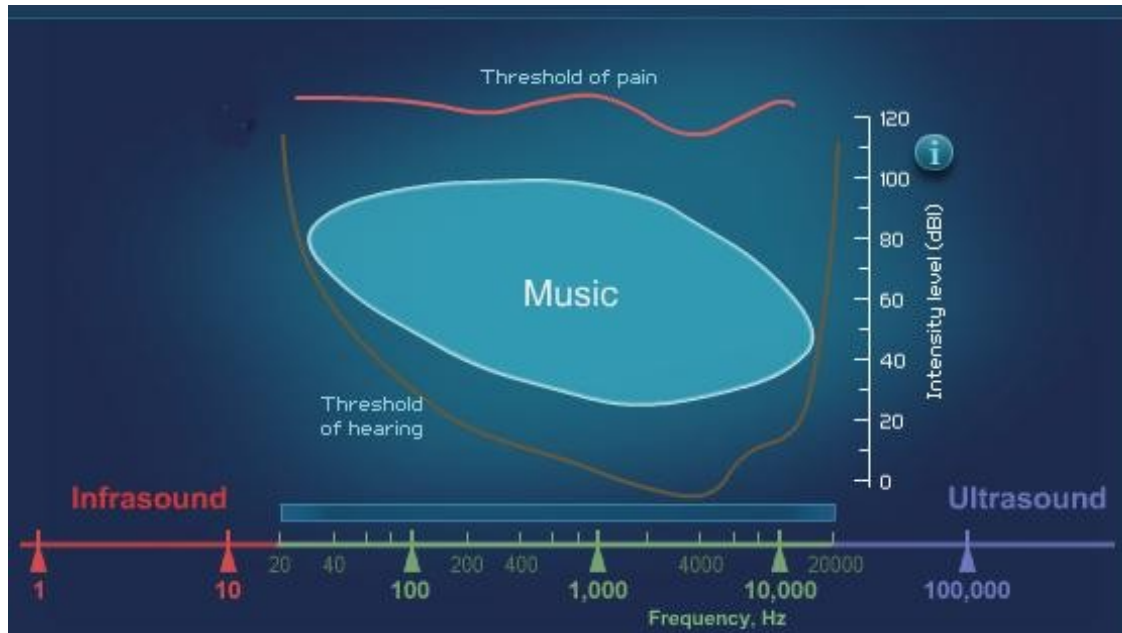


Part I

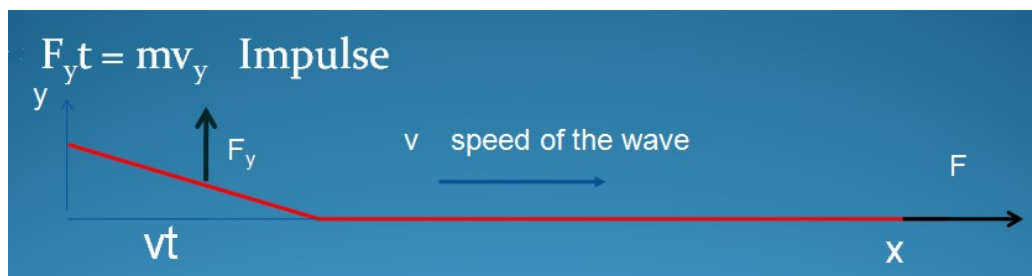
Chapter 2. Acoustics, sound waves, noise and tsunamis



Curve of human hearing

2.1 Waves and wave functions

When we want to describe waves and pulses, we start with the concept waves on a string, in order to introduce the parameters involved. In the figure below we pull a string with



a force F_y upwards thus creating a wave pulse travelling along the string. By studying the force vectors we want to determine the transverse impulse:

$$\frac{F_y}{F} = \frac{v_y t}{vt}$$

The transverse momentum, for the transverse impulse.

$$\mu vt \times v_y$$

leads to the following relation

$$F_y t = \frac{F v_y t}{v} = \mu vt \times v_y$$

This results in $v^2 = \frac{F}{\mu}$ where μ has the unit of mass/length [kg/m] and we get:

$$v = \sqrt{\frac{F}{\mu}}$$

The velocity of the wave is an important parameter as well as the velocity and acceleration of the particles on the string moving in the y-direction. When looking on waves, water waves, sound waves etc some of the most interesting parameters are the energy and power of the



waves.

We will first evaluate the mechanical power P that is given by the expression $P = \text{Energy}/\text{time} = \text{Force} \times \text{distance}/\text{time} \rightarrow P = F \cdot s / t = F \cdot v$.

We now will evaluate the average power of the wave. The power is $P(x,t) = F_y(x,t)v_y(x,t)$ where the relation between F_y and F can be found by the graph above.

$$\frac{F_y}{F} = - \frac{dy(x,t)}{dx}$$

Suppose, $y(x,t) = A \cos(kx - \omega t) \rightarrow dy(x,t)/dx = -kA \sin(kx - \omega t) \rightarrow$ The power, $P(x,t)$ will vary with time according to $P(x,t) = F k \omega A^2 \sin^2(kx - \omega t)$.

However, if we look at this expression over more than one period, we can calculate its mean value or average using the fact that $\sin^2 \alpha + \cos^2 \alpha = 1$. Integrated over one period we find that $\sin^2(kx - \omega t) = \sin^2 \alpha = 1/2 \rightarrow$

$$P_{ave} = F k \omega A^2 (1/2)$$

Using $v = f\lambda$ or $v = \omega/k$ we get and the expression between v and F :

$$v = \sqrt{\frac{F}{\mu}}$$

The expression for the mean power becomes:

$$P_{ave} = \frac{1}{2} \sqrt{\mu F} \omega^2 A^2$$

This means that the power is proportional to ω^2 and A^2 .

2.2 Sound waves as pressure variations. The speed of sound

Considering a longitudinal wave in a medium we can write:

$$\xi = A \cos(kx - \omega t)$$

Here, ξ is the momentaneous displacement of a particle from its equilibrium position.

We can show that the corresponding pressure wave can be written:

$$p = BkA \sin(kx - \omega t)$$

Here, p stands for the momentaneous pressure *deviation* from the average pressure. B is the *bulk modulus* and $k = 2\pi/\lambda$.

The *pressure amplitude* p_{\max} is thus:

$$p_{\max} = BkA$$

One can show that B is connected to the speed of sound v in the medium through:

$$v = \sqrt{\frac{B}{\rho}}$$

The speed of sound in air depends on the temperature. At 20°C the speed is 344 m/s. The following relation holds: $v = 331.4 + 0.606 \cdot T_C$ where T_C is the temperature in °C. Below is given a Table with different gases and sound speeds.

Gas	Temperature (°C)	Speed in m/s
Air	20	344
H ₂	0	1270
N ₂	0	373
O ₂	0	316
CO ₂	0	258
He	20	927

The lighter the gas, the higher the speed of sound.

One can also calculate the speed of sound in gases with the following expression:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Here $R = 8.314 \text{ J/mol K}$ is the general gas constant, T the absolute temperature, M is the molecular weight of the gas in kg/mol and γ is the adiabatic constant, characteristic of the specific gas. For calculations see Hyperphysics:

(<http://hyperphysics.phyastr.gsu.edu/hbase/sound/souspe3.html>).

Speed of sound in selected media:

Medium	Speed in m/s
Sea water	1530
Water	1480
Iron (bulk)	5950
Iron (rod)	5120
Aluminum (bulk)	6420
Aluminum (rod)	5000
Copper (bulk)	5010
Copper (rod)	3750
Lead (bulk)	1960
Lead (rod)	1210
Glass (bulk)	5640
Glass (rod)	5170

2.3 Sound intensity

The intensity $I = \text{Power}/\text{Area}$ has the unit W/m^2 and we get $I = P/S = Fv/S$ where S is the area. However, since the pressure $p = F/S$ we get $I = pv$.

$$I = p(x,t) v_{\text{particle}}(x,t) = BkA \sin(kx-\omega t) \omega A \sin(kx-\omega t) = B\omega k A^2 \sin^2(kx-\omega t)$$

But taking the time average we get $\langle \sin^2(kx-\omega t) \rangle = 1/2$

This leads to the following expression for the mean intensity:

$$I_{\text{av}} = B\omega k A^2 / 2$$

By using the formulae from section 2.2 we can rewrite this expression:

$$I_{\text{av}} = \frac{1}{2} \sqrt{\rho B} \omega^2 A^2 = \frac{1}{2} \rho v \omega^2 A^2$$

Here, ρ and B are material constants and ω and A depend on the wave. We can also write the mean intensity as

$$I_{\text{av}} = \frac{p_{\text{max}}^2}{2\rho v}$$

So, if we can measure the maximum pressure of the wave, we can calculate its mean intensity. However, sound intensity is not directly discussed when studying sound, but instead the so called Sound level, which is described below.

Example:

A sound wave with frequency 1000 Hz in air (temp. 20°C, density 1.3 kg/m³) has a pressure amplitude of 10 Pa. Calculate the displacement amplitude.

Solution:

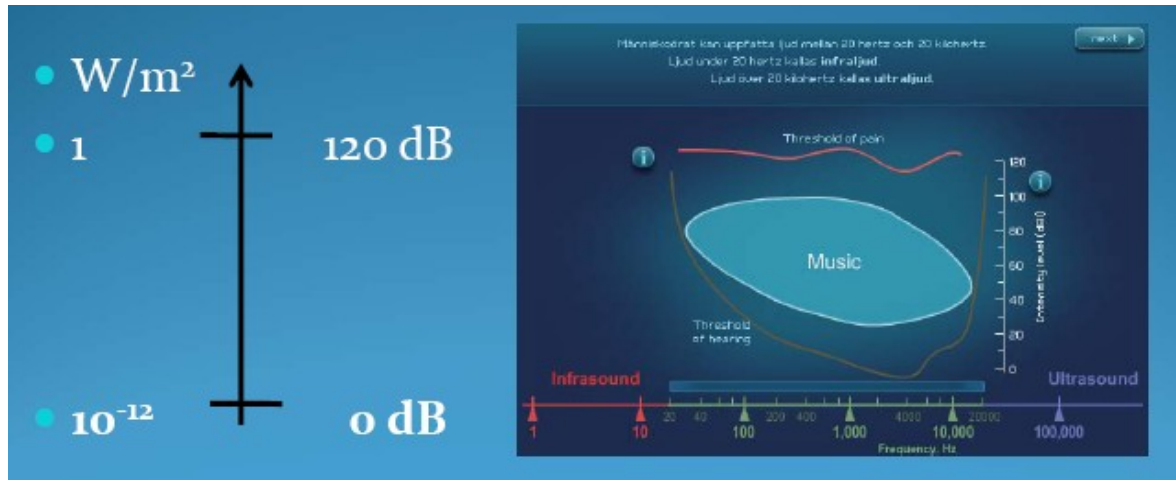
$$p_{\text{max}} = BkA \rightarrow A = p_{\text{max}}/Bk = p_{\text{max}}/\rho v^2 k = p_{\text{max}}/\rho v \omega$$

This gives (in SI-units): $A = 10/(1.3 \cdot 344 \cdot 2\pi \cdot 1000) = 3.6 \cdot 10^{-6} \text{ m}$

Answer: 3.6 μm .

2.4 Sound level. The dB scale

We define the so called sound level by $\beta = 10 \log(I/I_0)$ where I is sound intensity and $I_0 = 10^{-12} \text{ W/m}^2$ is the threshold of hearing. As can be seen in the figure 0 dB corresponds to the intensity 10^{-12} W/m^2 which is called the threshold for listening and 120 dB corresponds to 1 W/m^2 and is the level for pain. In the frequency region around 5 kHz man can hear the weakest sounds.



Example

We can describe the dB scale by an example. Suppose we have an airplane starting at Bromma airport around 7 km from KTH Campus. The sound level 10 m from the airplane is 120 dB . What is the theoretical sound level at KTH?

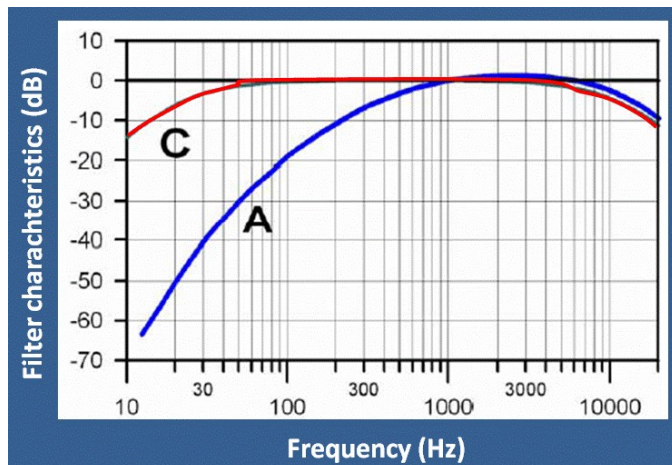
Solution

$$\beta_2 - \beta_1 = 10 \log(I_2/I_1) = 10 \log(r_1/r_2)^2 = 10 \log(10/7000)^2 = -57 \text{ dB}$$

This means that $\beta_2 = 63 \text{ dB}$, corresponding to normal speech level at 1 m distance.

However this is only the theoretically maximal sound level. There are different mechanisms such as water vapour causing absorption, density fluctuations causing scattering etc., which will cause the observed sound level to be undistinguishable from the background at the KTH campus.

dB(A). Since the human response is different for different sound frequencies vary considerably (look at the figure above) one has introduced the unit dB(A). Human response is highest between 1000-4000 Hz. In order to make it possible with one unit to describe the sound level one weights the frequency region with highest weight at 1000-4000 Hz and different weights at the ends of the human response curve. This is the background for the unit dB(A). Below are shown both the dB(A) and dB(C):

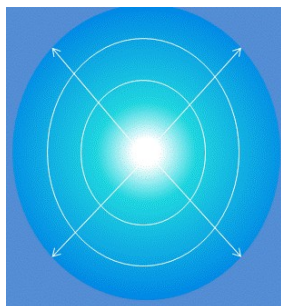


Above we have the characteristics for the dB(A) and dB(C) scales that are most commonly used. In order to prevent low-frequency noise, the dB(A) scale is used while the dB(C) scale is used for short pulse intensive noise.

Spherical sound waves

If we have a point source emitting spherical sound waves, we can describe how the sound intensity, I (W/m^2), changes with distance, r :

$$I = \frac{P}{4\pi r^2}$$



In the example above, about how the sound level changes with distance, we found that an aeroplane starting at Bromma airport could be heard in Stockholm 10 km away. The reason for the damping of the sound is by scattering and absorption by water vapour in the air. Thus, we don't observe the sound from distant sources. Let us introduce a damping factor k (m^{-1})

that damps the sound intensity exponentially with distance: $I_{damping} = e^{-kr}$. The damping constant k varies strongly with meteorological conditions, e.g. wind, rain or snowfall.

Thus, the intensity of spherical waves and damping of the intensity can be described as:

$$I = \frac{P}{4\pi r^2} e^{-kr}$$

Example

An X2000 train passes a train station. The acoustic power emitted from the train (both engine noise and speed noise) is assumed to be 0.44 kW. One has measured the damping coefficient to be $6.0 \times 10^{-4} \text{ m}^{-1}$ (at 20°C).

Calculate the noise level at a distance of 1 km.

Solution

Applying the formula $I = \frac{P}{4\pi r^2} e^{-kr}$ we obtain

$$I = 440 / (4\pi \times 1000^2) e^{-0.6} = 1.9 \times 10^{-5} \text{ W/m}^2.$$

The noise level is thus: $\beta = 10 \log(1.9 \times 10^{-5} / 10^{-12}) = 73 \text{ dB}$, which is in the same region as the noise from a vacuum cleaner at a distance of 1 m.

Note: Without damping we would get:

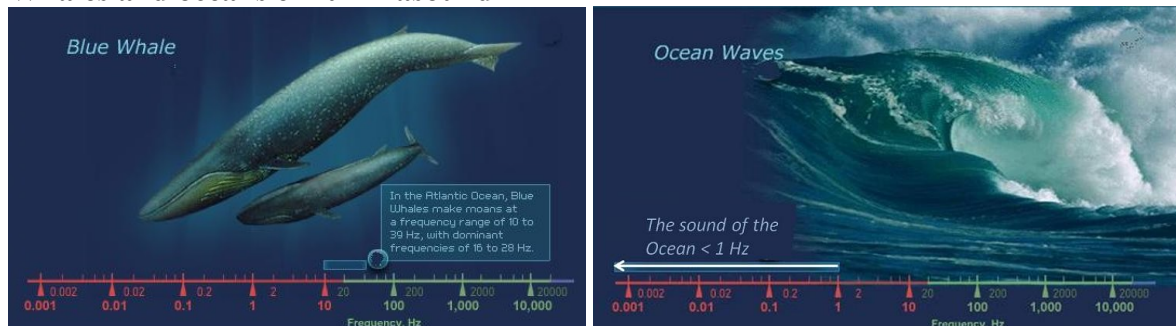
$$I = 440 / (4\pi \times 1000^2) = 3.5 \times 10^{-5} \text{ W/m}^2, \text{ giving a noise level of about } 75 \text{ dB}.$$

Other factors besides the distance influencing the damping are also temperatures and temperature gradients. The speed of sound is temperature dependent why a spherical wave can be focused just like light with optical lenses. If the temperature is higher at higher altitudes we can have high sound levels at longer distances.

2.5 Infrasound and the environment

Infrasound is sound with frequencies at 4 Hz and below. However, many infrasound sources emit sound in a wider frequency range also above 4 Hz. Infrasound is interesting in many ways. It can be detected at very long range by applying simple equipment. However, the sound waves are of course rather slow, since the speed of sound is around 340 m/s at normal temperature and pressure. In spite of that, it can be used to detect various phenomena, such as meteorites, bomb explosions, nuclear tests, avalanches, oil bursts, earth quakes, thunder and lightning, volcanoes etc. Many animals communicate with infrasound, such as whales and elephants.

Whales and oceans emit infrasound

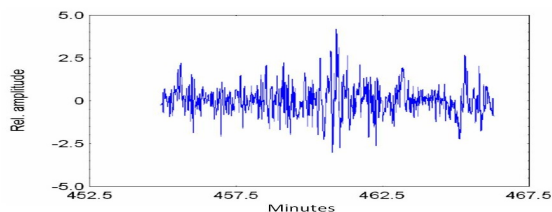


Blue whales are known to use infrasound wavelengths to communicate, since the infrasound can travel thousands of kilometres through the oceans. The oceans act as a super conductor due to its salty content. It is also a fact that both temperature and pressure variations found at certain depths will act as a kind of “voice tubes” that can channel the whale calls further than usual which makes it possible for whales on the other side of the ocean to hear the calls.

Meteorites emitting sound waves



Every day, Earth is hit by meteorites, although most of them are of small size. Sometimes they can be larger stones of the size around 1 m. A larger meteorite that hit the atmosphere



was supposed to have caused the death of the dinosaurs! Researchers claim that the impact site was found to be at the basin of the Yucatan peninsula that was fatal to many life forms 65 million years ago. A much smaller meteorite that could have caused a catastrophe happened during the summer 2002 over the

Mediterranean Sea. The meteorite had a diameter of 2 m and appeared in the atmosphere above Haifa. It continued in a steep trajectory towards a point south-west of Crete, where it exploded above the Mediterranean. The energy of the explosion was estimated to be around 25 kT (The energy release of 1 kg TNT, Trinitrotoluene corresponds to 4.2 MJ), i.e. more than the energy released in the Hiroshima bomb. Thus, large incident meteorites can be taken for nuclear blasts. The figure to the left shows the meteorite signal found in Lycksele in the northern part of Sweden at a distance of 4300 km from Crete.

2.6 Noise. Protecting against noise.

Noise is defined as all kinds of unwanted sound. It can be sound that is both dangerous for our hearing system or only disturbing sound. In Sweden “Arbetsmiljöverket” (www.av.se) is responsible for defining limits of high sound levels. The sound level limit is 80 dB(A) when one looks at exposure up to 8 hours. However, at working places, 40 dB is considered to be the limit as a background noise for making conversations without being hindered by noise.

Personal have to use Ear shield if;

- * If the sound level during an 8 hours work day is 85 dB(A) or more.
- * If the highest sound level is 115 dB(A) or more.
- * If the impulse peak value is 135 dB(C) or more.

Ear shields have to be marked CE meaning that the shields support the EU demands.

H – stands for damping of high-frequency noise.

M – stands for damping of mid-frequency noise.

L – stands for damping of low-frequency noise.



There are several types of devices to make a shield against noise. Small shields to be put directly in the ear normally damp the noise by 9-35 dB.

Sources of noise

The source in our surrounding that stands for the most noise is the traffic caused by cars. Secondly we have noise from railroad and aeroplanes. Almost 20% of the Swedish population is disturbed by traffic noise. According to the so called “Boverket” (www.boverket.se), which in Sweden is responsible for the construction of buildings and the rules of construction. The maximum sound level caused by traffic inside a building is set to 45 dB(A), 30 dB(A) in living rooms and 35 dB(A) in kitchens.

Looking at “Arbetsmiljöverket”, one discusses how to prevent sources of noise that can damage our hearing. In preventing large noise levels at working places will make a good development of the environment for the employees. This enables the employees working with full capacity. It is the employer that has a large responsibility for making the environment sound and healthy. Noise elevation at workplaces can cause various medical effects, such as hearing disturbances, hypertension (high blood pressure), different heart diseases besides annoyance and sleeping disorders.

Low frequency noise

Noise with low frequencies can result in disturbances already at levels just above the threshold for hearing. People can get tired and obtain headaches, difficulties in being concentrated etc. The distance between the threshold and the level where one feels the level to be unacceptable is less for low frequency sound than for high frequency sound. People with reduction of their hearing are most often disturbed by low frequency sound. Vibration in stomach and chest can be observed for low frequency sound since the resonance frequency in this area is around 50 Hz.

Example

When standing 45 m on the side of a highway when a car is passing by, one measures the maximum noise level to be $\beta = 75$ dB(A). What is the sound level when three identical cars pass simultaneously during the same conditions?

Solution

Since the noise from the cars is not coherent, we can just add their intensities at the listener's point. Thus the total intensity will be $3I_1$ where I_1 is the intensity caused by the first car. The dB scale gives: $\beta_1 = 10\log(I_1/I_0)$ and $\beta_2 = 10\log(3I_1/I_0) = 10\log 3 + 10\log(I_1/I_0) = 10\log 3 + 75 = 4.77 + 75$ dB = 80 dB

2.7 Tsunami waves

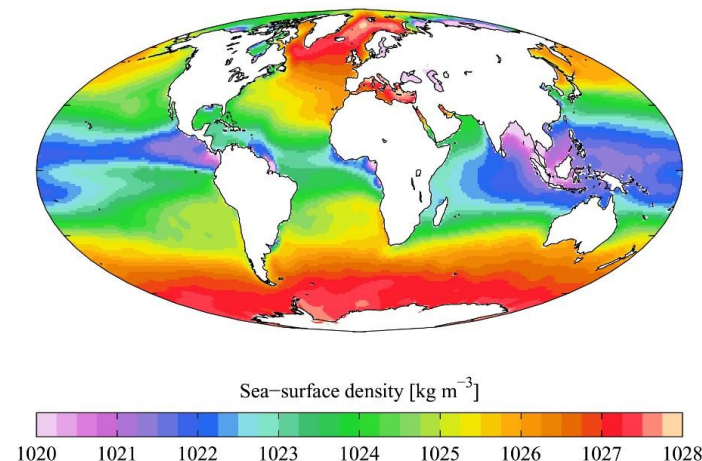


In Japanese, tsunami means harbor waves, often seen when waves from the deep ocean hits the shores around the coastline, most often after earthquakes from the bottom of the ocean. Information about Tsunami waves can be found on Tsunami research facilities, like [NEES Oregon State](#) and the general Network for Earthquake Simulation, [NEES](#).

Let us first study water temperatures and ocean water densities. In general, the ocean has a specific vertical structure. It has an upper layer with a thickness that ranges from around 20 m to 200 m in thickness. This layer consists of solar heated water, normally warmer than the deepwater below. This means that the density of the upper layer is less dense as compared to the deep water of the ocean. Only few areas have weak density stratification (North Atlantic).



The [NOAA](#) organization measures water temperatures around the World and the measurements results in figures showing surface water densities, as can be seen in the WOA atlas.



The densities are highest around the poles, especially in the South pole areas. We have the highest water density (1000 kg/m^3) at a temperature of 4.0°C for normal water. Salt water has a higher density than normal water, as can be seen.

It is not just the heating of stationary water that has to be discussed, but also the so called **thermohaline circulation** (See Environmental Science SK183N, Course I). This process results in a sinking of water in the polar areas followed by a deep transport of water to the subtropics, and in turn followed by a surface transport, again back to the Poles.

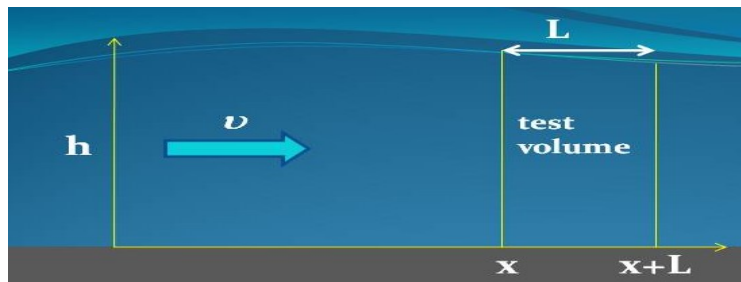
Tsunami wave models

However, we start with a simplified mathematical model of the structure of that does not take into account such aspects as varying ocean bottoms, density gradients that are continuous. Firstly, we will start with the simplest model leading to the shallow water equations. Then, secondly, we will assume an ocean with a flat bottom, two density zones like in the figure below. Still, this treatment will show the characteristics of Tsunami waves.

If we assume the bottom is flat and the different zones with depths h_1 and h_2 , with densities ρ_1 and ρ_2 the velocities of waves in the zones will be v_1 and v_2 .

Shallow water equations

We start with a model where we have a *flat ocean bottom* with the depth $h=h(x,y,t)$, and only one uniform density, ρ . We introduce a test volume with width L .



We will make some other assumptions, such as small depth variations compared to the depth, h , a flow velocity $v = v(x,y,t)$ that is independent of the depth, h . Other parameters, such as friction is neglected.

Then the pressure is obeying the hydrostatic pressure equation that with differentials is given by:

$$\frac{dp}{dz} = -\rho g$$

Integrating, we obtain $p = \rho g(h - z)$ when putting the pressure to 0 at the surface with $z = h$. This is the normal pressure relation where we have the pressure proportional to the depth.

Since both h and v are functions of (x,y,t) we use the following operator, $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$ to simplify the expression with p as a function of h . Thus, we can rewrite the expression above:

$$\nabla p = \rho g \nabla h$$

In order to derive an equation for the velocity of the wave, we start with Newton's law

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

Coriolis force

Since p is used for both momentum and pressure, we will only use p for pressure and mv for momentum. We add the Coriolis force due to the rotation of the Earth. The Coriolis force per mass unit can for a system moving with velocity v can be expressed as

$$-2\vec{\Omega} \times \vec{v}$$

Here Ω is just the rotational angle velocity of the Earth. Let us separate Ω in horizontal (Ω_H) and vertical (Ω_Z) components. We then end up with:

$$\vec{\Omega} = \Omega_H \times \Omega_Z \hat{z}$$

Doing so, we get the following expression for the Coriolis force:

$$- 2\Omega_H \times \vec{v} - 2\Omega_Z \hat{z} \times \vec{v}$$

However, since the velocity v is mostly directed in the horizontal direction, the first part of the Coriolis force will be: $- 2\Omega_H \times \vec{v} = 0$. Thus, only $- 2\Omega_Z \hat{z} \times \vec{v}$ remains of the Coriolis force.

We now introduce the **Coriolis parameter**, $f_C = 2\Omega_Z = 2|\Omega| \sin \phi$. Here, ϕ is the latitude.

This gives the expression for the Coriolis force on a mass unit:

$$- f_C \hat{z} \times \vec{v}$$

Now we apply Newton's law together with the pressure equation and end up with:

$$\frac{d\vec{v}}{dt} = -g\nabla h - f_C \hat{z} \times \vec{v}, \text{ that becomes } \frac{d\vec{v}}{dt} + g\nabla h + f_C \hat{z} \times \vec{v} = 0$$

We should remember that \vec{v} is the horizontal velocity moving in the x and y directions per mass unit. We can apply the *chain rule* and express $\frac{d\vec{v}}{dt}$ in the following way:

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + v_x \frac{\partial \vec{v}}{\partial x} + v_y \frac{\partial \vec{v}}{\partial y} \text{ which can be rewritten as } \frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \text{ for short.}$$

In order to determine v_x , v_y and h we need the expanded Newton's equation above and also the conservation of mass. We work with the square test volume in the figure above, with the sides L , height h and thus having a mass of $M_{test} = \rho hL^2$.

Let us study how the mass of the test volume changes with time. We get at the positions x and $x+L$:

$$\frac{dM_{test}}{dt} = \rho L [\Delta h v_x(x) + \Delta h v_y(y)], \text{ or more in detail:}$$

$$\frac{dM_{test}}{dt} = \rho L [h(x)v_x(x) - h(x+L)v_x(x+L) + h(y)v_y(y) - h(y+L)v_y(y) + h(y)]$$

However, this expression can be further simplified: $\frac{dM_{test}}{dt} = -\rho L \left[\frac{\partial h v_x}{\partial x} + \frac{\partial h v_y}{\partial y} \right]$ or with

$M_{test} = \rho hL^2$ that gives the total mass of the water column:

$$\frac{d(\rho hL^2)}{dt} = -\rho L \left[\frac{\partial h v_x}{\partial x} + \frac{\partial h v_y}{\partial y} \right]. \text{ This expression can be further simplified as}$$

$$\frac{\partial h}{\partial t} = -\nabla \cdot (hv), \text{ or with } \frac{\partial h}{\partial t} + \nabla \cdot (hv) = 0$$

This equation can be rewritten as a *differential equation* that determines both wave velocity and height of the water column:

$$\frac{dh}{dt} + h \nabla \cdot \mathbf{v} = 0$$

These types of differentials have solutions of the exponential type (See below).

Solution of the simplified shallow water equations

Let us start with $\mathbf{v} = \mathbf{v}_0 = 0$ and $h = h_0 = \text{constant}$. Then introduce a small disturbance of the height, η , giving $h = h_0 (1 + \eta)$ and assuming that $|\eta| \ll 1$.

The simplified shallow water equations regarding momentum and the continuity of mass as shown are:

$$\frac{dh}{dt} + h \nabla \cdot \mathbf{v} = 0$$

$$\frac{d\vec{v}}{dt} + g \nabla h + f_c \hat{z} \times \vec{v} = 0, \text{ that with } h = h_0 (1 + \eta) \text{ can be written as } \frac{\partial \vec{v}}{\partial t} + g h_0 \nabla \eta + f_c \hat{z} \times \vec{v} = 0$$

Let us assume that both \mathbf{v} and η are proportional to standard expressions as

$$\exp[i(k_x x + k_y y - \omega t)]$$

Here $k = 2\pi/\lambda$ is the wave number in the x and y-directions,, and $\omega = 2\pi f$ is the angular frequency and f the frequency $f = 1/T$.

The solution to this equation yields a polynomial expression with respect to ω :

$$\omega (\omega^2 - 1)(f_c^2 + k^2 g h_0) = 0$$

Here we have put the squared wave-number $k^2 = k_x^2 + k_y^2$.

The solution is as follows:

$$\omega = 0, \omega = \pm \sqrt{f_c^2 + k^2 g h_0}$$

Waves close to the Equator

Close to the equator, the Coriolis force equals zero why $f_c = 0$. We then get a simple expression:

$$\omega = \pm \sqrt{k^2 g h_0}$$

Since the wave velocity is $c = f\lambda = \frac{\omega}{k}$ we get the wave velocity (omitting the minus sign):

$$c = \frac{1}{k} \sqrt{k^2 g h_0} = \sqrt{g h_0}$$

This means that the deeper the ocean the higher the velocity of the wave, close to the equator.

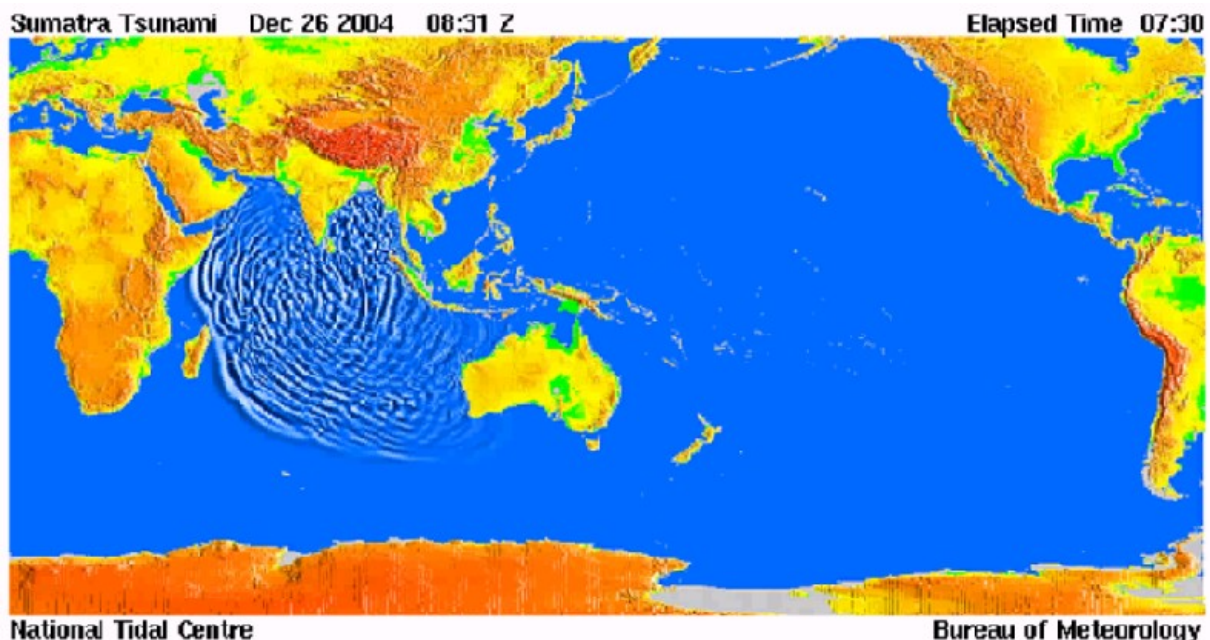
Example

A wave generated by an Earth quake at a depth of 2000 m, close to the equator, will generate a wave with velocity:

$$c = \sqrt{g h_0} = \sqrt{9.8 \times 2000} \text{ m/s} = 140 \text{ m/s} = 140 \times \frac{36}{10} \text{ km/h} = 504 \text{ km/h} \approx 500 \text{ km/h}$$

At a depth of 4000 m, the velocity will be around 700 km/h.

The development of the wave is shown on the figure below:



The simulation has been produced by the National Bureau of Metrology in Australia and can be found on the following links:

http://www.bom.gov.au/tsunami/info/media/sumatra_plan_topog.avi

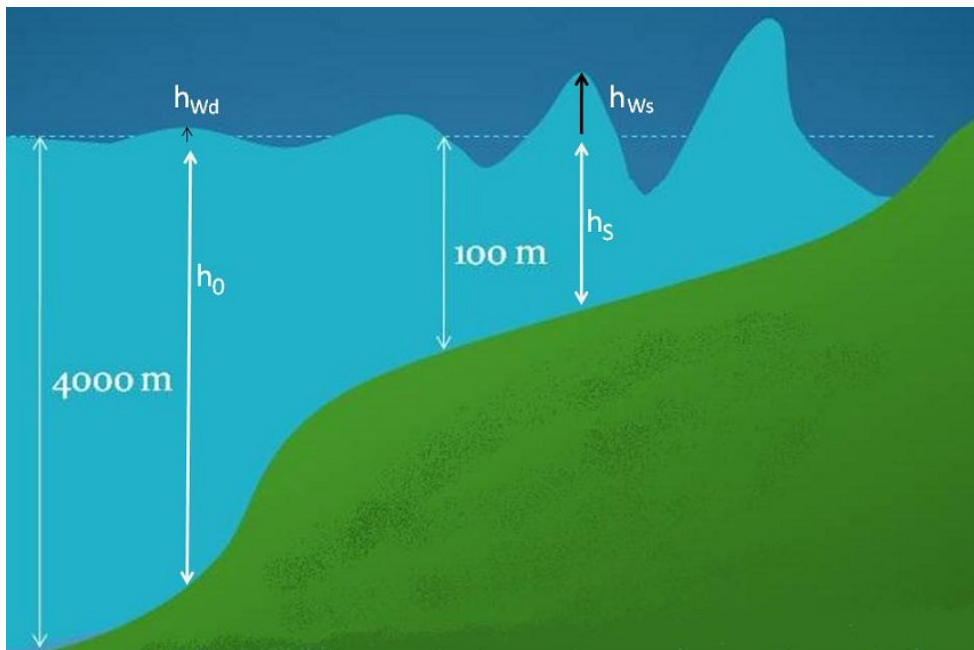
<http://www.bom.gov.au/tsunami/info/index.shtml>

The height of the wave in shallow waters

When the tsunami comes closer to shallow waters and leaves the deep waters of the ocean it changes in wave height. The speed of the tsunami is proportional to the square root of the depth, why it drops speed near the coastline. However, the energy flux of the tsunami depends both on wave height and wave speed and is almost constant. This means that it increases in wave height as the speed is reduced on shallower waters. This effect is called *shoaling*. Due to the shoaling effect, a tsunami can pass under ships on open sea unnoticeable. Near the coastline, it can increase its wave height by several meters, up to 5-10 meters. Calculations show that the increase of the tsunami's wave height as it enters shallow water is given by:

$$h_{ws} = h_{wd} \left(\frac{h_0}{h_s} \right)^{1/4}$$

where h_{ws} and h_{wd} are wave heights in shallow respectively deep water. Here h_s and h_0 are the depths of the shallow respectively the deep water. So a tsunami with a height of 1 m in the open ocean where the water depth is 4000 m would have a wave height of 4-5 m in water having a depth of 10 m.



Example

A wave generated by an Earth quake at a depth of 4000 m, close to the equator, has a wave-amplitude of 1 m. Calculate the height of the wave at the shore if the depth is 20 m.

$$h_{ws} = h_{wd} \left(\frac{h_0}{h_s} \right)^{1/4} = 1 \times \left(\frac{4000}{20} \right)^{1/4} m \approx 3.8m \approx 4m$$

How to detect tsunamis in the ocean

The tsunami has just a small amplitude, below 1 meter in the deep ocean. Still, the wavelength or width of the wave can be hundreds of kilometers. It is tough to detect a tsunami at sea, but wave height instrument can do this. By applying electromagnetic pulses, satellite altimeters are able to measure the height of the ocean surface by direct observations. Pulses are sent down to the surface of ocean from the satellite and the height of the ocean surface can be determined accurately, since the speed of the pulse (same as speed of light) is known as well as the altitude of the satellite. One just has to determine the time that it takes for a return of the pulse to the satellite. A satellite altimeter happened to be in the right place at the right time and measured the tsunami during the Indian Ocean tsunami of December 26th 2004.

