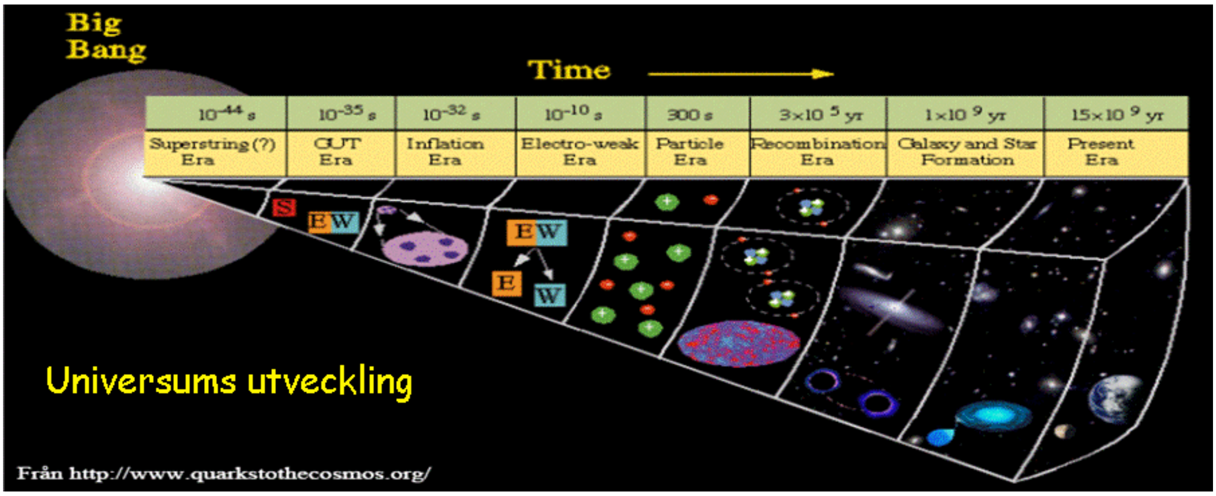
Chapter from the Internet course SK180N Modern Physics



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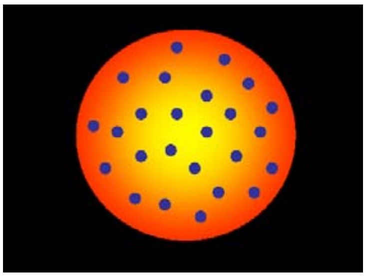
10.4 Nuclear physics

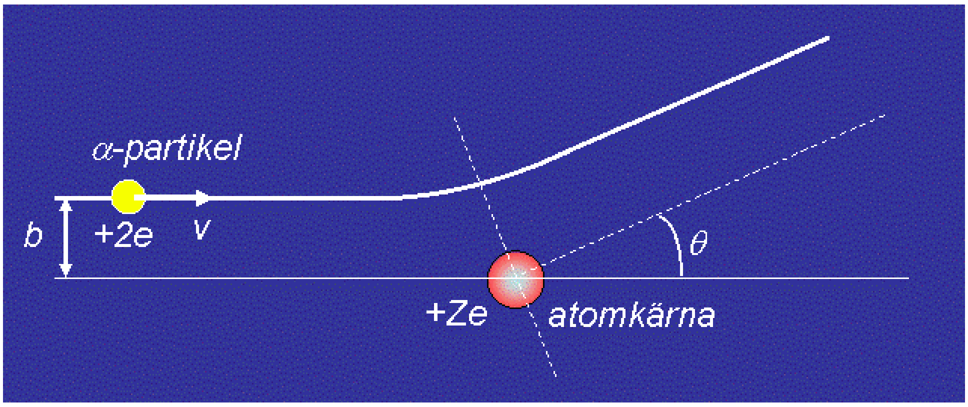
10.4.1 Introduction

Thomson discovered the electron in 1897 (but its mass was still unknown) and presented a model for how the atom was built



Thomsons ”plumpudding”-modell av atomen

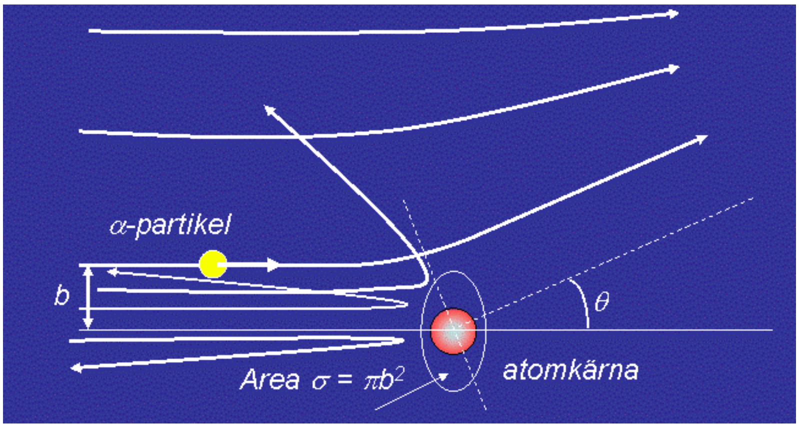
Rutherford (Geiger, Marsden) investigated his ”plum-pudding-model” and found that the positive charge of the atom (and also its mass) is concentrated to a very small region, a nucleus, with a width of around 1/10000 of the atom.



Since one expected very small scattering angles (”A bullet through a sack of snowballs”), the result of the experiment was extremely astonishing. With Rutherford’s own words:

”It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”

Rutherford, Geiger and Marsden investigated thin gold foils that were irradiated by α-particles and examined the angle dependence.



They studied different values on the parameter θ and derived the so called Rutherford’s scattering formula:

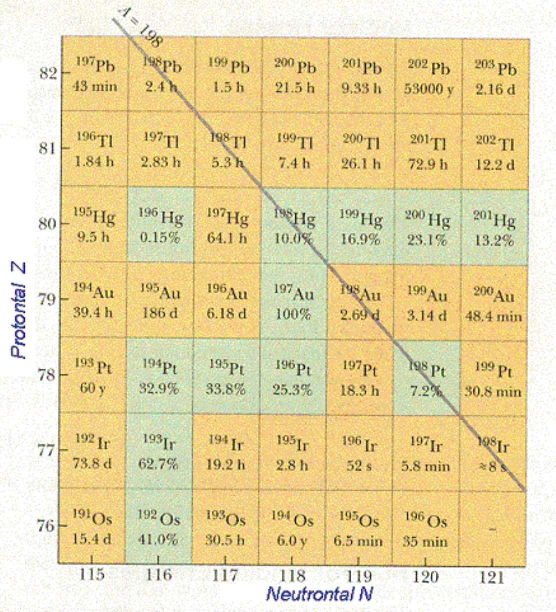
$$N(\theta) = \frac{N_{tot} n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2 \sin^4(\theta/2)}$$

Here N_{tot} is the number of α-particles per area unit that reached the folium, having the thickness t. The distance from the α-source to the folium is r. K is the α-particle.

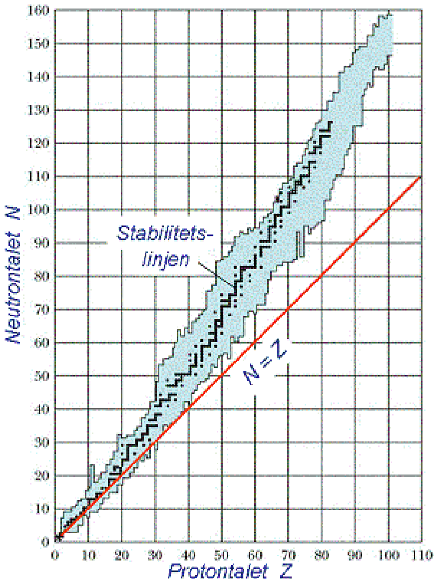
Rutherford suggested the following (1920) reaction p+e$^-$ to produce a neutral particle in the nucleus, needed since the mass of the number of protons is less than the total mass/2. But this solution is not good enough (according to the Heisenberg uncertainty relation). The *Neutron* was thus postulated (1928) and found in 1932 by Chadwick

The nucleus consists of protons and neutrons (called *nucleons*) and the total *mass number* is *A*.

The number of protons in the nucleus is given by the *atomic Z*
The number of neutrons in the nucleus is given by *N*
$A = Z+N$



Protontal Z	115	116	117	118	119	120	121
82	^{197}Pb 43 min	^{198}Pb 2.4 h	^{199}Pb 1.5 h	^{200}Pb 21.5 h	^{201}Pb 9.33 h	^{202}Pb 53000 y	^{203}Pb 2.16 d
81	^{196}Tl 1.84 h	^{197}Tl 2.83 h	^{198}Tl 5.3 h	^{199}Tl 7.4 h	^{200}Tl 26.1 h	^{201}Tl 72.9 h	^{202}Tl 12.2 d
80	^{195}Hg 9.5 h	^{196}Hg 0.15%	^{197}Hg 64.1 h	^{198}Hg 10.0%	^{199}Hg 16.9%	^{200}Hg 23.1%	^{201}Hg 13.2%
79	^{194}Au 39.4 h	^{195}Au 186 d	^{196}Au 6.18 d	^{197}Au 100%	^{198}Au 2.69 d	^{199}Au 3.14 d	^{200}Au 48.4 min
78	^{193}Pt 60 y	^{194}Pt 32.9%	^{195}Pt 33.8%	^{196}Pt 25.3%	^{197}Pt 18.3 h	^{198}Pt 7.2%	^{199}Pt 30.8 min
77	^{192}Ir 73.8 d	^{193}Ir 62.7%	^{194}Ir 19.2 h	^{195}Ir 2.8 h	^{196}Ir 52 s	^{197}Ir 5.8 min	^{198}Ir ≈8 s
76	^{191}Os 15.4 d	^{192}Os 41.0%	^{193}Os 30.5 h	^{194}Os 6.0 y	^{195}Os 6.5 min	^{196}Os 35 min	-



Chemical identical substances, but with different masses are called *isotopes.* They have different *atomic mass* (different A) but the same *atomic number* (same number of protons Z). Nuclides with the same number of neutrons (same N), but different Z are called *isotones*, while nuclei with the same A are called *isobars*.
Of the around 2000 known nuclei only 280 are stabile, the rest are *radioactive*, that is they decay spontaneously. In Nature, there are a few natural radioactive elements, the rest have been produced. The first are called *natural radioactive,* while the rest are called *induced radioactive*.

10.4.2 Natural radioactivity

Radiactive radiation was detected by Becquerel (1896) and can have three components,

- α-particles (that are helium nuclei)
- β- particles (that are electrons or positrons)
- γ- particles (that are photons)

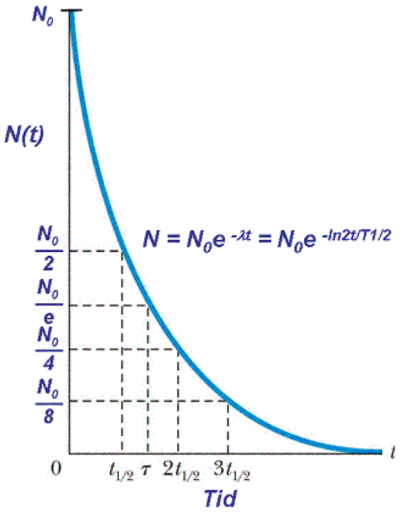
In rare cases, also neutrons and big nuclear fragments can be present.
The number of decays from a radioactive source decreases exponentially. There is a probability that a nucleus shall decay during the time *dt* (dt is supposed to be << 1). If there are *N* radioactive nuclei at time *t*, the number of nuclei will decay (the reason for the minus sign)

$$-dN/dt = N$$

with the solution

$$N = N_0 e^{-\lambda t}$$

N_0 is the number of nuclei at time t=0 and λ is the *decay constant* that varies for different nuclei. It depends on the interaction between the nucleus and the *inner* electrons of the atom and is thus independent of the atom if it is a part of a gas, fluid or a solid.



The activity (decay rate) R, is defined as the number of decays per time unit and can be written

$$R = - dN/dt = N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

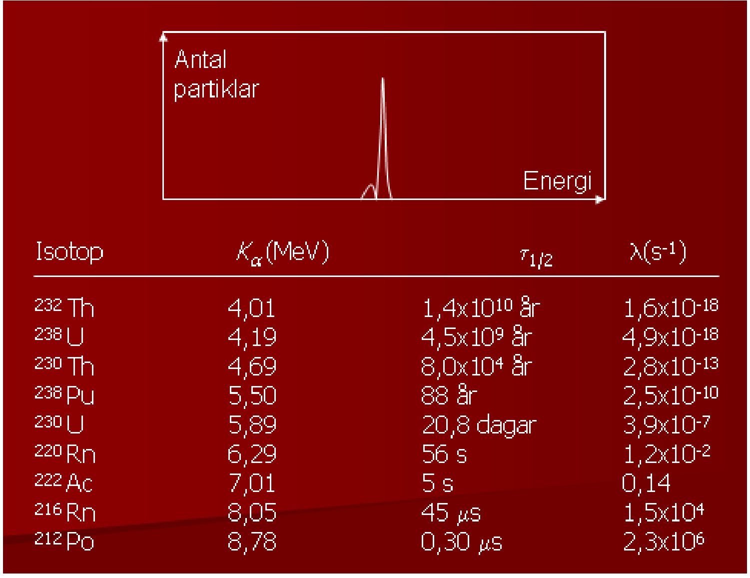
that also can be written

$$R = \lambda N$$

The activity is measured in the unit1 Bq (Becquerel) = 1 decay/s. Earlier the unit 1 Cu (Curie) = 3.7 10^{10} decay/s was used

10.4.3 α-decay

- α- particles are helium nuclei
- α- particles are only emitted of nuclei with A>200
- α- particles kinetic energy has a specified value (or perhaps a few determined values if the daughter-nucleus can exist in different energy levels) and is normally within the region 4-10 MeV, as can be seen in the figure below



Isotop	K_α(MeV)	$\tau_{1/2}$	$\lambda(s^{-1})$
^{232}Th	4,01	1,4x10^{10} år	1,6x10^{-18}
^{238}U	4,19	4,5x10^{9} år	4,9x10^{-18}
^{230}Th	4,69	8,0x10^{4} år	2,8x10^{-13}
^{238}Pu	5,50	88 år	2,5x10^{-10}
^{230}U	5,89	20,8 dagar	3,9x10^{-7}
^{220}Rn	6,29	56 s	1,2x10^{-2}
^{222}Ac	7,01	5 s	0,14
^{216}Rn	8,05	45 μs	1,5x10^{4}
^{212}Po	8,78	0,30 μs	2,3x10^{6}

The reaction formula can generally be written

$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$

as the uranium isotope ^{238}U decay can be written

$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

Example

^{238}U decays via α-decay according to

$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

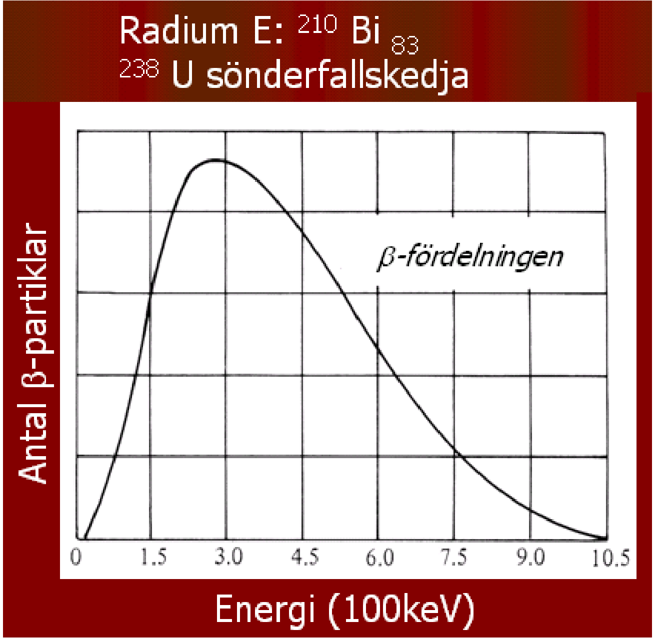
Calculate the liberated energy (Q-value) for the reaction if the atomic masses are the following (in atomic units, u)

238,05079 for ^{238}U
234,04363 for ^{234}Th
4,00260 for ^{4}He

$Q = \Delta mc^2 = \{238{,}05079 - (234{,}04363 + 4{,}00260)\}\ u \times 931{,}5\ MeV/u =$
$0{,}00456\ u \times 931{,}5\ MeV/u = 4{,}2849\ MeV$

10.4.4 β-decay

In the β-decay an electron (or a positron) is released. If you measure its kinetic energy, one obtains a continuous spectrum although one would expect just *one single value*.



In order to fulfil the laws about the conservation of energy and momentum, one has to make one more assumption, namely that there must be still one more particle emitted together with the electron, the so called the *neutrino* (electron neutrino) ν_e. Pauli suggested the existence of the neutrino around 1930 to explain the β-decay. It is extremely difficult, but not impossible to detect. In a famous experiment from 1953 Reines and Cowan succeeded in doing this by using the neutrino flow from a reactor. The probability, *cross section*, for a neutrino reaction to take place is extremely small, of the order of 10^{-20} barn, giving a free mean distance of many thousands of light years in water. 1 *barn* = 10^{-28} m^2.

The reaction formula for the β-decay can generally be written

$^A{}_ZX \rightarrow {}^A{}_{Z+1}Y + e^- +$ anti-n_e for the β^--decay

$^A{}_ZX' \rightarrow {}^A{}_{Z-1}Y' + e^+ + n_e$ for the β^+-decay

For certain nuclei there is another process present, so called *electron capture* that is a kind of β-decay. The nucleus captures a K-electron (inner shell). In this case just only one neutrino is ejected, but it is followed by X-ray radiation from the electronic transition when the temporary free place of the K-shell is filled.

10.4.5 γ-decay

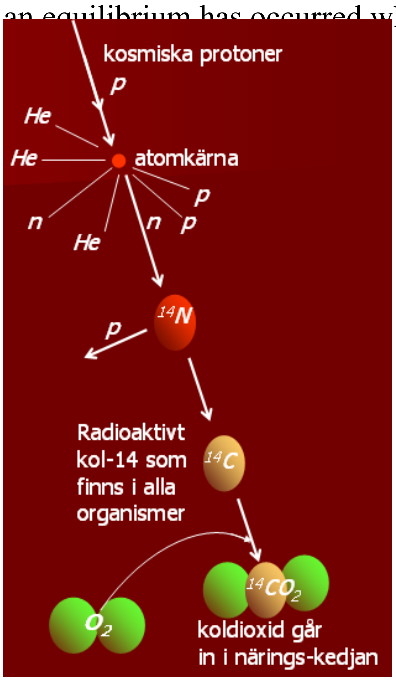
At γ-decay electromagnetic radiation is emitted. The number of nucleons does not change in this process, why the reaction formula generally can be written

$A_ZX^* \rightarrow A_ZX + \gamma$

The nucleus goes from an excited state (denoted *) to a state of lower energy. The spectrum is discrete why the emitted radiation only has just one energy (or a few energies), or one can say that the nucleus energy is ruled by quantum physics. A competing process is the so-called *inner conversion*. In this case, the nucleus transfers its excess energy to an electron (in the K-shell) and is ejected from the atom, thus giving X-ray radiation. Within archaeology, the isotope ^{14}C is used, having a suitable half-life of 5730 years. This isotope is produced in the atmosphere where ^{14}N-nuclei are hit by thermal neutrons and then the following reaction occurs

$^{14}N + n \rightarrow {}^{14}C + H$

The production of this isotope is supposed to have been constant during a long time and that an equilibrium has occurred where the number of created ^{14}C-atoms = number decays.



All living organisms contain carbon and are thus to some extent radioactive. When the organism is alive the activity is constant, but when it dies, it decays according to

$$R = R_0 \cdot e^{-(\ln 2)t/5730}$$

By measuring R one can determine the time of the organisms death.

Perhaps it can be worth noticing that $uc^2 = 931{,}494$ MeV and in the same way we get
$m_pc^2 = 938{,}3$ MeV $m_nc^2 = 939{,}6$ MeV $m_ec^2 = 0{,}511$ MeV
The nuclear mass, M, is less than the sum of the masses of the protons and the neutrons. This *massdefect* Δm corresponds to an energy ΔE_{be} (>0), the nuclear *binding energy* $\Delta E_{be} = \Sigma(mc^2) - Mc^2$ $(= \Delta mc^2)$
If one adds energy to the nucleus, one can split it into its parts. If the electrons binding energies of the atom can be neglected, the binding energy E_{be} for a nucleus with Z protons and $(A-Z)$ neutrons be written
$E_{be} = Z(M_H - m_e)c^2 + (A-Z)m_nc^2 - (M-Zm_e)c^2$
M is the ATOMIC MASS and m_e the mass of the electron. The electron masses vanish in the expression, why
$E_{be} = ZM_Hc^2 + (A-Z)m_nc^2 - Mc^2$
The binding energy/nucleon then becomes $\varepsilon = E_{be}/A$
and ε depends on A according to the diagram below.