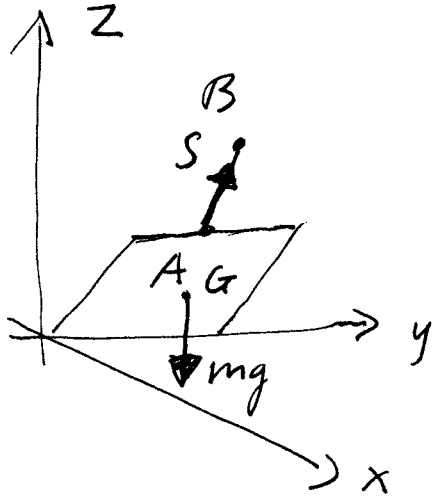
Lösningar till tentan 2/6 2014, SG1109

1



$$\bar{r}_{AB} = a\left(-\frac{1}{\sqrt{2}}, \frac{1}{2}, 1-\frac{1}{\sqrt{2}}\right)$$

$$\bar{r}_G = \frac{a}{2}\left(\frac{1}{\sqrt{2}}, 1, \frac{1}{\sqrt{2}}\right)$$

$$\bar{e}_{AB} = \frac{\bar{r}_{AB}}{r_{AB}} =$$

$\bar{r}_B = a(0,1,1)$

$\bar{r}_A = a\left(\frac{1}{\sqrt{2}}, \frac{1}{2}, \frac{1}{\sqrt{2}}\right)$

$$= \frac{(-\sqrt{2}, 1, 2-\sqrt{2})}{(9-4\sqrt{2})^{1/2}}$$

$$S = S\bar{e}_{AB} \quad ; \quad M_y = \bar{e}_y \cdot \bar{M}_0 = 0 \Rightarrow$$

$$\bar{e}_y \cdot \left(\bar{r}_G \times (-mg\bar{e}_z) + \bar{r}_A \times S\bar{e}_{AB}\right) = 0 \Rightarrow$$

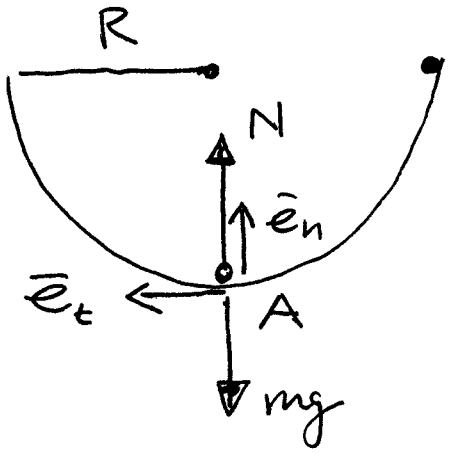
$$\frac{a}{2\sqrt{2}} mg + \left[\left(\frac{a}{\sqrt{2}}\bar{e}_x + \frac{a}{\sqrt{2}}\bar{e}_z\right) \times \frac{S}{(9-4\sqrt{2})^{1/2}} \cdot \right.$$

$$\left(-\sqrt{2}\bar{e}_x + (2-\sqrt{2})\bar{e}_z\right) \cdot \bar{e}_y = 0 \Rightarrow$$

$$\frac{mg}{2\sqrt{2}} - S\left(\frac{(2-\sqrt{2})}{\sqrt{2}} + 1\right) \frac{1}{(9-4\sqrt{2})^{1/2}} = 0$$

$$\frac{2S}{(9-4\sqrt{2})^{1/2}} = \frac{mg}{2} \Rightarrow S = \frac{(9-4\sqrt{2})^{1/2}}{4} mg$$

2.



Energiprincipen ger: $\frac{1}{2}mv^2 = mgR \Rightarrow$

$v^2 = 2gR$

$\bar{a} = \dot{v}\bar{e}_t + \frac{v^2}{R}\bar{e}_n$

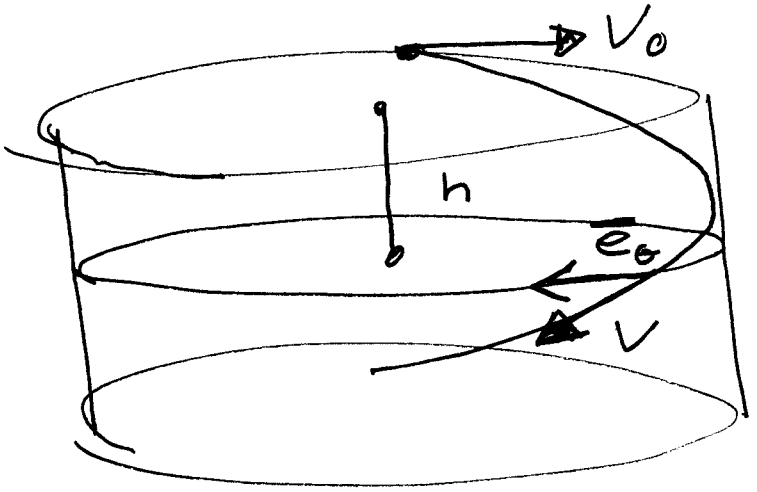
NII: $\bar{e}_t$: $\dot{v} = 0$ i punkten A

$\bar{a} = \frac{v^2}{R}\bar{e}_n = \frac{2gR}{R}\bar{e}_n = 2g\bar{e}_n$

NII: $\bar{e}_n$: $m\frac{v^2}{R} = N - mg$

$N = m\frac{v^2}{R} + mg = 2mg + mg = 3mg$

3.

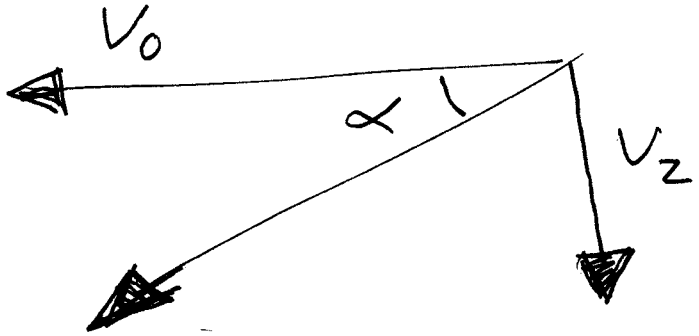


NII $\bar{e}_\theta$: $mR\ddot{\theta} = 0$

$$R\dot{\theta} = C$$

B.V. $\begin{cases} t = 0 \\ R\dot{\theta} = V_0 \end{cases} \Rightarrow C = V_0$

$$V_\theta = V_0$$



Energiprincipen ger

$$\frac{1}{2} m V_0^2 = \frac{1}{2} m (V_0^2 + V_z^2) - mgh \rightarrow$$

$$V_z = \sqrt{2gh}$$

$$\tan\alpha = \frac{V_z}{V_0} = \frac{\sqrt{2gh}}{V_0}$$

$$\alpha = \arctan\left(\frac{\sqrt{2gh}}{V_0}\right)$$

4. En planets massa ges av

$$M = \frac{4\pi R^3}{3}\rho$$

a) Flykthastigheten fås då totala energin är lika med noll:

$$\frac{1}{2}mv^2 - G\frac{mM}{R} = 0 \Rightarrow v = \sqrt{\frac{2GM}{R}}$$

$$v_A = \sqrt{\frac{2GM_A}{R_A}} \qquad v_B = \sqrt{\frac{2GM_B}{R_B}} \Longrightarrow \frac{v_A}{v_B} = \frac{R_A}{R_B}$$

b) Keplers tredje lag: $T = \frac{2\pi a}{\sqrt{GM}}$

$$T_A = \frac{2\pi a}{\sqrt{GM_A}} \Longrightarrow$$

$$T_B = \frac{2\pi a}{\sqrt{GM_B}}$$

$$\frac{T_A}{T_B} = \sqrt{\frac{M_B}{M_A}} = \left(\frac{R_B}{R_A}\right)^{3/2}$$