Original provided by Linda Lundström
Revised and translated by Per Jansson
Tel: 08-5537 8212
Mail: per.jansson@biox.kth.se

Optical Physics: Summary Session 4

Geometrical optics: Principal planes

Thick Lenses: All lenses can not be approximated as thin, i.e. their thickness, d, has to be taken into account. A thick lens optical properties in air are described by two **principal planes**, H_1 and H_2, and an **effective focal length**, f_{eff}:

$\frac{1}{f_{eff}} = (n_l - 1)[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l-1)d}{n_l R_1 R_2}]$

Vertex: point where the lens surface intersect the optical axis

The distance from the left vertex to $H_1 = -\frac{f_{eff}(n_l-1)d}{R_2 n_l}$
(positive direction from OS towards IS)

The distance from the right vertex to $H_2 = -\frac{f_{eff}(n_l-1)d}{R_1 n_l}$
(positive direction from OS towards IS)

If one "disregards" the distance between H_1 and H_2, one can perform calculations and ray constructions according to the same formulas as for thin lenses. One thus pretends that H_1 is the left vertex of a thin lens and H_2 the right vertex. f_{eff} is the focal length of the thin lens, measured as positive from H_1 towards OS and from H_2 towards IS, exactly as for a thin lens with positive focal length.

System of thin lenses: A system of thin lenses can be described by principal planes in the same way as a thick lens. It has a front **principal plane**, H_1 and a back principal plane H_2, and an **effective focal length**, f_{eff}:

$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$

The distance from the first lens to H_1: $\overline{H_{11}H_1} = \frac{f_{eff}d}{f_2}$
(positive direction from OS towards IS)

The distance from the second lens to H_2: $\overline{H_{22}H_2} = -\frac{f_{eff}d}{f_1}$
(positive direction from OS towards IS)
Observe sign! Error in equation 6.10

Superposition of waves

Sum of two harmonic waves:
$\vec{E_{tot}} = \vec{E_1} + \vec{E_2} = E_{01}e^{i\varphi_1} + E_{02}e^{i\varphi_2}$ where $\varphi = \vec{k} \cdot \vec{r} - \omega t + \epsilon$
The intensity of the total wave is then:
$I_{tot} = |\vec{E_{tot}}|^2 = \vec{E_{tot}}\vec{E_{tot}}^* = E_{01}^2 + E_{02}^2 + E_{01}E_{02}[e^{i(\varphi_1-\varphi_2)} + e^{-i(\varphi_1-\varphi_2)}]$
This can be written as:
$I_{tot} = |\vec{E_{tot}}|^2 = \vec{E_{tot}}\vec{E_{tot}}^* = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\varphi_1 - \varphi_2)$

If the two harmonic waves have the same frequency, i.e. $\omega_1 = \omega_2 = \omega$ (and thus also $k_1 = k_2 = k$), the total wave is also harmonic and can be written as $\vec{E_{tot}} = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\epsilon)}$. The new phase factor ϵ is given by:

$tan\epsilon = \frac{E_{01}sin\epsilon_1 + E_{02}sin\epsilon_2}{E_{01}cos\epsilon_1 + E_{02}cos\epsilon_2}$.

The total amplitude is:

$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}cos(\epsilon)$

The last term in the expression of the amplitude is called the interference term and is the one giving rise to destructive and constructive interference.

Sum of multiple harmonic waves:
When two harmonic waves with different amplitudes and phases are added, it results in a non harmonic wave, but the wave will still be periodic. This is exactly what **Fourier's theorem** states: *A* periodic function can always be viewed as a superposition of multiple harmonic waves, i.e. it can always be divided into its frequency components.
Addition of periodic waves in general can therefore be performed by adding the frequency components. The frequency components are found by **Fourier transformation** of the wave. The Fourier transform of $E(t)$ is given by:

$\widehat{E}(\omega) = \int_{-\infty}^{\infty} E(t)e^{-i\omega t}dt$

Observe that **convolution** of two functions in time space is a simple multiplication in frequency space and vice versa:

$f * g(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau = \widehat{f}(\omega) \cdot \widehat{g}(\omega)$
$f(t) \cdot g(t) = \widehat{\widehat{f} * \widehat{g}}(\omega)$

Laser cavity:
Frequency difference between the longitudinal modes in a laser cavity: $\Delta\nu = \frac{v}{2L}$ where v is the speed of light in the cavity and L is the cavity length.

Heisenberg's uncertainty principle:
$\Delta t \Delta E >= \frac{\hbar}{2}$
where Δt is the spread in time and ΔE the spread in energy ($E = h\nu$).