Environmental Science, Solutions Chapter 8

8.1

If we look through a telescope with an opening of 22 cm at two objects close to each other at a wavelength of 550 nm. What is the minimum angle between the objects under, in radians?

The Rayleigh criterion gives $\theta_C = \dfrac{1.22\lambda}{D} = \dfrac{1.22 \cdot 550 \cdot 10^{-9}}{0.22}$ rad $= 3.05{\cdot}10^{-6}$ rad

Answer: 3.0 μrad

8.2

We are using a telescope with an opening lens with diameter D, at two objects close to each other at a wavelength of λ, giving a minimum angle of resolution between the two objects. If we double the lens diameter to $2D$. How will the resolution change?

The Rayleigh criterion gives $\theta_C = \dfrac{1.22\lambda}{D}$. If we change the diameter to $2D$ we get $\theta_{C2} = \dfrac{1.22\lambda}{2D} = \dfrac{1}{2} \cdot \dfrac{1.22\lambda}{D} = \dfrac{\theta_C}{2}$

Answer: The angle of resolution will drop to half of its initial value

8.3

A Landsat satellite is studying the Earth. The opening of the observing telecope is 45 cm and one uses infrared light with $\lambda = 1.5$ μm. The satellite travels 706 km above the ground. Is it possible to see an object of the size of a car, around 5.0 m?

The Rayleigh criterion gives $\dfrac{1.22\lambda}{D} = \dfrac{1.22 \cdot 1.5 \cdot 10^{-6}}{0.45}$ rad $= 4.07 \cdot 10^{-6}$ rad.

The car is seen from under the angle $\theta = \dfrac{x}{L} = \dfrac{5.0}{706 \cdot 10^3}$ rad $= 7.1 \cdot 10^{-6}$ rad.

The Rayleigh angle is smaller, why we can deduce that the car is observable.

Answer: It is possible to observe the car.

8.4

A satellite is orbiting around the Earth with an orbiting period of $T = 12$ h. What is the satellite's height above the ground?

The acceleration towards the Earth is $a = \frac{GM}{R^2}$, which shall equal the centrifugal acceleration $a_C = R\omega^2$ where $\omega = \frac{2\pi}{T}$, why we get $R = \left(\frac{GM}{\left(\frac{2\pi}{T}\right)^2}\right)^{1/3}$.

$R = \left(\frac{6.67428 \cdot 10^{-11} \cdot 5.972 \cdot 10^{24}}{\left(\frac{2\pi}{12 \cdot 60 \cdot 60}\right)^2}\right)^{1/3}$ m = 26610 km from the centre of the Earth and 26610 km − 6378 km = 20232 km $\approx 2.0 \cdot 10^4$ km.

Answer: $2.0 \cdot 10^4$ km

8.5

A satellite is orbiting around the Earth with an orbiting period of T and orbits with a radius R from the center of the Earth. If the orbiting period would be $8T$, what would the radius of the orbit be?.

The acceleration towards the Earth is $a = \frac{GM}{R^2}$, which shall equal the centrifugal acceleration $a_C = R\omega^2$ where $\omega = \frac{2\pi}{T}$, why we get $R = \left(\frac{GM}{\left(\frac{2\pi}{T}\right)^2}\right)^{1/3}$. If the period increases to $8T$ we get the new radius:

$$r = \left(\frac{GM}{\left(\frac{2\pi}{8T}\right)^2}\right)^{1/3} = \left(64\frac{GM}{\left(\frac{2\pi}{T}\right)^2}\right)^{1/3} = 4\left(\frac{GM}{\left(\frac{2\pi}{T}\right)^2}\right)^{1/3} = 4R$$

Answer: 4R

8.6

Satellites often use interference filters to observe special bands in the spectra. Construct a thin trnasmission filter that operates at 550 nm and uses a material with refractive index 1.48, surrounded by air. Give the thickness of the filter as an answer.

For maximum transmission we need to minimize reflection. The optical path length and the phase difference equals the condition for minimum in reflection which reads $2nd + \left(\frac{\lambda}{2} + 0\right) = (m + \frac{1}{2})\lambda$, which with $m = 1$ gives $2nd = \lambda$ and $d = \frac{\lambda}{2n} = \frac{550}{2 \cdot 1.48}$ nm ≈ 186 nm

Answer: 186 nm