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# Optical Physics: Summary Session 6

### Interference

Interference happens when waves with the same frequency add up. As for the case of superposition one gets:
$\vec{E_{tot}} = \vec{E_1} + \vec{E_2} = \vec{E_{01}}\cos(\varphi_1) + \vec{E_{02}}\cos(\varphi_2)$ where $\varphi = \vec{k}\cdot\vec{r} - \omega t + \epsilon$

The intensity in the total wave is thus:
$I_{tot} = |\vec{E_{tot}}|^2 = \frac{|\vec{E_{01}}|^2}{2} + \frac{|\vec{E_{02}}|^2}{2} + \vec{E_{01}}\cdot\vec{E_{02}}\cos(\vec{k_1}\cdot\vec{r} - \vec{k_2}\cdot\vec{r} + \epsilon_1 - \epsilon_2)$

The phase difference between the waves is $\delta = \vec{k_1}\cdot\vec{r} - \vec{k_2}\cdot\vec{r} + \epsilon_1 - \epsilon_2$. Observe that the interference term contains $\vec{E_{01}}\cdot\vec{E_{02}}$, which means that one has to take the state of polarization into account (if they are orthogonal to each other, the interference term is zero). In case of parallel states of polarization the expression reduce to:
$I_{tot} = \frac{|\vec{E_{01}}|^2}{2} + \frac{|\vec{E_{02}}|^2}{2} + E_{01}E_{02}\cos(\delta)$

If $\delta = 2\pi m,\ m = 0, 1, 2, ...$ **constructive interference** occurs.
If $\delta = \pi(m + 1),\ m = 0, 1, 2, ...$ **destructive interference** occurs.

When two waves interfere, they give rise to a fringe pattern called **interference fringes**.

One distinguishes different techniques to create interference:

- Wavefront splitting interferometers, e.g. Young's experiment: Two thin slits in a opaque screen, separated by a distance $a$, split the light from a point source in two coherent sources. The phase difference between one ray from each slit to a screen is $\delta = a\sin\theta$, where $\theta$ is the angle from the optical axis.

- Amplitude splitting thin film interferometers: A dielectric plate with thickness $d$ and refractive index $n$. A part of the light is reflected at the first surface and the other part transmitted into the plate where it is reflected by the other surface, producing interference between the first and second reflection. The phase difference is $\delta = \frac{2\pi}{\lambda_0}2n_{skikt}d\cos\theta_t \pm \pi$, where the extra term $\pi$ is due to the phase shift when light is reflected by a denser medium.

- Amplitude splitting mirroring interferometers, e.g. Michelson: A beam splitter divides the light into two parts which are directed by one mirror each towards another beam splitter where they meet and produce an interference pattern on a screen behind the beam splitter. The path for one of the mirror arms, from beam splitter to mirror, is $d$ longer than for the other. The phase difference between the two waves is $\delta = 2d\cos\theta$ where $\theta$ is the angle to the optical axis.

- Multiple ray interferometers, e.g. Fabry-Perot: Light is incident between two mirrors separated by $d$ and is reflected back and forth while a fraction is being transmitted through the mirrors. The transmitted waves are caught by a lens and interfere with each other. The phase difference is $\delta \approx \frac{2\pi}{\lambda_0} 2nd\cos\theta_t + 2\phi$ where $n$ is the refractive index of the medium between the mirrors and $\theta_t$ is the angle of the transmitted rays with the optical axis. The transmission is given by:

  $$\frac{I_t}{I_i} = \frac{1}{1+Fsin^2(\delta/2)},\ F = \frac{4R}{(1-R)^2}$$

  When a Fabry-Perot interferometer is used for spectroscopic purposes the **chromatic resolving power**, $R$, and the **free spectral range**, $(\Delta\lambda_0)_{fsr}$, are important quantities.

  $$R = \frac{\lambda_0}{(\Delta\lambda_0)_{min}} \approx F\frac{2n_f d}{\lambda_0},\ \ (\Delta\lambda_0)_{fsr} \approx \lambda_0^2/2n_f d$$