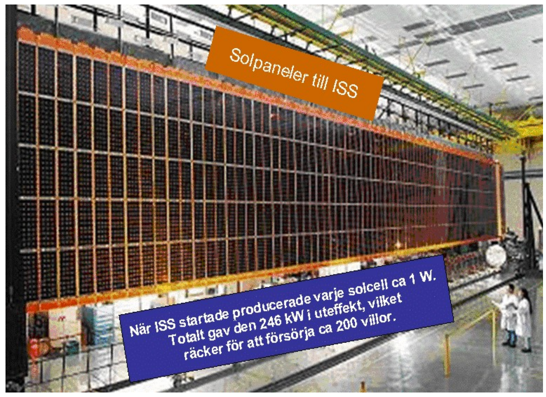
7.2.7 Solar cells

A *solar cell* is a thin layer of doped semi-conductor material that converts light to electricity. A typical material is silicon that has an efficiency of 15 – 20 %, which means that an area of a square decimetre will give 1.5 W. With a voltage of 0.5 V, this means 3 A. The solar cells are then connected parallel and in series to obtain the right voltage and high enough current.

Solar cells

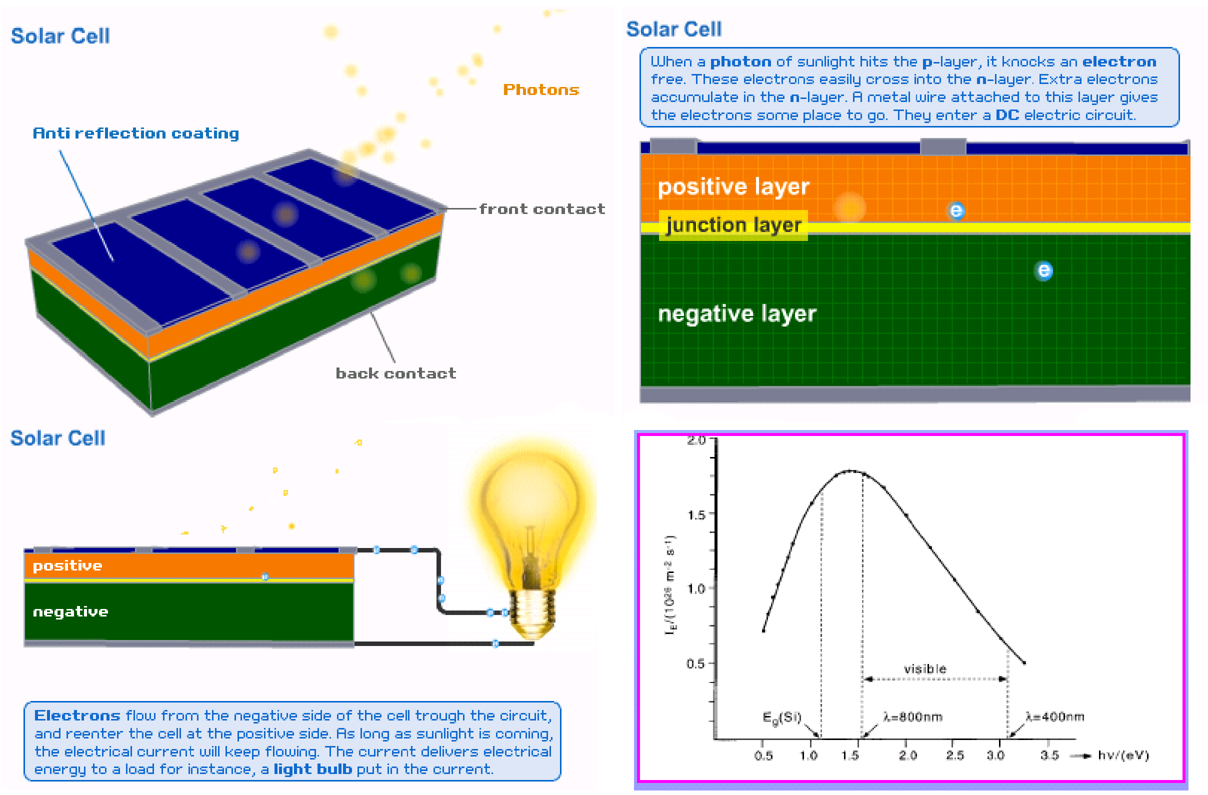


Construction of a solar cell



Solar panels of the ISS space programme

One important parameter regarding solar cells is the band gap, E_g, since only that part of the solar spectrum exceeding, will contribute to useful energy. In the figure below, we observe the band gap of silicon around 1.2 eV.

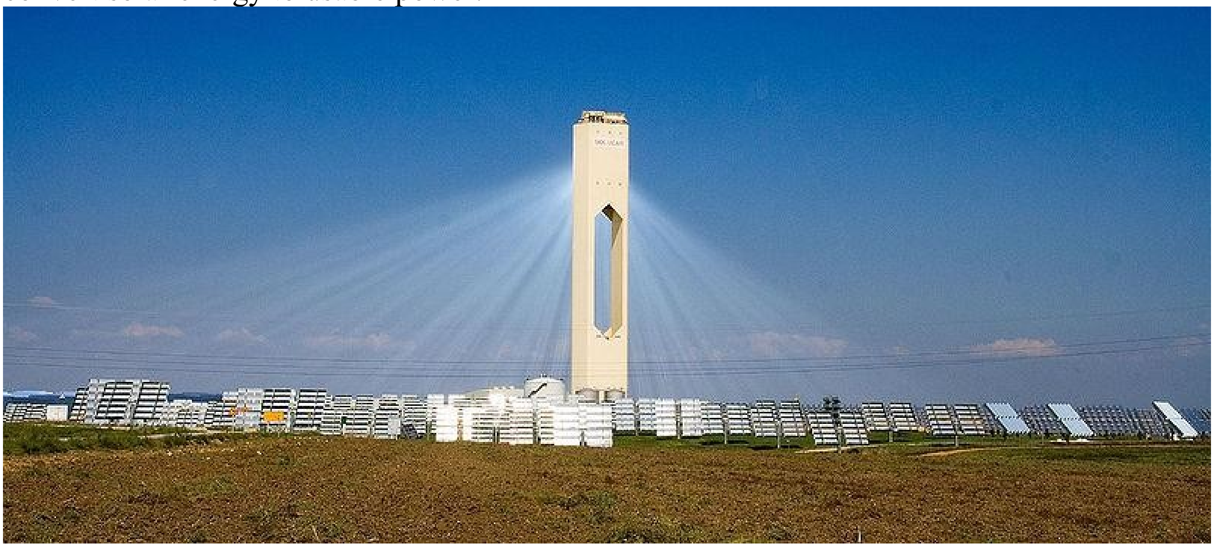


The photons, hit the *p*-layer making the electrons pass the junction layer into the *n*-layer

The rest of the energy is lost in the form of heat. This is why the efficiency of silicon is rather low, just above 20 %. However, as discussed in the beginning of the chapter there seems to be an improvement in technology increasing the efficiency to around 40 %.

7.2.8 Solar power plants

A *solar reflector* can be used to produce energy via an absorbing medium. Using a parabolic mirror one can focus the solar light on the absorbent, normally a tube with circulating water. Near Sevilla in Andalusia in Spain the Worlds largest solar plant has been constructed to convert solar energy to usable power.



The picture shows the solar tower, where the parabolic mirrors on the ground are focused on.

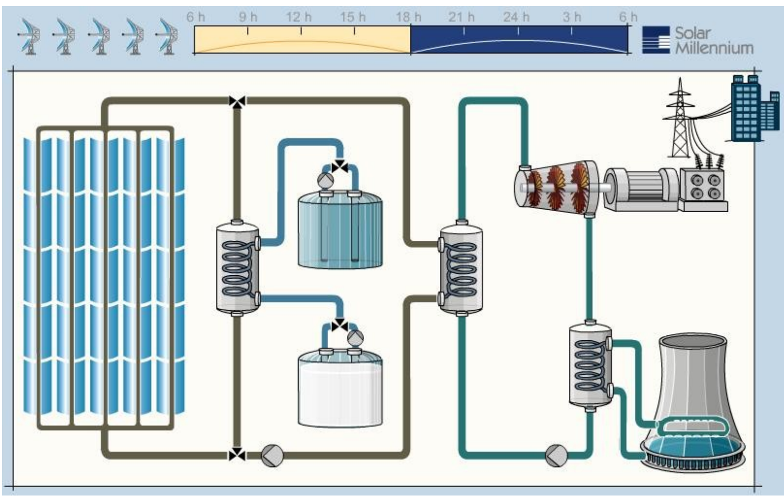
The solar park with the two towers receiving the sunlight from the large parabolic mirrors.
(by courtesy of Solar Millenium)

Example

Each parabolic mirror has an area of 14x6 m^2 and the large solar park to the left consists of 400 mirrors. How much power can we deliver to the tower?

Solution

The solar constant is $I = 1.38$ kW/m^2. We get $P = I \times A = 1.38 \times 10^3 \times 400 \times 14 \times 6$ W = 46 MW.



Here, the principle of operation can be seen. The sunlight is focused on water filled pipes with circulating water going to a heat exchanger and after some steps to a steam turbine for electricity production. The parabolic mirrors can turn, making the in-coming light always perpendicular to the sunlight.

7.3 Wind Energy

Windmill Park outside the coastline of Denmark

7.3.1 Maximum power of a wind turbine

When looking at wind power, one can use it in many ways; to sail, to generate energy in windmills and to generate electric power. The first question to ask is how much energy can be extracted from the wind at a certain wind velocity *u*? Betz has investigated this and we will try to perform a simple calculation here.

We start with the *kinetic energy* of a moving volume of 1 m^3 air with mass *m* and velocity *u*:

$$T_{wind} = \frac{1}{2} m u^2$$

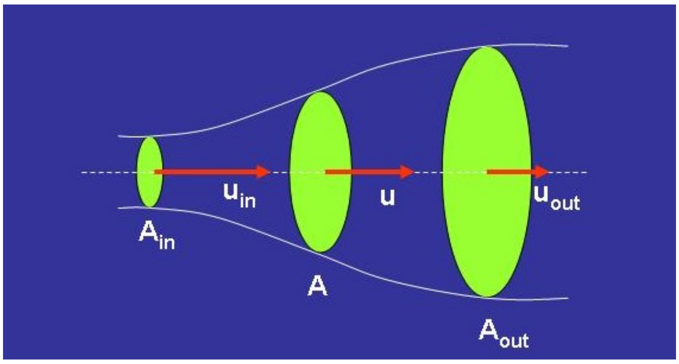
We can then calculate the *kinetic energy density* of the wind by using $\rho = \frac{m}{V}$ where $V = 1$ m^3.

$$T_{wind} = \frac{1}{2} \rho u^2$$

With $\rho \approx 1.2\ \mathrm{kg/m^3}$, the density of air, we can calculate the total *power per m^2* of the wind:

$$P_{wind} = T_{wind} \times u = \frac{1}{2} \rho u^3$$

Of course the whole power cannot be used since the air that has been slowed down and is doing the work, has to vanish somewhere. Let us look at a turbine used in airplanes. The principle of continuity says that at constant density, air has to expand and get a larger cross section after having left the turbine. In the picture below, we see that expansion.



The stream of air passes through an area A_{in} at the speed u_{in} and after passing the turbine; the cross section area is A_{out} with velocity u_{out}. Let us introduce an effective area A with velocity u_{eff}.
We can lean on the continuity relation where the mass flow into the turbine equals the mas flow out of the turbine.

Let us define the mass flow as, $J_m = m/t$ kg/s. with $\rho = m/V$, why we obtain

$$J_m = \rho A_{in} u_{in} = \rho A_{ut} u_{ut} = \rho A u_{eff}$$

This expression can be used to show that the value of the effective speed will become
$u_{eff} = \frac{u_{in} + u_{out}}{2}$, i.e a mean value of the velocities.

Let us now introduce α coefficient a where $u_{eff} = u_{in}(1-\alpha)$ and $u_{out} = u_{in}(1-2\alpha)$
The parameter α can be allowed to vary between $0 \le \alpha \le 1/2$

Now let us concentrate on the efficiency of the turbine and the maximum power P_{max} we can extract from the wind

Earlier we derived the following expressions $T_{wind} = \frac{1}{2}\rho u^2$ and $P_{wind} = T_{wind} \times u$

The power $P_{extracted}$ we can extract will thus depend on the kinetic energy of the air after and before passing through the turbine:

$$P_{exctr} = \frac{\rho A}{2}\left(u_{in}^2 - u_{out}^2\right)u_{eff}$$

Let us now use the expression with α above and replace u_{eff} and u_{out}, which gives

$$P_{exctr} = \frac{\rho A}{2}\left(u_{in}^2 - u_{\text{int}}^2(1-2\alpha)^2\right)u_{in}(1-\alpha) = 2\rho A u_{in}^2(1-\alpha)^2\alpha$$

We can now study the maximum power we can extract by looking at the function

$$f(\alpha) = (1-\alpha)^2\alpha$$

We can get the maximum by putting $\frac{\partial f}{\partial \alpha} = 0$. First we simplify f(α) and we get

$$f(\alpha) = \alpha - 2\alpha^2 + \alpha^3$$

$\frac{\partial f}{\partial \alpha} = 1 - 2 \cdot 2\alpha + 3\alpha^2$. We have to solve $1 - 2 \cdot 2\alpha + 3\alpha^2 = 0$. We rewrite this second-degree equation and get $\alpha^2 - \frac{4}{3}\alpha + \frac{1}{3} = 0$ with the solution $\alpha = \frac{2}{3} \pm \frac{1}{3}$

Since the condition on α was: $0 \le \alpha \le 1/2$, there can only be one solution, namely $\alpha = \frac{1}{3}$

Earlier we had $P_{exctr} = 2\rho A u_{in}^3 (1-\alpha)^2 \alpha$. With $\alpha = \frac{1}{3}$ we get

$P_{exctr} = 2\rho A u_{in}^3 (1 - \frac{1}{3})^2 \frac{1}{3} = \rho A u_{in}^2 \frac{8}{27} = \frac{\rho A u_{in}^3}{2} \frac{16}{27} = P_{wind} \frac{16}{27}$, the so-called Betz limit.

So, finally, the maximum power we can get from the wind is

$$P_{exctr} = \rho A u_{in}^3 \frac{8}{27} = P_{wind} \frac{16}{27}$$

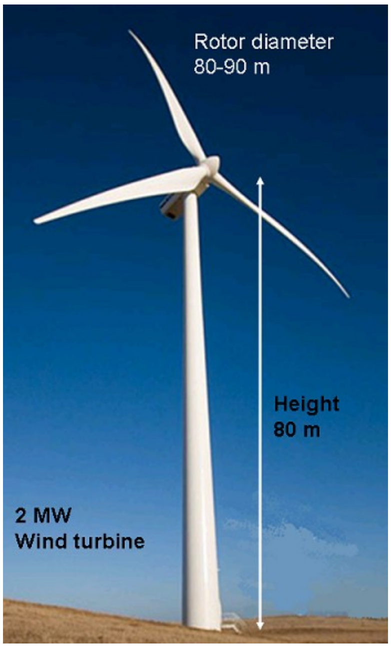
Example

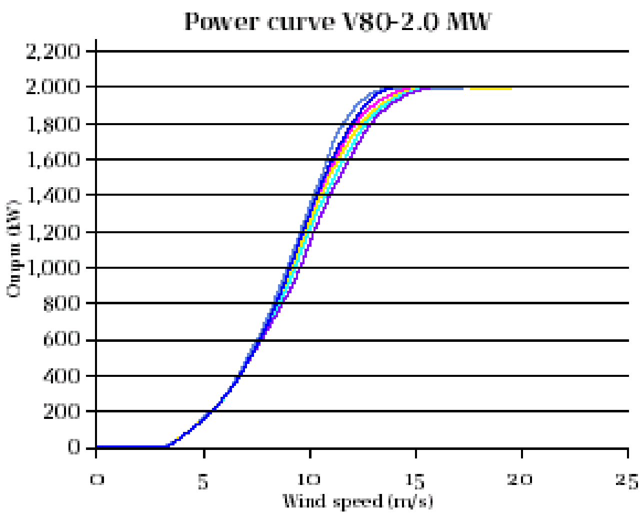
Suppose the wind is blowing with a strong breeze, around 11 m/s (Beaufort 6, or 22 knots). We have a windmill with a rotor diameter of 44 m. Estimate the power of the windmill.

Solution

The maximum power we can obtain from the wind according to Betz law is

$$P_{exctr} = \rho A u_{in}^3 \frac{8}{27} = 1.2 \times \pi \times 22^2 \times 11^3 \frac{8}{27} W \approx 720\ kW \approx 0.72\ MW$$



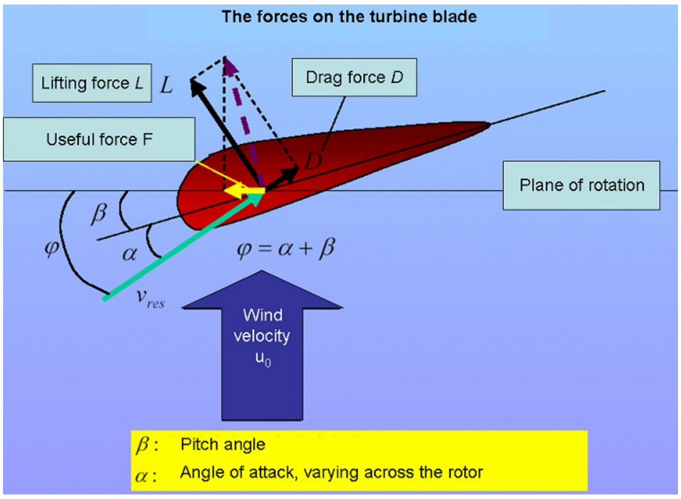


Above is shown a large wind turbine with a maximum power of 2.0 MW. The tower has a weight of 160 tons. It starts to deliver power at a wind speed of 4 m/s and has a cut-off at wind speeds of 25 m/s.

Wind-to-Hydrogen Project Animation: This wind-to-hydrogen animation demonstrates how electricity from wind turbines is used to produce hydrogen at NREL's National Wind Technology Centre: http://www.nrel.gov/hydrogen/proj_wind_hydrogen_animation.html

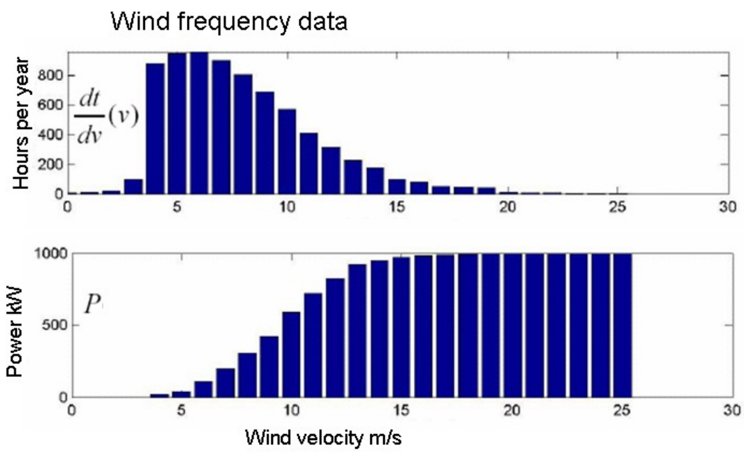
7.3.2 Aerodynamics

When constructing a turbine blade, at first sight, it looks just like a propeller of an aeroplane. The aerodynamics gained, when building propellers, have been used in the design of turbine rotor blades. If we look at a blade rotating as in the figure below, the wind attacks the blade resulting in both lifting, dragging and useful forces.



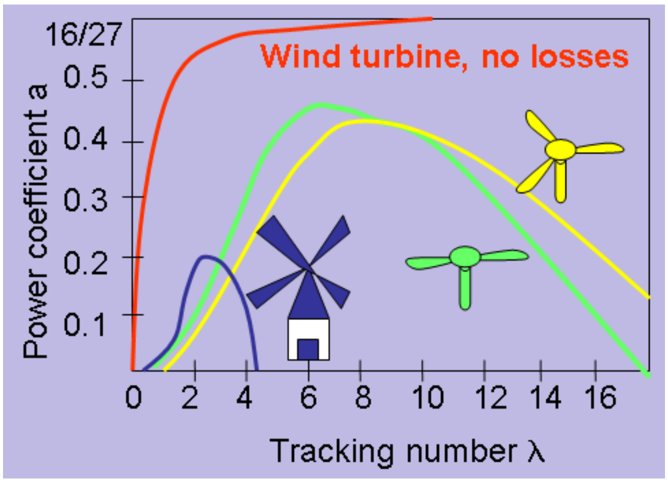
Turbine rotor blade and forces

Since the blades velocity is smaller closer to the centre and moves much faster at the tip, one changes the form of the blade.
When looking at wind velocities at different parts of the World, there is of great importance to know the wind velocities and wind frequency data when putting up new wind turbines.



In chapter 7.3.1 we discussed the maximum power of a wind turbine where we found the *coefficient of performance*, *a*, to have a maximum of 16/27. One often discusses the velocity of the outer part of the turbine blade, the tip, and compares it with the non-disturbed wind velocity u_0. The turbine rotates with frequency *f* and circulation time *T*.
In this comparison we introduce the dimensionless *tracking number* λ and define it as:

$$\lambda = \frac{u_{tip}}{u_0} = \frac{2\pi R \times \frac{1}{T}}{u_0} = \frac{2\pi Rf}{u_0}$$



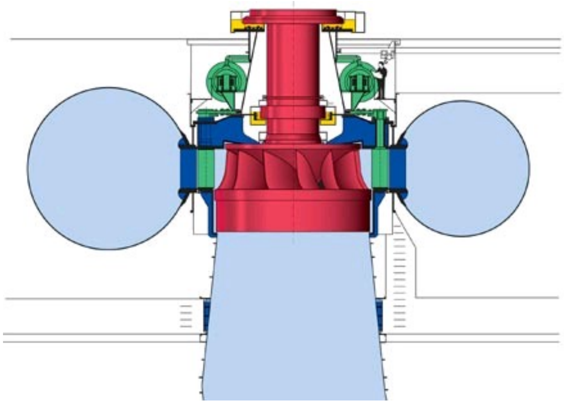
Example
Suppose we have a power coefficient $a = 0.33$. The wind velocity $u_0 = 10$ m/s, the rotation frequency is 0.25 Hz and the rotor radius is 44 m. Calculate the tracking number λ.
Solution
The tracking number $\lambda = \frac{2\pi Rf}{u_0} = \frac{2\pi\ 44 \times 0.25}{10} \approx 6.9$

7.4 Hydro energy
7.4.1 Power from waterfalls
Describing the energy gained in a *waterfall* is rather simple. We just look at how the potential energy of the system converts to kinetic energy and is converted to electric energy via a generator. The flow Q [m^3/s] is of course of great importance as well as the height h of the fall. The potential energy of a volume having the mass m will then become $W_p = mgh$. The density of water is ρ = m/V. Thus we can determine the mass passing the turbine to be ρQ [kg/s]. The mechanical power P the waterfall produces will then be
$P = \rho Qgh$ J/s
Since the density of water is close to 1 kg/dm^3 and $g = 9.8$ m/s$^2 \approx 10$ m/s^2 we get
$P = 10\ hQ$ kW


Power plant at the Trollhättan waterfalls.

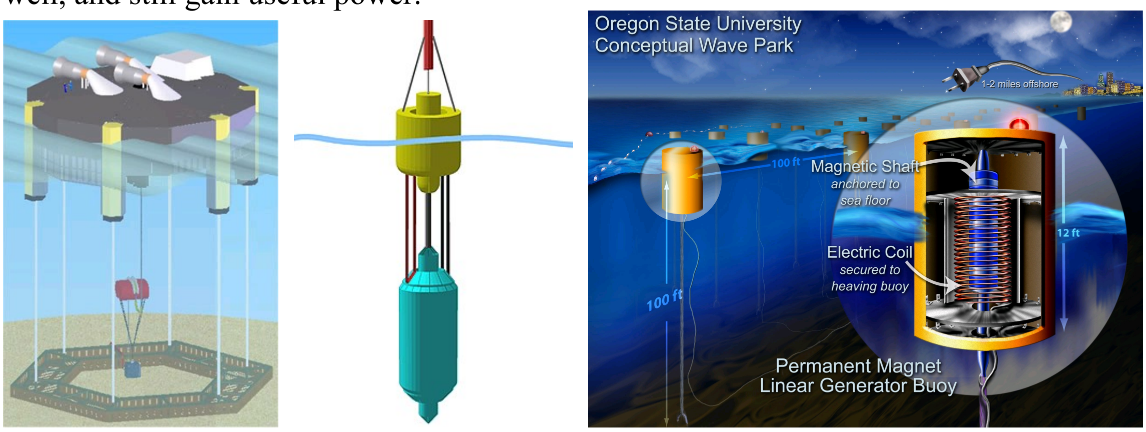

Waterflow around 300 m^3/s, falling height 32 m.

Trollhättan in Sweden has a spectacular waterfall where the flow is 300 m^3/s and with a height of 32 m we get the maximum power: $P = 10 \times 32 \times 300 \text{ kW} \approx 96$ MW
There are several types of turbines used in waterfalls, such as the Francis turbine (Siemens):



It is a turbine of the so-called reaction type developed by J. Francis and is mostly used at high waterfalls (http://en.wikipedia.org/wiki/Francis_turbine).
In Sweden, about 65 TWh is generated by waterpower of totally 130 TWh, if the supply of water is normal. The total power generated is P = 1.6 GW. The Swedish parliament has stated that no more large scale waterfalls can be built. The rivers, Torne, Kalix, Pite and the Vindel Rivers are fully protected from exploiting. However, there is still a potential for exploiting the rivers, almost 30 TWh. The largest plants in Sweden are *Harsprånget* in the *Lule älv* river (830 MW) and *Stornorrfors* in the *Ume älv* river (580 MW). Most waterfall plants are small with powers around some tens or hundreds kW.

7.4.2 Power from the oceans
When looking around the World there are some regions, where wave energy can be converted to electric energy. There are especially rich wave energy areas that can be found at *i*) the Atlantic coast of Europe, *ii*) the West Coast of North America, *iii*) at Hawaii and, *iv*) at New Zealand.
The energy generating bobs are designed to recover useful power from the oceans wave energy. They can be deployed in large arrays offshore. The electrical power of a bob can be around 500 kW in average at the North Atlantic coast. Some bob's natural frequency can be set to match the typical ocean waves or swell, where the periodicity of the Atlantic is T = 10 s, or at the Pacific where the time constant it is longer, around T = 15 s. By matching the periodicity one obtains good energy absorption. Some of the bobs can ride large waves as well, and still gain useful power.



Commercial wave energy generators (Orecon, Wavebob) and an Oregon State University buoy.

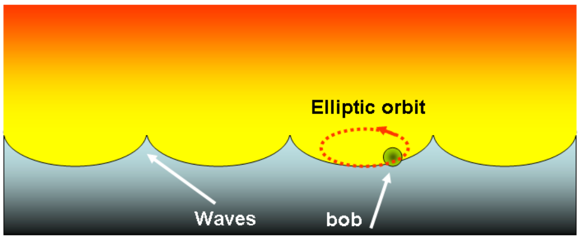
The blowing wind passing the surface of the Oceans and turns the waves into an organized system and generates Ocean waves. An object floating in the seas moves, or bobs, up and down and performs a trajectory showing an elliptical orbit.

7.4.3 Physical concepts

The size of the waves is determined, firstly, by the speed of the wind and, secondly, by the distance over which the wind has the possibility of exciting the waves, the so-called *fetch*. Thirdly, the wave size is given by the depth and topography of the bottom of the sea. (which can focus or disperse the energy of t he waves). A given wind speed has a matching practical limit over which time or distance will not produce larger waves. This limit is called a *completely developed sea*. Let us study the physics behind the waves and look at the Bernoulli equation (p = air pressure, h height over the sea level, u wind speed, ρ air density):

$$p + \rho g h + \frac{1}{2}\rho u^2 = const.$$

This equation shows that when the wind velocity increases, the air pressure decreases, leading to a net force on the surface of the water lifting it upwards, and vice versa, generating ocean waves.



A bob moving in an elliptical trajectory

Example

Look at an area of the sea that covers 100 m^2. Air flows just above the sea level at a speed of 200 m/s. What is the net force on this area of the sea? Also calculate the force per area unit-

Solution

Applying Bernoulli's equation we get (h = 0):

$$p_1 + \frac{1}{2}\rho u^2 = p_2 \Rightarrow \Delta p = \frac{1}{2}\rho u^2 = \frac{1}{2} 1.2 \times 200^2 \text{ Pa} = 24 \text{ kPa}$$

With $p = \frac{F}{A}$ we obtain the net force $F = \Delta p \times A = 24 \times 10^3 \times 100 \text{ N} = 2.4 \text{ MN}$

This is the net force over the whole area. The force on 1 m^2 is $F_1 = \frac{F \times 1}{100} \text{ N} = 24 \text{ kN}$

In the example above we can look on a volume of 1 m^3 including 1 m^2 of the surface. It has the weight mg = 1000g N = 10^4 N, why the lifting force is 2.4 times bigger, so the water moves upwards and a wave is generated.

When studying ocean waves in general, large waves are of course more powerful and the wave power is determined by several parameters, such as wave height, wave speed, wavelength, and water density. The wave motion is highest at the surface and diminishes exponentially with depth. However, wave energy is also present as pressure waves in the deep water. We can take the Tsunamis as an example, where in many cases Earthquakes generate pressure waves that can be transported over large distances, such as over a whole Ocean.

One can show that the potential energy of a set of waves is proportional to the square of the wave height multiplied by the period *T* of the wave, i.e. the time between wave crests. Studies also show that waves with longer period have relatively longer wavelengths λ. They also achieve greater speed *u*.

The potential energy is expected to be equal to the kinetic energy. Wave power is often expressed in kW/m, a useful unit when studying a location such as a shoreline. The following formula expresses the power in kW/m:

$P = const. \times H^2 \times T$ kW/m

Here *H* is the wave height in meters, and *T* the wave period in seconds. The value of the constant is roughly 0.5.

The formula above shows how we can calculate the power of the waves. If we exclude waves generated by great storms, we find that the largest waves reach a height about *H* = 15 m. In the Pacific the period is about *T* = 15 s, whereas the period is around *T* = 10 s in the Atlantic Ocean.

Example
Calculate the wave power in kW/m of an Pacific Ocean wave with wave height 15 m and a period of 15 s.
Solution
Looking at the wave front, we find using $P = 0.5xH^2xT$, that the power will become P=0.5x15^2x15 kW/m = 1688 kW/m ≈ 1700 kW/m = 1.7 MW/m.

In average, those kind of large powers are not to be expected. A more realistic average will be around 50 kW/m.

7.5 Bio Energy

In the earlier days of man when our ancestors made fires from wood, they actually used biomass where energy had been stored from the photosynthesis. We have already given the formulae for this reaction:

$$6CO_2 + 6H_2O + 4.7 \times 10^{-18} \text{ J} \Rightarrow C_6H_{12}O_6 + 6O_2$$

Actually, several other carbon molecules are formed, why this is just a simplified model.

Some 60 photons are participating in the reaction when the incoming photons initiate the reaction. When we look at the reduction of CO_2 to carbon hydrates as well as the oxidation of water to oxygen, O_2, there is actually two different reactions going on simultaneously. These two reactions occur across membranes in the bio domain, where a proton gradient is created through the membrane and the ATP-molecule (adenosine-thriphosphate) is formed from the molecule ADP, which can be found in the literature where the Krebs cycle or citric acid cycle is described (http://en.wikipedia.org/wiki/Citric_acid_cycle).

If we look at the efficiency of the reaction above, where we needed 4.7 x 10^{-18} J for making the reaction pass to the right, we can compare with the energy of the 60 electrons involved. If we calculate the energy of the 60 photons, having an energy of around 2 eV we get 60x2 eV = 120 x 1.6×10^{-19} J = 19 $\times 10^{-18}$ J, to be compared with the 4.7 $\times 10^{-18}$ J of the reaction, the efficiency is 4.7/19 = 24 %.

The main application of biomass is for heating homes, where nowadays, wood pellets are used. However, using wood to generate electric power is very small. Biomass, generally, refers to plant matter grown for use as bio-fuel. However, it also includes plant or animal matter, which can be used to produce fibres as well as chemicals, and also directly, heat. Waste that can be converted to biomass can be used as fuel and is a growing industry. Fossil fuel however, is not considered as biomass, although being “old” biomass with carbon out of the carbon cycle for ages.

A biomass pellet burner

For the moment the prices differ quite a lot when comparing prices between houses heated by electricity respectively heated by pellets. The prices for electricity are around 0.90 SEK/kWh and for biomass pellets around 2000 SEK/ton. An estimate of a house consuming 32000 kWh annually would be slightly above 8 tonnes biomass pellets. The cost of electricity ends up around 29000 SEK to be compared with 16000 SEK for pellets.

7.6 Nuclear energy.

Chapter from Internet course Modern Physics SK180N

7.6.1 Introduction

In
α-particle scattering towards nuclei, one gets a deviation from Rutherford’s theory for high energies of the α-particle (and light nuclei). This gives us a possibility to determine the nuclear radius. Corresponding experiments have been done with electrons, but this was done muck later since a rather high energy is required to give the electron a deBroglie-wavelength of the same order as the diameter of the nucleus. These result showed that the nucleus at a first approximation can be regarded as spherical with a radius *R*, depending on its mass number *A* according to

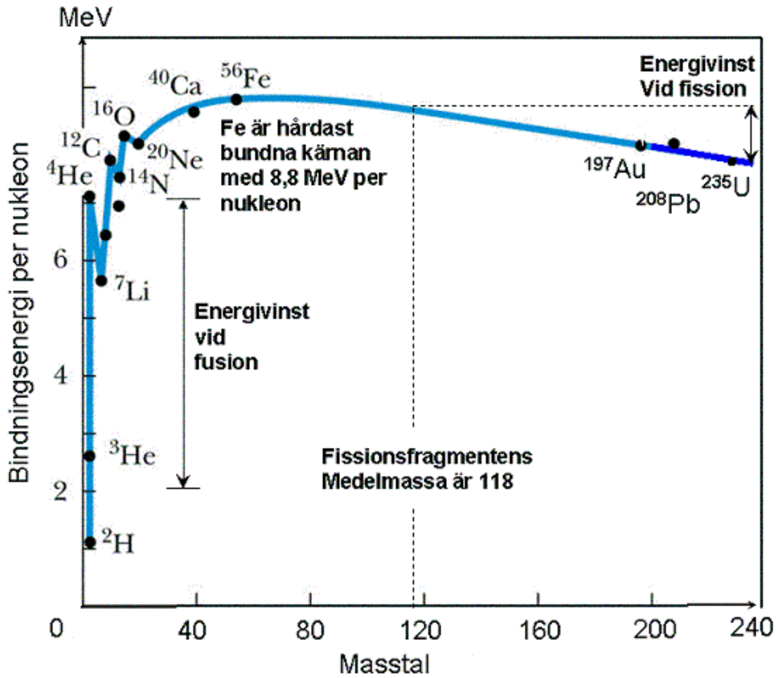
$$R = R_0 A^{1/3}$$

Here R_0 = (1,1-1,4) 10^{-15} m and the lower value is from electron scattering, whereas the higher from α-particle scattering.
The nuclear mass, that can be measured with high accuracy is given in **atomic** mass units defined by the mass of 1/12 of the ^{12}C-atom and has the value

1 u = 1,66056 10^{-27} kg
Perhaps it can be worth noticing that uc^2 = 931,494 MeV and in the same way we get
m_pc^2= 938,3 MeV m_nc^2= 939,6 MeV m_ec^2= 0,511 MeV
The nuclear mass
, M, is less than the sum of the masses of the protons and the neutrons. This *mass defect* Δm corresponds to an energy ΔE_{be} (>0), the nuclear *binding energy*

$$\Delta E_{be} = \Sigma(mc^2) - Mc^2 \ (= \Delta mc^2)$$

If one adds energy to the nucleus, one can split it into its parts. If the electrons binding energies of the atom can be neglected, the binding energy E_{be} for a nucleus with Z protons and $(A-Z)$ neutrons be written
$E_{be} = Z(M_H - m_e)c^2 + (A-Z)m_nc^2 - (M-Zm_e)c^2$
M is the ATOMIC MASS and m_e the mass of the electron. The electron masses vanish in the expression, why
$E_{be} = ZM_Hc^2 + (A-Z)m_nc^2 - Mc^2$
The binding energy/nucleon then becomes $\varepsilon = E_{be}/A$
and ε depends on A according to the diagram below.



We observe from the diagram that one can obtain *kinetic energy* from *rest energy* if
1) two light nuclei add to form a heavier nucleus, in a process called ***fusion***
or
2) a heavy nucleus is split into two lighter nuclei, a process called ***fission***

7.6.2 Fission
If a nucleus with mass number A and atomic number Z decays into two equal parts (a possible decay, but not the most common) the sum of the rest energies before the reaction is:

$$ZM_Hc^2 + (A-Z)m_nc^2 - A\varepsilon_A$$

and after:

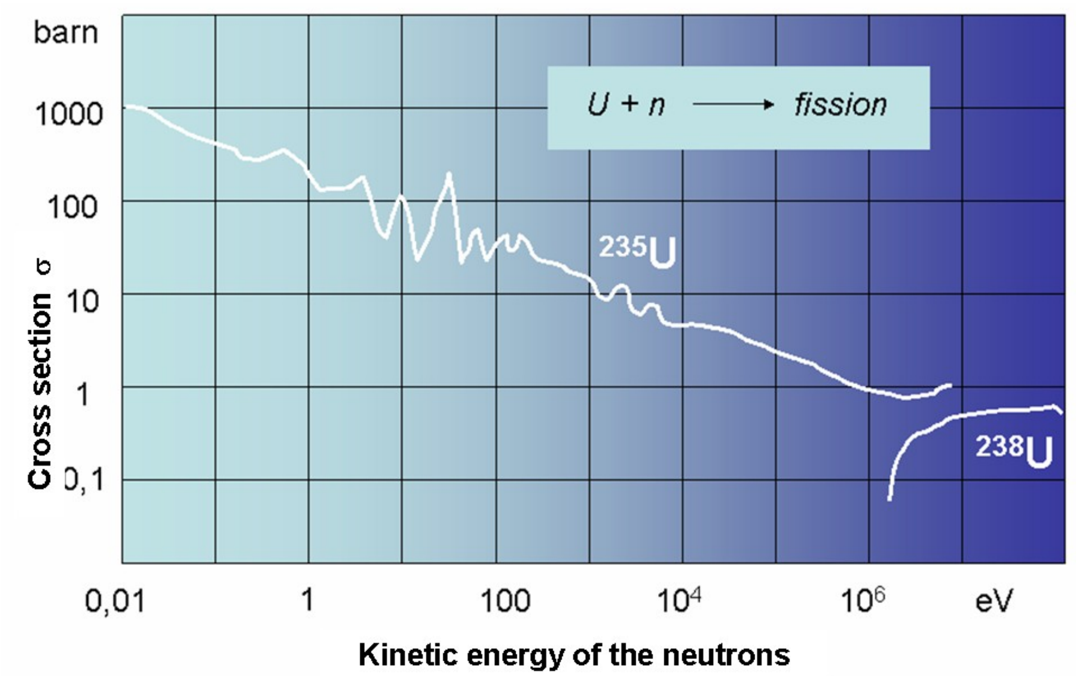
$2(ZM_Hc^2/2 + (A\text{-}Z)m_nc^2/2 - A\varepsilon_{A/2}/2)$

The difference in rest energies (fore-after) becomes

$A(\varepsilon_{A/2} - \varepsilon_A) > 0$

With A = 238 and $\varepsilon_{A/2}$ = 8,5 and ε_A = 7,6 MeV/nucleon respectively, we get
238(8,5-7,6) MeV/nucleus = more than 200 MeV/nucleus!

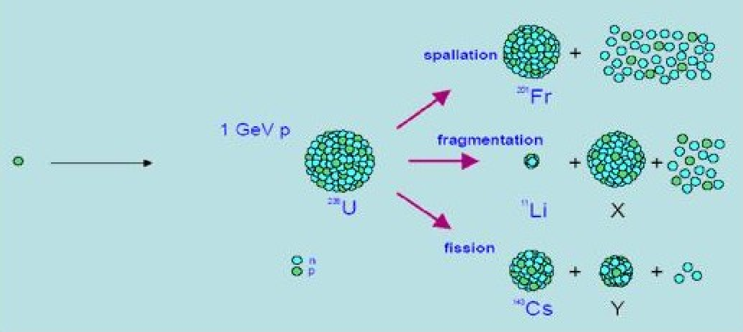
In order to achieve fission, by making the nucleus to become unstable in some way, which can be done, by sending neutrons to hit the nucleus. If one studies normal fission elements like ^{235}U and ^{238}U, we can look at the diagram below, showing the probability or the cross section, for fission to occur. From the diagram we see that it for ^{235}U is enough with slow (thermal) neutrons for the reaction to start. For ^{238}U on the other hand, neutrons with kinetic energies in the MeV-region are needed.



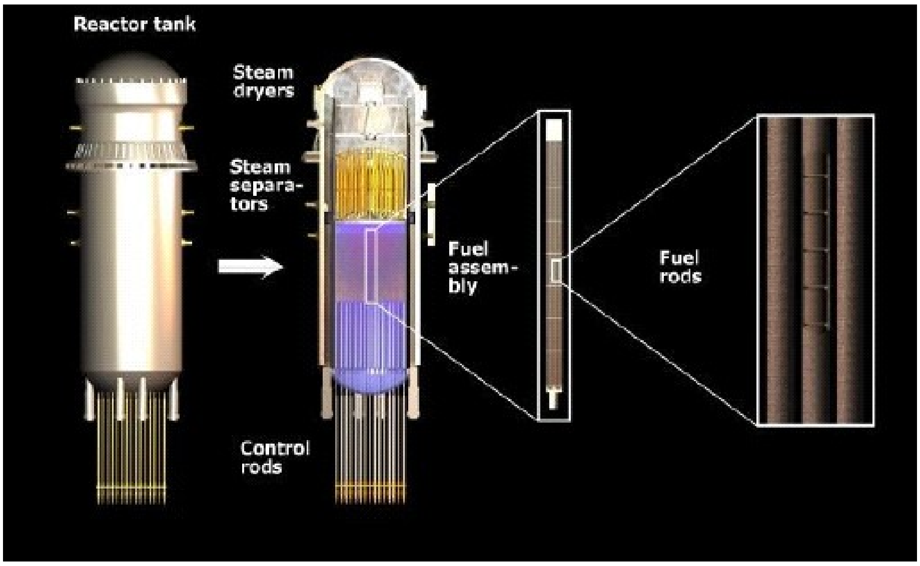
As we have mentioned, normally two equally large fission fragments are not created at nuclear reactions. An example is shown below where ^{235}U captures a neutron:

$$^1n + {}^{235}U \rightarrow {}^{236}U^* \rightarrow {}^{144}Ba + {}^{89} + {}^{89}Kr + 3^1n \text{ or } {}^1n + {}^{235}U \rightarrow {}^{236}U^* \rightarrow {}^{140}Xe + {}^{89} + {}^{94}Sr + 2^1n$$

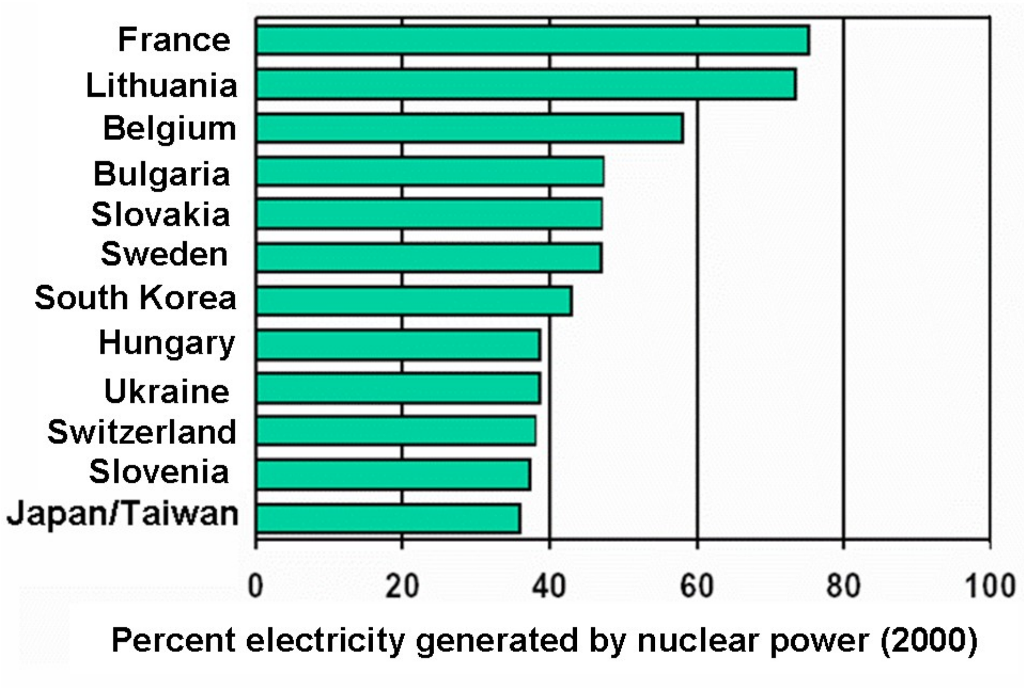
In the figure below, we see different alternatives for fission when a high-energy proton (1 GeV) falls upon a uranium nucleus:



Since there are more neutrons created in the nuclear reaction, all these secondary neutrons can react with another nucleus, thus creating still more neutrons, why a *nuclear chain reaction* can follow. The energy release is due to the kinetic energy of the participating daughter nuclei.

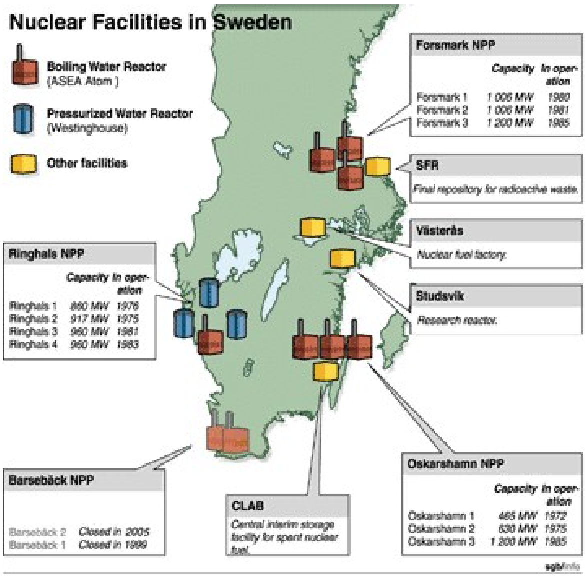


Reactor with control rods, fuel rods, steam part for electricity production



As we can see from the diagram, France and Lithuania are dominating the picture where more than 70% of their electric energy is produced by nuclear power plants.

Totally there are 12 nuclear reactors with a capacity of 10.4 MW. In the year 1997, the reactors produced more than 46% of the Swedish power production.
The nuclear power facilities in Sweden are shown below (www.ski.se) :



However, the two Barsebäck plants have been closed, one in 1999 and one in 2005.

Nuclear Waste in Sweden

The companies that are responsible for nuclear power plants are also responsible for the handling and final disposal of its nuclear waste, according to Swedish law. In order to fulfil this requirement, the nuclear companies formed a joint company SKB, the Swedish Nuclear Fuel and Waste Management Co.
The nuclear fuel that has been spent, and other high-level nuclear waste are temporary placed in storage the Central Interim Storage Facility for Spent Nuclear Fuel (CLAB), near to the Oskarshamn Nuclear Power Plant.

Bedrock disposal
There has not so far been constructed a repository for spent nuclear fuel. However, the industry focuses on a final disposal in the bedrock. In June 2009, a decision was made to select *Forsmark* north of Stockholm to be a disposal site, where to place the repository. The bedrock is more stable than the other possible sites and the water flow is also smaller. In a near future, the nuclear industry will start to construct the depository.

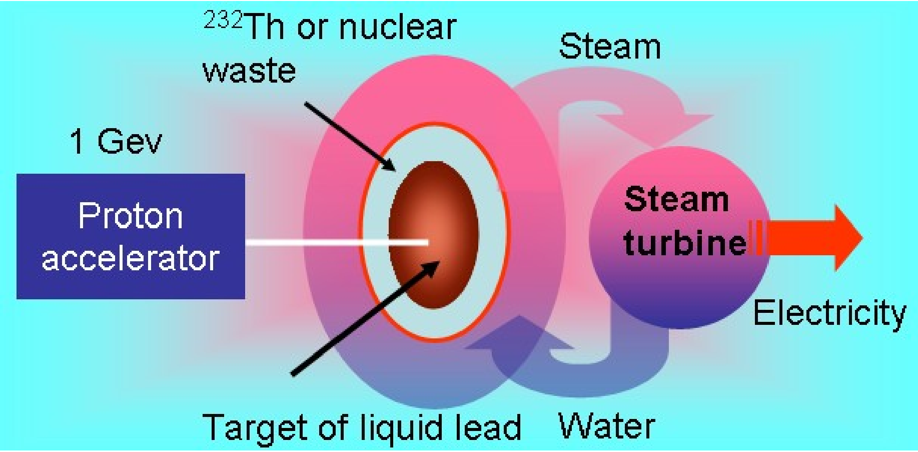
There is a advanced research programme for final disposal going on by the industry. SKI is constantly reviewing the programme. SKI is also developing an independent expertise and know-how in order to be ready when the industry submits the application for constructing the repository. Besides, SKI supervises the existing waste facility in operation.

Some 8000 tonnes of radioactive waste from used nuclear fuel will have been produced by the Swedish nuclear plans if we look at the year 2010. The most dangerous and long-lived isotopes will be left for large amounts for at least 1 000 000 years. This is a huge problem to deal with. The existing model we have for the moment is to keep it in the bedrock, after storage elsewhere for some tens of years.

Example of how CLAB will dispose the used nuclear fuel in the bedrock around 600 m below the surface.

Transmutation
When uranium nuclei catch neutrons, as we have seen, either a nuclear fission process occurs or a heavier element is produced. These heavier elements are mostly highly radioactive and emit dangerous radiation for normally longer times. However, if these elements are put under radiation one can obtain a faster decay and a shortening of the storage time in the bedrock. This process is named *transmutation*.

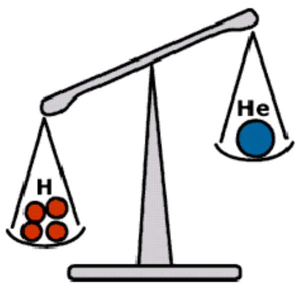
However, this technique is not simple, and there exist no really good solution yet. One uses a particle accelerator and accelerates protons hitting lead, which emits neutrons that are used to radiate the heavy radioactive waste. These ADTW processes (Accelerator Driven Transmutation of Waste) lead to a shorter half-life, and a transmutation has occurred.



Another use of the ADTW process is to work on burning nuclear weapons waste for energy production. One advantage with the process is that it is self-closing; it cannot work without an accelerator pumping in neutrons in the waste. Another advantage is that it is possible to use the enormous energy resources there is in nature regarding ^{238}U and ^{232}Th. However, there are still technical problems that have to be solved.

7.6.3 Fusion

We have already realized that one can extract energy by letting two lighter nuclei form a heavier nucleus thus gaining kinetic energy due to the mass defect. The fusion reaction of the Sun is based on the figure below:



The repulsive Coulomb barrier has to be overcome in order to make the two positive nuclei come close enough to each other, making the distance very short to reach the region where the strong attractive force acts. One can make a simple estimation of how large the Coulomb barrier is for two colliding hydrogen nuclei.

Example:

What average kinetic energy must each proton have if they are to overcome the repulsive Coulomb barrier making two protons cause a fusion?

The Sun radiates isotropically the energy 3.9 10^{26} J/s. The fusion process in the inner part of the Sun is a process containing several steps, where hydrogen is converted to helium. In the first step, two hydrogen nuclei (protons) collide and create deuterium under the emittance of one positron and one (electron)-neutrino according to (*Q*-value is liberated energy)

$^1H^+ + {}^1H^+ \rightarrow {}^2H^+ + e^+ + \nu_e$ (Q = 0.42 MeV)

(After this reaction, the positron is of course annihilated by an electron $e^+ + e^- \rightarrow \gamma + \gamma$($Q$ = 1.02 MeV))
This is a rare process. Normally the protons touch each other elastically, but once in 10^{26} collisions, deuterium is created. It is this "bottleneck" making the Sun burn "slowly" (but still there are some $10^{7.6}$ kg/s deuterium nuclei created depending on the high number of available protons).
When a deuterium nucleus has been created, instantly a ^{3}He-nucleus is created according to
$^2H^+ + {}^1H^+ \rightarrow {}^3He^{2+} + \gamma$ (Q = 5,49 MeV)
During 10^5 years in average (What makes the process take so long?) there are two ^{3}He-nuclei created and one ^{4}He-nucleus:

$^3He^{2+} + {}^3He^{2+} \rightarrow {}^4He^{2+} + {}^1H^+ + {}^1H^+$ (Q = 12.86 MeV)

In this way one can summarize the Suns total *p-p*-cycle to

$2({}^1H^+ + {}^1H^+) \rightarrow {}^4He^{2+} + 2(e^+ + n_e) + 2\gamma$

and if one adds four electron masses on each side, one realizes that one can use atomic masses and can calculate the Q-value of the process:

$Q = \Delta mc^2 = 4(1.007825)uc^2 - 4.002603\ uc^2 = 0.028697\ uc^2 = 26.7$ MeV

Both neutrino particles carry around 0.5 MeV of this energy from the Sun. When all the hydrogen of the Sun has burnt (in about 5×10^9 years), the Sun's mass will not be large enough to start a helium cycle, but the Sun will end its phase as a star to become a Red Giant. Other massive stars can continue the fusion reactions. However, they cannot create fusion elements heavier than ^{56}Fe.
Element with masses larger than 56 are believed to be created in Supernovae explosions. One has tried to copy the elegant fusion reaction of the Sun, but it has failed since the reaction is too "slow". Tempting alternative fusion reacions that would possibly work are:

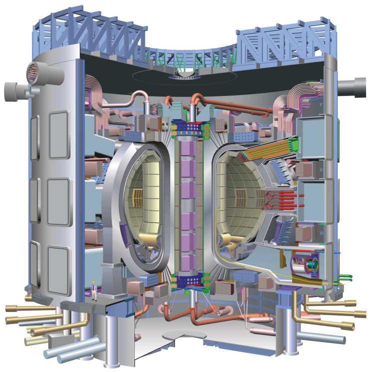
$^2H^+ + {}^2H^+ \rightarrow {}^3He^{2+} + n$ (Q = 3.27 MeV)
$^2H^+ + {}^2H^+ \rightarrow {}^3H^+ + {}^1H^+$ (Q = 4.03 MeV)
$^2H^+ + {}^3H^+ \rightarrow {}^4He^{2+} + n$ (Q = 17.59 MeV)

There are three criteria that have to be fulfilled in order to obtain a successful (Earth based) fusion reactor.

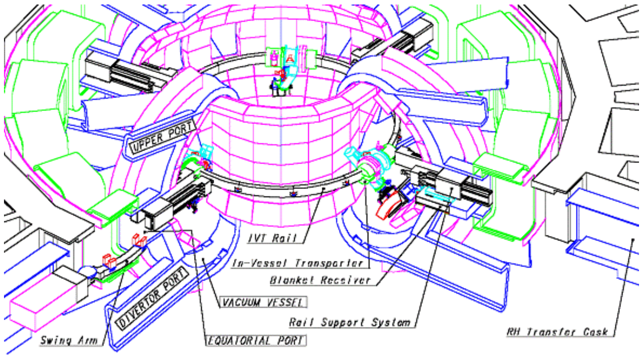
- High particle density (a neutral plasma) n
- High plasma temperature T
- Long enough confinement time (τ) of the plasma. One can show that a fusion reactor using the

Deuterium-Tritium reaction must obey the Lawson criterion: $n\tau > 10^{20}$ s/m^3

Magnetic confinement is shown below in a so-called Tokamak. ITER is the name of the last research plant that is planned to be constructed and in operation 2026.



ITER

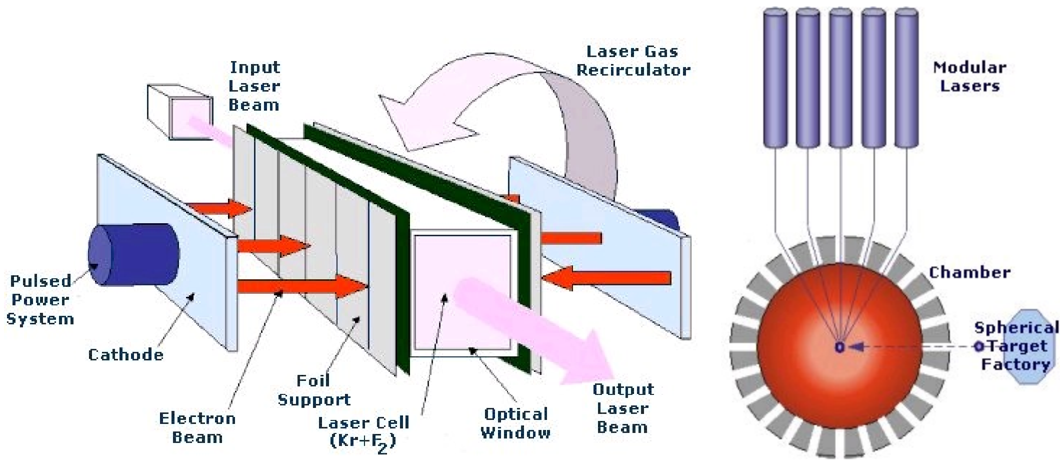


Cross section through ITER

Parameters for ITER:

Total power	**500 MW (700MW)**
***Q* = fusion power/auxiliary heating power**	**≥10 (inductive)**
Plasma burning time	**≥ 300 s**
Plasma max radius	**6.2 m**
Plasma min radius	**2.0 m**
Plasma current	**15 MA (17.4 MA)**
Magnetic field "Toroidal" @ 6.2 m radius	**5.3 T**
Plasma volume	**840 m^3**
Plasma surface	**680 m^2**
Heating/Driving power	**73 MW (100 MW)**

There are different ways in trying to fulfill the Lawson criterion. Another method is to shoot powerful laser beams on to pellets of Tritium (http://www.llnl.gov/str/Petawatt.html) in order to obtain fusion.



A krypton fluoride laser is based on the reaction: (Electron energy) + Kr + F_2 => KrF* + F => Kr + F_2 +light

There are a number of Laser Parameters needed to achieve a fusion reaction in a pellet. The Laser energy has to be around 1.5 MJ. The laser pulse time has to be $4x10^{-9}$ s. This gives an output power of P = (Energy/burning time) = 4 x 10^{14} W.

Contents Chapter 7

Learning goals

Be able to discuss basics of nuclear physics, proton number, mass, neutron number
Be able to discuss the introduction of the decay constant
Calculate the *Q*-value
Be able to discuss the size of the nucleus and nuclear models
Be able to discuss about the nuclear bonding energy
Be able to roughly discuss the bonding energy per nucleon
Describe fission and nuclear reactors
Describe fusion
Give a view of ITER
Give a view of Laser fusion

Advices for reading

Think of the methods used in physics are often based on scattering
Rutherford's contribution was important in the study of the atom
Also think of the neutron beeing difficult to detect, one had first postulated its existence
Make some thoughts over the nuclear decay. It was at first not easy to understand since there were several mechanisms
Think about the difference between α-and β-decay.
Study the decay spectrum at α-and β-decay. Notice the difference and explain it
Discuss the neutrino's role in the decay
Think of the importance of the *Q*-value for reactions. Study it closely
Think of how the γ-decay differs from α-and β-decay
Think of how the γ-decay can be used for practical age determinations
Try to describe the nucleus dimensions with a simple model
Think of the curve for the binding energy per nucleon decides how to obtain fission and fusion
Think of that the *Q*-value plays an important role and the need for new neutrons in the fission process

Readings

- Thornton, Rex, Modern Physics, Saunders
- Krane, Modern Physics, Wiley

- Beiser, Concepts of Modern Physics, McGraw-Hill
- Serway, Moses, Moger, Modern Physics, Saunders
- Eisberg, Resnick, Quantum Physics of Atoms, Molecules, Solids and Particles, Wiley
- Blatt, Modern Physics, McGraw-Hill
- Halliday and Resnick, Fundamentals of Physics, Wiley
- Benson, University physics, Wiley

WEB-readings

- http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html
- http://nobelprize.org/nobel_prizes/physics/articles/lecuyer/index.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1935/index.html
- http://www.llnl.gov/str/Petawatt.html
- http://nobelprize.org/nobel_prizes/physics/laureates/1967/index.html