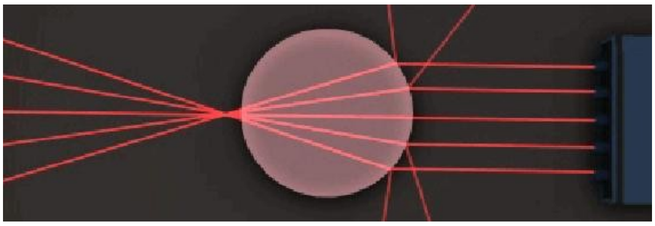
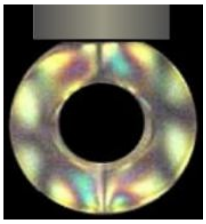
Basic Waves and Optics


1. Electromagnetic Waves

The electromagnetic wave consists of oscillating electric ($\vec{E}$) and magnetic ($\vec{B}$) fields. The electromagnetic spectrum is formed by the various possible frequencies (*ω=2πf*) of the electromagnetic waves. An electromagnetic wave has an electric field $\vec{E}$ and a magnetic field $\vec{B}$ travelling along the positive x-axis and depends on x and t:

$$E = E_0 \sin(kx - \omega t) \quad \text{and} \quad B = B_0 \sin(kx - \omega t).$$

The speed of an electromagnetic wave in vacuum can be written as

$$c = \frac{E}{B} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad c = f\lambda$$

2. Energy flow

The so-called *Poynting's* vector, the rate at which energy is transported is given by

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The intensity (*I*) of a wave [W/m^2] is written by

$$I = \frac{1}{c\mu_0} \frac{E_0^2}{2} = \frac{1}{c\mu_0} E_{RMS}^2, \text{ where } E_{RMS} = \frac{E_0}{\sqrt{2}}$$

The intensity of a wave at distance r from a point source is given by

$$I = \frac{P}{4\pi r^2}$$

3. Radiation pressure

An electromagnetic wave that hits a surface (area *A*), exerts a force (*F*) and a pressure on the surface according to

$$F = \frac{IA}{c} \quad \text{(Total absorption)}$$

If the radiation is totally reflected we have

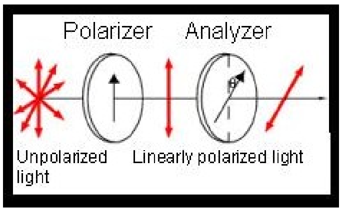
$$F = \frac{2IA}{c} \quad \text{(Total reflection backwards)}$$

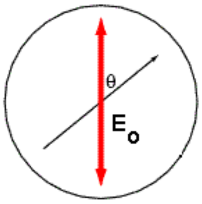
The corresponding radiation pressures will then be

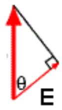
$$p = \frac{I}{c} \text{ and } p = \frac{2I}{c} \text{ respectively}$$

4. Polarization

The electromagnetic waves are polarized if their filed vectors are all in a single plane. Light waves from ordinary sources are not polarized; that is, they are unpolarized or randomly polarized.







A polarization sheet, a Polaroid, can make unpolarized light become polarized, with the intensity

$$I = \frac{1}{2} I_0$$

Malu's law: If the original light is polarized linearly, the transmitted light through a Polaroid tilted an angle θ between the polarization direction of the incoming beam and the polarization direction of the Polaroid, we have

$$I = I_0 \cos^2 \theta$$

Polarization by reflection, Brewster angle θ_p

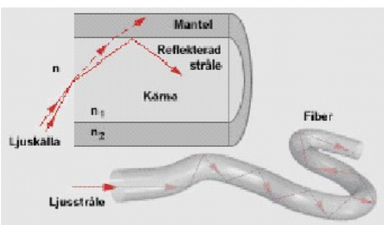
$$\tan\theta_p = \frac{n_2}{n_1}$$

A reflected wave will be fully polarized with its ***E***-vectors perpendicular to the plane of incidence if it hits the boundary at the Brewster angle, θ_p.

5. Geometrical Optics

Snell's law: The refractive index, $n = \frac{c}{v}$, for a material, is defined as the factor with which the speed of light in vacuum is reduced, when entering the material. This causes the light to change direction according to Snell's law.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



Total internal reflection, critical angle θ_c:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Example.

Let the angle of incidence be θ within the fiber at reflection against the mantle. Determine the crucial angle θ_c for total reflection. Let n_1 = 1,53 and n_2=1,28.

Solution

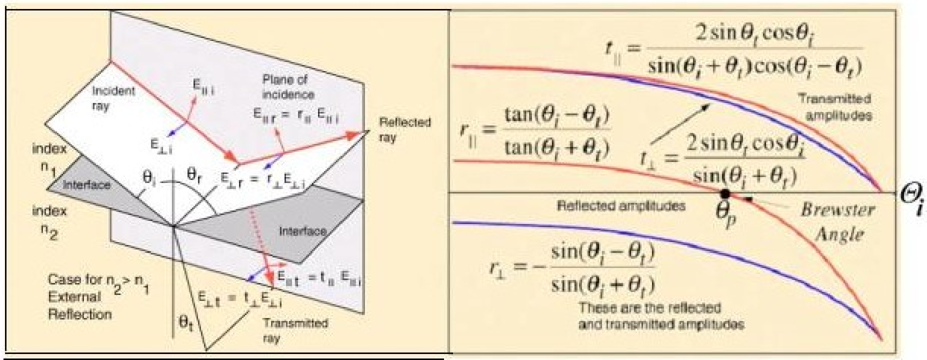
Snells' law with the angle of refraction β = 90º gives:

1,53 sinθ_c = 1,28 sin 90º

sinθ_c = 0,8366

θ_c = 56,9º

6. Fresnel formulae for reflection and transmission



The Components of the Electric field | Fresnel formulae for reflection and transmission

Here we have split the electromagnetic field into two components, $E_\perp$ and E_{II} . The coefficients describing the reflected and transmitted parts can be seen in the picture to the right. R = $r_{//}^2$ gives the coefficient of reflection.

Example

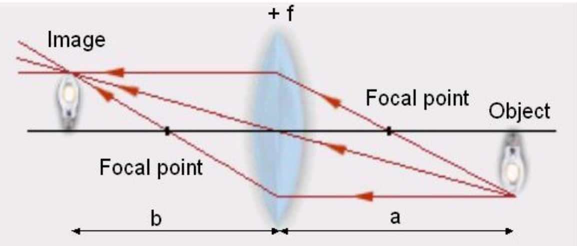
At an angle of incidence θ_i = 50º and refractive angle of θ_p=30º we have $R_{//} = \left(\frac{\tan(50-30)}{\tan(50+30)}\right)^2$ = 0,4%

The normal component has the coefficient $R_\perp = \left(\frac{\sin(50-30)}{\sin(50+30)}\right)^2$ = 12%. The rest of the light is then transmitted; T = $T_{//}$ + $T_\perp$ = 87,6%.

7. Lens equations

Gaussian lens equation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}$$



Newton lens equation

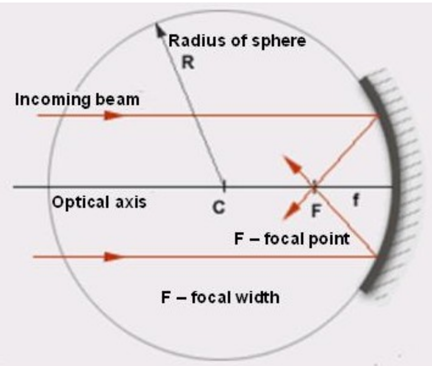
$$xy = f^2$$

, where $x = f - a$ and $y = f - b$

Real and virtual images

An image can be seen as a reproduction of an object via light and if the image can be formed on a surface, it is a *real* image. If the image requires the visual system of an observer, it is a *virtual* image.

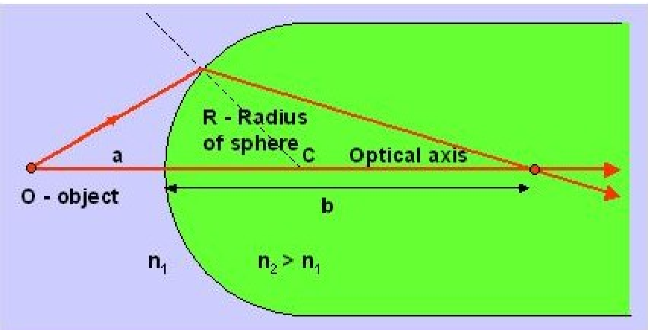
8. Spherical mirror equation

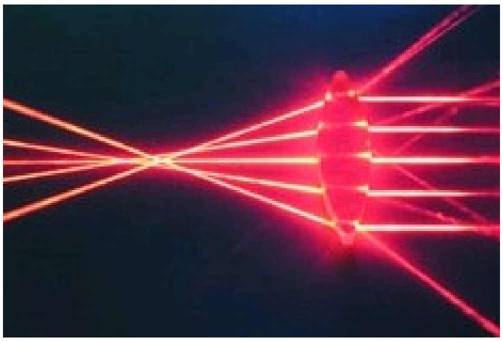


$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \frac{2}{r}$$

, where *r* is the radius of a spherical mirror

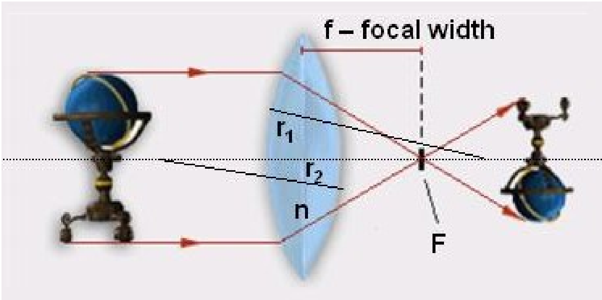
9. Spherical refracting surfaces





$$\frac{n_1}{a} + \frac{n_2}{b} = \frac{n_2 - n_1}{R}$$

10. Thin lenses



$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f} = \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

11. Magnification

The lateral magnification by a thin lens or spherical mirror is

$$m = -\frac{b}{a}$$

The magnitude of *m* is given by

$$|m| = \frac{H}{h}$$

, where *H* is the height of the image and *h* of the object.

12. Optical instruments

A simple *magnifying lens* produces an angular magnification given by

$$m_\theta = \frac{\gamma}{f} = \frac{25cm}{f}$$

γ = 25 cm is the distance for clear seeing

A *compound microscope* produces a total magnification *M* given by

$$M = mm_\theta = -\frac{s}{f_{obj}}\frac{\gamma}{f_{eye}}$$

, where *s* is the tube length, and f_{obj} and f_{eye} are the focal widths of the objective and eyepiece.

A refracting telescope produces an angular magnification m_θ of

$$m_\theta = -\frac{f_{obj}}{f_{eye}}$$

13. Huygens' principle

All points of a wavefront serve as a point sources for secondary wavelets.

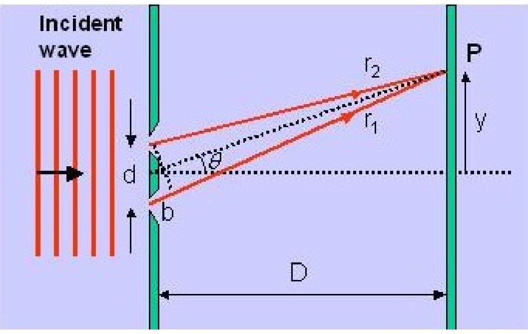
14. Wavelength and refractive index

The wavelength λ_n of a wave in a medium with refractive index n is related to the wavelength in vacuum λ by

$$\lambda_n = \frac{\lambda}{n}$$

15. Interference

Young's double slit experiment



$$d\sin\theta = m\lambda \quad \text{Maxima for } m = 0, 1, 2, 3, \ldots$$

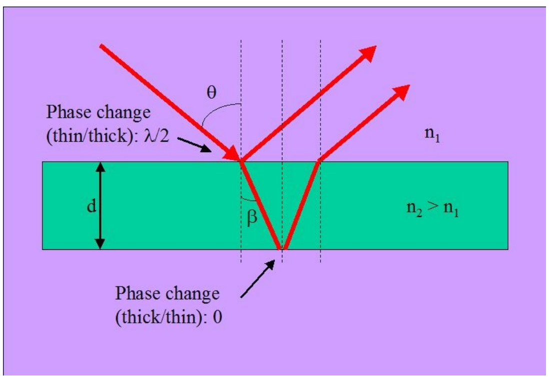
Coherence

It two waves at a point will interfere, they have to be coherent, i.e. their phase difference has to be constant in time.

The double slit intensity

$$I = 4I_0 \cos^2\frac{\phi}{2}, \text{ where } \phi = \frac{2\pi d}{\lambda}\sin\theta$$

Interference in thin films



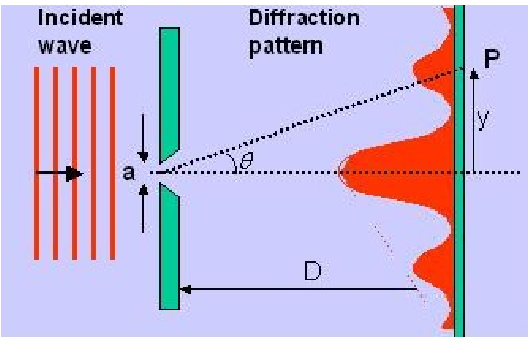
Optical path difference + phase difference = Condition for maximum

$$2n_2 d\cos\beta + (0 - \lambda/2) = m\lambda$$

If normal incidence ($\theta = 0$), we have: $2n_2d = (m-1/2)\lambda$, $m = 0, 1, 2, ...$

16. Diffraction

Single-slit diffraction



Diffraction Minima occurs when $a \sin\theta = m\lambda$, $m = 1, 2, 3, ...$

Single slit intensity

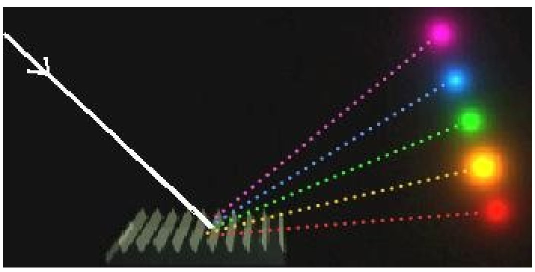
$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2$$ where $\alpha = \frac{\pi a}{\lambda}\sin\theta$ and I_0 is the intensity at the pattern centre

Rayleigh's criterion

First maximum for circular apertures with diameter D

$$\theta_c = 1.22\frac{\lambda}{D}$$

Diffraction gratings



Maxima occur when $n\lambda = d\sin\theta$ and $n = 1, 2, 3, ..$ is the order, d is the slit width

Resolving power $R = \frac{\lambda}{\Delta \lambda} = Nn$, where N is the total number of grooves

Dispersion $\frac{d\theta}{d\lambda} = \frac{n}{d \cos\theta}$

Links:
http://www.biox.kth.se/Education/gm/bok/inledning/index.htm
http://hyperphysics.phyastr.gsu.edu/hbase/geoopt/lenscon.html#c1