Throughput Analysis of ARQ in Interference with Nakagami-$m$ Block Fading Channels

Peter Larsson, Lars K. Rasmussen, Mikael Skoglund

ACCESS Linnaeus Centre,
KTH Royal Institute of Technology,
Stockholm, Sweden
A Simplified Motivating Example

Scenario (for the motivating example)
- ARQ in interference.
- Block Rayleigh fading.

Main problem
- Optimize ARQ throughput in interference!
- Closed-form opt. rate point: \( R^* = f_R(\Gamma_0, \Gamma_1) \)
- Closed-form opt. throughput: \( T^* = f_T(\Gamma_0, \Gamma_1) \)
- Various usage of the results: See paper!

Solution (for the motivating example)
- Decoding probability:
  \[
P = \frac{e^{-\Theta}}{1 + \Theta \Gamma_1}, \quad \Theta \triangleq \frac{e^R - 1}{\Gamma_0}
\]
- ARQ Throughput:
  \[
  T = R \cdot P
\]
- Optimality condition:
  Try to solve for the optimal rate!
  \[
  \frac{d \ln (T)}{dR} = 0 \Rightarrow
  \frac{1}{R^*} - \frac{e^{R^*}}{\Gamma_0} - \frac{\Gamma_1 e^{R^*}}{\Gamma_0 + \Gamma_1 (e^{R^*} - 1)} = 0
  \]

Conclusion (for the motivating example)
- Opt. rate unsolvable in a closed-form!
- But, we want to find closed-form opt. solutions for even more general cases!
In the Paper We....

Solve the “unsolvable” problem!
• Propose a parametric closed-form optimization framework.
• Find closed-form expressions for
  • Outage (decoding) probability.
  • Throughput.
  • Optimal rate point.
  • Optimal throughput value.

Generalize the problem!
• Include arbitrary # of interferers.
• Use per-user Nakagami-m fading.

Consider also other ARQ problems!
• Scaled-power case. See paper!
• Interf.-limited case. See paper!

The main contribution is:
A parametric optimization framework (2 methods!) giving closed-form expressions for the optimal throughput value and the optimal rate point for ARQ operating in interference.
System Model

Communication Scenario
- ARQ in interference.

Nakagami-\(m\) block fading channel:
\[
\gamma_k = g_k \Gamma_k, \quad \Gamma_k \text{ is the mean SNR}
\]
\[
f_{g_k}(g_k) = m_k^m \left( \frac{m_k}{\Gamma(m_k)} \right) g_k^{m_k-1} e^{-m_k g_k}, m_k \geq 1/2
\]

Motivations:
- Wide range of fading conditions.
- \(m=1\) is Rayleigh fading.
- \(m=2\) is 2-branch MRC/ TX div.
- Converge to a non-fading ch.
- Good fit to measurements.

Assumptions:
- AWGN & Capacity achieving codes.
- Inter- and intra-user i.i.d. block fading.
- Error-free ACK & No overhead.
- Always a packet to send.
Decoding Probabilities

Generic channel fading - Decoding prob.

\[ P = \mathbb{P}\left\{ \ln \left( 1 + \frac{g_0 \Gamma_0}{1 + \sum_{k=1}^{K} g_k \Gamma_k} \right) > R \right\} \]

Own ch. Rayleigh, other ch. Nakagami-\( m \)

\[ m_0 = 1 \]

\[ P_1 = \frac{e^{-\Theta}}{\prod_{k=1}^{K} (1 + \Theta \tilde{\Gamma}_k)^{m_k}} \]

\[ \tilde{\Gamma}_k \triangleq \Gamma_k / m_k \]

Own ch. Rayleigh+MRC, other ch. Nakagami-\( m \)

\[ m_0 = 2 \]

\[ P_2 = P_1 \left( 1 + \Theta + \sum_{k=1}^{K} \frac{m_k \Theta \tilde{\Gamma}_k}{1 + \Theta \tilde{\Gamma}_k} \right) \]

All fading channels are Nakagami-\( m \)

- See paper!

Analytical and simulation results

- A check against Monte Carlo simulation results. Ok!
The Key Idea: Parametric Optimization

Parameterization method (1) in $R$
- Throughput expressed as
  \[ T = R \cdot P, \quad P = f(R, \tilde{\Gamma}_0, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \]
- Optimality condition
  \[ \frac{d \ln(T)}{dR} = 0 \]
  \[ \Rightarrow \frac{1}{R} + \frac{P'_R}{P} = 0 \]
- Idea: Parameterize in $R^*$. Solve for $\Gamma_0$. (if at all possible). Insert in $T$.
- Solution
  \[ \Gamma_0 = f_\Gamma(R^*, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \]
  \[ T^* = f_T(R^*, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \]
  \[ R^* \]
- Insight: It is hard to solve for $R^*$ vs. $\Gamma_0$, but generally easy to solve $\Gamma_0$ vs. $R^*$.

Parameterization method (2) in $\Theta$
- Throughput expressed as
  \[ T = \ln(1 + \tilde{\Gamma}_0 \Theta)P, \quad P = f(\Theta, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \]
- Optimality condition
  \[ \Theta \frac{d \ln(T)}{d\Theta} = 0 \]
  \[ \Rightarrow (1 + \Theta \tilde{\Gamma}_0) \ln(1 + \Theta \tilde{\Gamma}_0) / \Theta \tilde{\Gamma}_0 = t \]
  \[ t(\Theta, \tilde{\Gamma}_1, ..., \tilde{\Gamma}_K) \overset{\Delta}{=} -\Theta P'_\Theta / P \]
- Idea: Parameterize in $\Theta$. Solve for $\Gamma_0$. Insert in $R$ and $T$.
- Solution
  \[ \Gamma_0(\Theta) = m_0 \frac{W_0(-te^{-t}) - 1}{\Theta} \]
  \[ T^*(\Theta) = R^* \cdot P \]
  \[ R^*(\Theta) = (t + W_0(-te^{-t})) \]
- Insight: It is easier to solve for $\Gamma_0$ if only $R$, but not $P$, is expressed in $\Gamma_0$. 

Lambert’s $W$-function
Example: Parametric Optimization

Parameterization method (1):
Own ch. Rayleigh, 1 Rayleigh Interf.
- Decoding probability \[ P = \frac{e^{-\Theta}}{1 + \Theta \Gamma_1}, \quad \Theta \triangleq \frac{e^R - 1}{\Gamma_0} \]
- Optimality condition
\[ \frac{1}{R^*} - \frac{e^{R^*}}{\Gamma_0} - \Gamma_0 \Gamma_1 (e^{R^*} - 1) = 0 \]
can be rewritten as
\[ \frac{a}{1 + x} + \frac{b}{x} = 1 \quad \text{where} \]
\[ x \triangleq \frac{\Gamma_0}{\Gamma_1 (e^{R^*} - 1)} \]
- Solving for a positive \( x \)
\[ x_+ = \frac{a + b - 1 + \sqrt{(a + b - 1)^2 + 4b}}{2} \]
- Back substitution yields
\[ T^* = \frac{R^* e^{\frac{1}{\Gamma_1 x_+}}}{1 + x_+^{-1}} \]
\[ \Gamma_0 = x_+ \Gamma_1 (e^{R^*} - 1) \]

Parameterization method (2):
Own ch. Rayleigh, \( K \) Nakagami-m Interf.
- Decoding probability \[ P = \frac{e^{-\Theta}}{\prod_{k=1}^{K} (1 + \Theta \tilde{\Gamma}_k)^{m_k}} \]
- Optimality condition
\[ (1 + \Theta \tilde{\Gamma}_0) \ln(1 + \Theta \tilde{\Gamma}_0) / \Theta \tilde{\Gamma}_0 = t, \]
where
\[ t(\Theta) \triangleq \frac{1}{\Theta + \sum_{k=1}^{K} \frac{m_k \Theta \tilde{\Gamma}_k}{1 + \Theta \tilde{\Gamma}_k}} \]
- Insert in
\[ \Gamma_0(\Theta) = m_0 \frac{-t}{W_0(-te^{-t}) - 1} \]
\[ T^*(\Theta) = R^* \cdot P \]
\[ R^*(\Theta) = t + W_0(-te^{-t}) \]

Parametric closed-form for optimal throughput!

Note that method 2 handles any arbitrary interferences case!
Selected Results

Optimal throughput and optimal rate vs. SNR

\[(x, y) = (10 \log_{10} \Gamma_0 (R^*), T^* (R^*))\]

Throughput Analysis of ARQ in Interference with Nakagami-m Block Fading Channels.

P. Larsson, L. K. Rasmussen, M. Skoglund

Apart from plotting curves, we can e.g. also study asymptotes parametrically. See paper!
Summary and Conclusions

Summary

• Studied ARQ in interference.
• Proposed parameterized optimization for closed-form results.
  • Method 1: Parameterized in $R$. Handles some interference cases.
  • Method 2: Parameterized in $\Theta$. Handles any interference case.
• Derived some closed-form expressions.
• Studied other problems in the paper
  • Scaled-power case
  • Interf.-limited case

Conclusions

• Solved an “unsolvable” opt. problem.
• Noted large losses with interference.
• Use as reference cases for benchmarking new ARQ+MCS schemes.

The material was adapted for the interactive presentation format. Please see the paper for more general assumptions and analysis, as well as other studied problems.
Related papers

• Please also consider the following related papers: