EP2200 Queuing Theory and Teletraffic Systems

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Allowed help: Calculator, Beta mathematical handbook or similar, provided sheets of queuing theory formulas, Laplace transforms and Erlang tables.

1. A call service-desk has 20 clerks answering the incoming calls. The call inter-arrival times are modeled as i.i.d exponentially distributed random variables with an average of 1 minute. The call durations are exponential with an average of 15 minutes.
a) Assume that calls arriving when all clerks are occupied, are dropped. Give the call blocking probability and calculate the average number of calls that are served per hour. (2p)
b) Assuming that the clerks are selected randomly, what is the percentage of time a clerk is busy? (2p)
c) Calculate the average duration of the periods, in which the service-desk is able to accept arriving calls. (2p)
d) Assume now that arriving calls can wait in an infinite queue, if all clerks are busy. Calculate the probability that a call has to wait, the average waiting time, and the probability that a call has to wait for more than 3 minutes. Give the average number of calls served per hour. (4p)

2. Consider an M/M/3 loss system with two buffer positions. Call arrivals originate from two groups of subscribers, A and B: each group has five subscribers. Each subscriber generates calls according to a Poisson process with intensity $\lambda = 5$ calls/hour, if it does not have a call already in progress. Call durations have exponential distributions with mean $1/\mu = 6$ min.
a) Give the Kendall notation of the system and draw the state diagram. (2p)
b) Calculate the percentage of time the buffer positions are both full. (2p)
c) Calculate the mean number of blocked calls per hour. (2p)
d) Calculate the expected waiting time for non-blocked subscribers. (2p)
e) Assume that the calls from group B are only allowed to use a server, if upon arrival, all servers are idle (otherwise they are dropped). Calls from group A can use any of the servers, if available. Draw the modified state diagram! (2p)

3. Jobs arrive to a server according to a Poisson process with intensity $\lambda = 2$ jobs per time unit. The service time has Hyper-Exponential distribution, with Laplace transform $S^*(s) = \frac{1}{s+2} + \frac{1}{s+4} + \frac{2}{s+8}$. Jobs that arrive when the server is busy are dropped.
a) Give the Kendall notation and draw the state transition diagram for the system. (2p)
b) Calculate the utilization of the server, and the probability that an arriving job is dropped. (3p)
Assume now that the jobs that arrive when the server is busy are placed in a queue, with infinite buffer capacity.
c) What is the utilization of the server in this case? Is the utilization decreased or increased, compared to case (c)? Why? (2p)
d) Calculate the average waiting time of the jobs. (3p)

4. Customers arrive to a single server system with an infinite queue accord-

ing to a Poisson process with intensity λ. Arriving customers belong to class 1 with probability a, or to class 2 with probability $1-a$. The service time is $1/b$ time units for class 1 customers, and it is Erlang-c distributed with mean c/b for class 2 customers. Consider the following values: $a = 0.75$, $b = 1$, $c = 2$, $\lambda = 0.5$.

a) Give the mean, the second moment and the variance of the service times of the class 1 customers, of the class 2 customers, and considering all the customers arriving to the server. (2p)

b) Consider the case where the customers are served without priorities and give the average waiting and system times for an arbitrary customer. (3p)

c) You decide to introduce preemptive priority scheduling, and give high priority to customers of class 1. Calculate the average time spent in the system for customers of class 1, class 2, and for an arbitrary customer. How do the average system times change compared to case (b)? (3p)

d) A customer of high priority arrives at the system when the server is busy and there are 2 low priority customer and 1 high priority customer waiting for service. What is the probability that this customer has to wait for more than 3 time units before its service starts? (2p)

5. a) The receptionist of the "Best service in town" hotel goes to have a coffee in the next door cafe, whenever there are no guests at the reception desk, and stays there for 10 minutes. If no one is waiting when he returns, he goes back for some more chat (for 10 minutes again). In average, 10 guests attend the reception desk per hour, and the receptionist needs on average 5 minutes to help each of them. What is the probability that an arriving guest finds the receptionist at his desk? How much time does the receptionist spend in the cafe per hour, on average? (3p)

b) Consider a queuing network with two nodes. Customers enter the network at node 1. All customers served at node 1 proceed to node 2. After service at node 2, they return again to node 1 with probability p, or leave the network. Both of the nodes are M/M/1 systems. Consider $\lambda = 1$, $\mu_1 = \mu_2 = 3$, and $p = 0.5$. Calculate the average time the requests spend in the queuing network. (2p)

c) Jobs arrive to a system with two processors. There is a dedicated queue for each of the processor and the arriving jobs are assigned to the processors in a round-robin manner. Jobs arrive at the system according to a Poisson process. What is the arrival process to each of the queues and why? What would be the arrival process to a queue in a system with n processors? (2p)

d) Cars arrive to a highway toll booth according to a Poisson process, on average 5 cars per minute. We model the time needed for paying the toll with an exponential distribution with mean 30 seconds. Assume that cars queue up at a common queue. How many parallel lanes should there be if you would like the average waiting time to be less than 15 seconds? What is then the probability that an arriving car has to wait for more than 1 minute? (3p)