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Optical Physics: Summary Session 2

Fresnels equations: reflection and transmission at interface
The **plane-of-incidence** is the plane that contains the $\vec{k}$-vector (ray of light) and the normal of the surface, i.e. the plane that is normally drawn when marking the angles of incidence, reflection and refraction. All light, polarized and unpolarized, can be described by two polarization compontents: polarization normal to the plane-of-incidence and polarization parallel to the plane-of-incidence. Both these components follow the laws of reflection and refraction, the amplitude of the fields follows Fresnels equations:

- $r_\perp = \frac{E_{0r\perp}}{E_{0i\perp}} = -\frac{sin(\theta_i-\theta_t)}{sin(\theta_i+\theta_t)}$
- $t_\perp = \frac{E_{0t\perp}}{E_{0i\perp}} = \frac{2sin\theta_t cos\theta_i}{sin(\theta_i+\theta_t)}$
- $r_\parallel = \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{tan(\theta_i-\theta_t)}{tan(\theta_i+\theta_t)}$
- $t_\parallel = \frac{E_{0t\parallel}}{E_{0i\parallel}} = \frac{2sin\theta_t cos\theta_i}{sin(\theta_i+\theta_t)cos(\theta_i-\theta_t)}$

The sign of r indicates the change in direction of the $\vec{E}$-field: a negative sign for $r_\perp$ means that the $\vec{E}$-field has changed direction, a positive sign for $r_\parallel$ means that the $\vec{E}$-field has changed direction (this is due to the choice of coordinate system).
The reflected and transmitted fraction of the intensity given by:
Reflectance $R = \frac{I_r}{I_i} = r^2$
Transmittance $T = \frac{n_t cos\theta_t}{n_i cos\theta_i}\frac{I_t}{I_i} = \frac{n_t cos\theta_t}{n_i cos\theta_i}t^2$.
For a non-absorbing material it follows that $R + T = 1$. Observe that also $R_\perp + T_\perp = 1$ och $R_\parallel + T_\parallel = 1$.

Total Reflection:
Total reflection at a surface happens if: $\theta_i \geq \theta_c = arcsin\frac{n_t}{n_i}$
This means that all light is reflected and nothing is transmitted if the angle of incidence is bigger than or equal to the critical angle θ_c.
This is however not entirely true! If $\theta_i > \theta_c$ one obtains complex values for $r_\perp$ and $r_\parallel$ which actually gives a transmitted wave with exponentially decreasing amplitude. This transmitted wave is called **evanescent** wave and can not normally be observed due to its rapidly decreasing amplitude. However, if two materials are placed close to each other with a very small gap, one can obtain **frustrated total reflection**. This means that a part of the light is tunnelling over the gap to the other material even though the condition for total reflection is fulfilled on the first surface. This happens if the gap is so small that the amplitude of the evanescent wave has not been reduced to zero. Frustrated total reflection is used for example in beam splitters.

Geometrical optics: Thin lenses

Geometrical optics is an approximation where the wave properties of light are neglected. Only refraction and reflection of light is taken into account. This approximation works well as long as all dimensions are $\gg \lambda$. One often uses a **Paraxial approximation** assuming that the incidence angles are small. This means that $sin\theta \approx \theta$.

These approximations combined with Fermat's principle imply that refraction in a spherical surface can be described by:

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

	OS	IS
s_o	+	-
s_i	-	+
R	-	+

OS (object space) is defined as the area to the left of the refracting surface
IS (image space) is defined as the area to the right of the refracting surface
n_1 and n_2 are the refractive indeces in OS and IS respectively
s_o is the distance object - surface (positive if object is in OS)
s_i is the distance surface - image (positive if image is in IS)
R is the radius of curvature of the surface (positive if the center of curvature is in IS)

This formula gives rise to a few relations:

- Thin lens surrounded by different media: $\frac{n_{m,o}}{s_o} + \frac{n_{m,i}}{s_i} = \frac{n_l - n_{m,o}}{R_1} - \frac{n_l - n_{m,i}}{R_2}$
- Thin lens surrounded by air: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2})$
  f=focal length of lens
- Newton's formula: $x_o x_i = f^2$
  x_o and x_i are the distances from front focal plane to object and back focal plane to object
- Transverse magnification: $M_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$
  y_o and y_i are the heights of the object and image (positive if above optical axis)
- Longitudinal magnification: $M_L = \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$