Environmental Science II

Problems Chapter 4

4.2.1 A radioactive sample has a half-life of 2.2 days. What fraction of the decaying nuclei remains after 5.0 days?

Solution
Applying the formula

$$N = N_0 e^{-\ln 2 t/T}$$

we obtain with T = 2.2 days and t = 5.0 days.

$$\frac{N}{N_0} = e^{-\ln 2 \times 5.0/2.2} = 0.207 \approx 0.21$$

Answer: 21% remains

4.2.2 Two similar nuclides with the same number of nuclei decay with a half-life of 1.2 minutes respectively 14 hours.
How much stronger activity has the most dangerous nucleus?

Solution
The higher the activity the higher is the risk of radiation damage. The Activity $R = \lambda N$.
If N is the same in both nuclei, we get with T_2 = 14 hours = 14x60 minutes:

$$\frac{R_1}{R_2} = \frac{\lambda_1 \times N}{\lambda_2 \times N} = \frac{T_2}{T_1} = \frac{14 \times 60}{1.2} \approx 700$$

Answer: 700 times stronger

4.2.3 A radioactive sample decays with a half-life of 15 h. What fraction of the samples radioactive nuclei remain after 18 half-lifes?

Solution
Here we have $t = 18T$ why we get:

$$\frac{N}{N_0} = e^{-\ln 2 t/T} = e^{-\ln 2 \times 18T/T} = e^{-\ln 2 \times 18} = 3.8 \times 10^{-6}$$

Answer: A fraction of 3.8×10^{-6} remains

4.2.4 ^{212}Po has a half-life 0.30 μs. How long time does it take until only 1/16 of the original nuclei in a sample remain?

Solution
After one half-life, $T_{1/2}$ there are ½ of the nuclei left. After another half-life, ¼ remains, and after still one more half-life there are 1/8 of the nuclei left and finally after still one more, 1/16 is left. Thus, we have 4 half-lifes totally, why it takes 4 x 0,30 μs = 1,2 μs.

Answer: 1,2 μs

4.3.1 Two radioactive samples with exactly the same number of nuclei, one that contains ^{235}U ($T_{1/2}$ = 7.037x10^{8} years) and one with ^{238}U ($T_{1/2}$ = 4.468x10^{9} years). Compare the activities.

Solution
$R_{235} = \lambda_{235}N_{235}$ and $R_{238} = \lambda_{238}N_{238}$, where $N_{235} = N_{238}$. This gives
$R_{235} / R_{238} = \tau_{238}/\tau_{235} = 4.468\text{x}10^{9} / 7.037\text{x}10^{8} = 6.349$

Thus, the activity of the ^{235}U sample is 6.349 times higher.

Answer: 6.349 times higher

4.3.2 Calculate the number of atoms in exactly one gram (1 g) of ^{226}Ra. The Avogadro number is 6.022 x 10^{23} atoms per mole.

Solution

The number of nuclei in one gram of ^{226}Ra is

$$N = \frac{1}{226} \times 6.022 \times 10^{23}\, nuclei = 0.026646 \times 10^{23}\, nuclei$$

Answer: 2.66 x 10^{21} atoms

4.3.3 The half-life of ^{226}Ra is 1620 years. Use the problem above to calculate the activity of 1,0 g radium.

Solution

The activity of a radioactive sample is given by $R = \lambda N$. The connection between the decay constant λ and the half-life is given by $\lambda = ln2/T_{1/2}$.

$$R = \lambda N = \frac{\ln 2}{T_{1/2}} N = \frac{\ln 2}{1620 \times 365 \times 24 \times 3600} 2.66 \times 10^{21} \approx 3{,}6 \times 10^{10}\, decays/s$$

This was earlier the activity in the unit 1 Ci = 1 Curie

Answer: 3,6 x 10^{10} decay/s, i.e. 1,0 Curie, the earlier definition of activity.

4.3.4 One measures 735 α-particles/minute from 1,0 mg ^{238}U.

Determine the half-life of ^{238}U.

Solution

Measured activity *R = 735/60* decay/s. *R = (ln2/$T_{1/2}$)N* where *N* = $\frac{m}{M}N_0$

The half-life becomes: $T_{1/2} = \frac{\ln 2}{R} \times \frac{m}{M} N_0 = \frac{\ln 2}{735/60} \times \frac{1{,}0 \times 10^{-3}}{238} \times 6{,}023 \times 10^{23} s \approx$

4,5x10^9 years (We have divided by 365x24x60x60 s/year)

Answer: 4.5 x 10^9 years

4.4.2 You are supposed to make an age determination by the carbon-14 method on a sample with 1.0 gram carbon. You measure an activity of 0.415 decays/minute that was performed during 24 hours. The original activity was 0.233 decays per gram and second.
Determine the age of the sample.

Solution

The activity per gram = *S* = 0.415 decays/minute g. We get the equation:
$0.415 = 0.233x60 \times e^{-0.693t/5730}$ We take the logarithm and get $t = 2{,}91x10^4$ years.

Answer: 2,9 x 10^4 years

4.7.1 α -particles from ^{210}Po have a kinetic energy of 5.3 MeV. How large is the decay energy *Q*?

Solution
$M^{210}{}_{Po}$ = 209, 982 876 u, $M^{206}{}_{Pb}$ = 205, 974 455 u, m_α = 4,00260 u.

The *Q*-value is given by: $Q = M^{210}{}_{Po}c^2 - (M^{206}{}_{Pb}c^2 + m_\alpha c^2) - K =$

(209, 982 876 – 205, 974 455 – 4,00260) x 931,4 – 5,3 MeV =

0,005821x931,4 MeV – 5,3 MeV = 5,4216794 – 5,3 MeV = 0,122 MeV

Answer: 0,122 MeV

4.7.2 We study a free neutron that decays to a proton. The half-life is 28 minutes.
The maximum kinetic energy for the electron involved is 782 keV.
Determine the mass of the neutron.

Solution

$m_n = m_p + m_e + \frac{E_{K\max}}{c^2} = M_H + \frac{E_{K\max}}{c^2} =$

1,007825 u + (0,782 MeV)(1/931,5 u/MeV) = 1,008665 u

Answer: 1,008665 u

4.7.3 ^{60}Co decays by β^- - decay to an excited state of nickel, (^{60}Ni). Then, a γ-photon with energy 1,173 MeV, and thereafter a γ-photon with energy 1,333 MeV decay to the ground state of ^{60}Ni. Determine how great γ-power 1,0 gram ^{60}Co generates. The half-life of ^{60}Co is 5.3 years.

Solution

$\lambda = \ln 2/(5.3x365x24x3600) = 4.147x10^{-9}\ s^{-1}$

$N = 6.023x10^{23}/60 = 1.004x10^{22}$ nuclei

$R = \lambda N = 4.164x10^{13}$ decays/s

Each decay gives a total γ-energy of 2.506 MeV, i.e. $4.015x10^{-13}$ J

So the power is 16.7 W

Answer: 17 W

4.7.4 You try to get fusion between deuterium and tritium, i.e., between ^{2}H and ^{3}H and gets ^{4}He and one neutron.
Calculate how much kinetic energy we get in the reaction.

Solution

The reaction will be: $^2H + {}^3H \rightarrow {}^4He + {}^1n$

$Q = (M_{before} – M_{after})c^2$ = *(3,016 050 + 2,014 102 – 4,002 603 – 1,008 665)x931,5 MeV*

=17,6 MeV

Answer: 17.6 MeV

4.7.5 ^{235}U is bombarded by slow neutrons to obtain ^{236}U. Calculate the *Q*-value.

Solution

The reaction will become: $^{235}U + {}^{1}n \rightarrow {}^{236}U$

Q = (M_{before} – M_{after})c^2 = (235.1170 + 1.008 665 – 236.1191) x 931.5 MeV =

= 6.1153 MeV.

The potential barrier is only 5.3 MeV why it is enough with thermal neutrons

4.8.1 Calculate how thick layer (in mm) of lead is needed to stop 96% of incident 50 keV X-rays. Lead has an absorption coefficient $\mu = \mu_m\rho(cm^{-1})$ =1.22 cm^{-1}.

Solution

Only *(1-0.96)I(0)* of the intensity remains, why the formula for absorption gives

(1-0.96) I(0) =I(0) exp(-μx) which means that

*0.04 = exp –1.22**x*** where we obtain *x* in cm. We take the logarithm and get:

*ln(1/0.04) = 1.22**x*** and *x* = 3.2189/1.22 cm = 2.64 cm

Answer: 26 mm

4.8.2 Calculate how thick layer (in mm) of aluminum is needed to stop 96% of incident 50 keV X-rays.

Aluminum has an absorption coefficient $\mu = \mu_m\rho(cm^{-1})$ =0.21 cm^{-1}.

Solution

Only *(1-0.96)I(0)* of the intensity remains, why the formula for absorption gives

(1-0.96) I(0) =I(0) exp(-μx) which means that

*0.04 = exp –0.21**x*** where we obtain x in cm. We take the logarithm and get:

*ln(1/0.04) = 0.21**x*** and *x* = 3,2189/0.21 cm = 15.3 cm.

Answer: 15 cm

4.10 Looking at the FP (fission products) and the TRU (trans-uranium) one finds that the FP decays much faster than the TRU. How many timed faster, if we want to reach the radioactivity level of comparing to Nature?

Solution

Looking at the picture at page 28 in the internet-book we find that $T_{1/2}$ for FP is around $6x10^2$ years and for TRU $5x10^5$ years, why we get $5x10^5/6x10^2$ = 830. FP decays around 800 times faster than TRU.

Answer: 800 times faster