Formulas and theory for SK2300 Optical Physics

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This document contains useful formulas and a brief summary of the theory for the course SK2300 Optical Physics. The theory is split up in seven parts in order to match the problem classes. More information about the course can be found at KTH Social.

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Constants and units

Constants

Speed of light	$c = 299792458$ m/s
Electric constant (vacuum permittivity)	$\epsilon_0 = 8.8541 \cdot 10^{-12}$ F/m
Magnetic constant (vacuum permeability)	$\mu_0 = 4\pi \cdot 10^{-7}$ H/m
Planck constant	$h = 6.6261 \cdot 10^{-34}$ Js
Planck constant/2π	$\hbar = 1.0546 \cdot 10^{-34}$ Js
Elementary charge	$e = 1.6022 \cdot 10^{-19}$ C
Electron mass	$m_\mathrm{e} = 9.1094 \cdot 10^{-31}$ kg
Atomic mass unit	$1\ \mathrm{u} = 1.6605 \cdot 10^{-27}$ kg
Density of air	$\rho_\mathrm{luft} = 1.2\ \mathrm{kg/m^3}$
Density of water	$\rho_\mathrm{vatten} = 1000\ \mathrm{kg/m^3}$
Resistivity of copper	$\rho_\mathrm{Cu} = 1.7 \cdot 10^{-8}\ \Omega\ \mathrm{m^2/m}$
Wavelength of red light	$\lambda_\mathrm{r} = 650$ nm
Wavelength of green light	$\lambda_\mathrm{g} = 550$ nm
Wavelength of blue light	$\lambda_\mathrm{b} = 450$ nm

SI units

Quantity	**Unit**	**Symbol and expression**
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mol	mol
Luminous intensity	candela	cd
Angle	radian	$\mathrm{rad} = \mathrm{m} \cdot \mathrm{m}^{-1}$
Solid angle	steradian	$\mathrm{sr} = \mathrm{m}^2 \cdot \mathrm{m}^{-2}$
Frequency	hertz	$\mathrm{Hz} = \mathrm{s}^{-1}$
Force	newton	$\mathrm{N} = \mathrm{m\ kg\ s^{-2}}$
Pressure, stress	pascal	$\mathrm{Pa} = \mathrm{N/m^2} = \mathrm{m^{-1}\ kg\ s^{-2}}$
Energy	joule	$\mathrm{J} = \mathrm{N\ m} = \mathrm{m^2\ kg\ s^{-2}}$
Power	watt	$\mathrm{W} = \mathrm{J/s} = \mathrm{m^2\ kg\ s^{-3}}$
Electric charge	coulomb	$\mathrm{C} = \mathrm{s\ A}$
Voltage	volt	$\mathrm{V} = \mathrm{W/A} = \mathrm{m^2\ kg\ s^{-3}\ A^{-1}}$
Electric capacitance	farad	$\mathrm{F} = \mathrm{C/V} = \mathrm{m^{-2}kg^{-1}\ s^4\ A^2}$
Electric resistance	ohm	$\Omega = \mathrm{V/A} = \mathrm{m^2\ kg\ s^3\ A^{-2}}$
Electric conductance	siemens	$\mathrm{S} = \mathrm{A/V} = \mathrm{m^{-2}\ kg^{-1}\ s^{-2}\ A^2}$
Magnetic flux	weber	$\mathrm{Wb} = \mathrm{V\ s} = \mathrm{m^2\ kg\ s^{-2}\ A^{-1}}$
Magnetic field strength	tesla	$\mathrm{T} = \mathrm{Wb/m^2} = \mathrm{kg\ s^{-2}\ A^{-1}}$
Inductance	henry	$\mathrm{H} = \mathrm{Wb/A} = \mathrm{m^2\ kg\ s^{-2}\ A^{-2}}$

Session 1: EM waves

Wave equation

The wave equation is given by

$$\nabla^2\Psi(\vec{r},t) = \frac{1}{v^2}\frac{\partial^2\Psi(\vec{r},t)}{\partial t^2}$$

and has the following solutions:

- Generic
  $\Psi(\vec{r},t) = f(\vec{r} \mp vt)$
- Harmonic plane wave
  $\Psi(\vec{r},t) = A\cos(\vec{k}\cdot\vec{r} \mp \omega t) = A\frac{1}{2}\left(e^{i(\vec{k}\cdot\vec{r}\mp\omega t)} + e^{-i(\vec{k}\cdot\vec{r}\mp\omega t)}\right)$
- Harmonic spherical wave
  $\Psi(\vec{r},t) = \frac{A}{r}\cos(kr \mp \omega t) = \frac{A}{r}\frac{1}{2}\left(e^{i(kr\mp\omega t)} + e^{-i(kr\mp\omega t)}\right)$

All waves that satisfy the **wave equation** can be expressed as a combination of plane waves. The combination or **superposition** means that the wave amplitudes add up. If they are in phase (i.e. same $\vec{k}\cdot\vec{r}\mp\omega t$) **constructive interference** occurs, and if they are out of phase by π, **destructive interference** occurs.

Maxwell's equations

Electromagnetic radiation consists of varying electric $\vec{E}(\vec{r},t)$ and magnetic $\vec{B}(\vec{r},t)$ fields that satisfy Maxwell's equations:

$\nabla\cdot\vec{D} = \rho$	(Gauss's law)
$\nabla\cdot\vec{B} = 0$	(Gauss's law for magnetism)
$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$	(Faraday's law of induction)
$\nabla\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$	(Ampère's circuital law)

$\vec{D} = \epsilon_0\vec{E} + \vec{P}$ is the electric displacement field
$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$ is the magnetizing field
$\vec{P} = \epsilon_0\chi\vec{E}$ is the polarization density
$\vec{M}$ is the magnetic polarization
ρ is the free charge density
$\vec{J}$ is the free current density

Common notations

Amplitude: A, E_0
Spatial frequency: $\kappa = 1/\lambda$
Angular spatial frequency: $k = 2\pi/\lambda$
Temporal frequency: $\nu = 1/\tau$
Angular temporal frequency: $\omega = 2\pi\nu$
Wave speed: $v = \nu\lambda$

Important properties for light propagation in free space

- The $\vec{E}$ and $\vec{B}$ fields are perpendicular to each other and generate each other when they vary.
- $\vec{E} \times \vec{B}$ gives the direction of propagation of the light, i.e. $\vec{k}$.
  For light propagating in the x direction: $E_y = cB_z = \frac{1}{\sqrt{\mu_0 \epsilon_0}} B_z$
- **Poynting vector:** $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = c^2 \epsilon_0 \vec{E} \times \vec{B}$
  Gives the energy transport per unit time and area.
- The time average of the Poynting vector is the **intensity:**
  $I =< \vec{S} >_T = \frac{c\epsilon_0}{2} \vec{E}_0^2 = \frac{c}{\mu_0} < \vec{B}^2 >_T = c\epsilon_0 < \vec{E}^2 >_T$
  This implies that the intensity from a point source decreases with the distance from the source by $1/r^2$.

Refractive index

The refractive index, $n = \frac{c}{v}$, of a material represents the reduction/increase in speed compared with the speed of light in vacuum. When the speed changes so does the wavelength, but not the frequency! The refractive index, n, for a material varies with the frequency of the light (**dispersion**). For some materials the refractive index is a complex number, which means that the material attenuate/amplify the amplitude of the light. All materials absorb strongly at the light frequencies which match the resonance frequencies of the material. Near a resonance frequency n is big. The refractive index for normal glass increases with the frequency in the visual spectra.

Light rays

Light rays are a convenient way of describing light. They point in the direction of propagation of the light wave, i.e. in the same direction as $\vec{k}$, and hence they change direction when n changes. How light travels and changes direction for different n is described by **Fermat's principle:**

A ray follows the optical path , $\int_{s_1}^{s_2} n(s)ds$, which is stationary, i.e. a small perturbation from this path gives only a small change in phase and constructive interference.

From Fermat's principle it follows that:

- Ray propagation is reversible
- Reflection at an interface gives $\theta_i = \theta_r$
- Refraction at an interface gives $n_i \sin\theta_i = n_t \sin\theta_t$

Session 2: Reflection, thin lenses

Fresnels equations: reflection and transmission at interface

The **plane-of-incidence** is the plane that contains the $\vec{k}$-vector (ray of light) and the normal of the surface, i.e. the plane that is normally drawn when marking the angles of incidence, reflection and refraction. All light, polarized and unpolarized, can be described by two polarization compontents: polarization normal to the plane-of-incidence and polarization parallel to the plane-of-incidence. Both these components follow the laws of reflection and refraction, the amplitude of the fields follows Fresnels equations:

$$\left| \begin{array}{l} r_\perp = \frac{E_{0r\perp}}{E_{0i\perp}} = -\frac{\sin(\theta_i-\theta_t)}{\sin(\theta_i+\theta_t)} \\ \\ t_\perp = \frac{E_{0t\perp}}{E_{0i\perp}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)} \\ \\ r_\parallel = \frac{E_{0r\parallel}}{E_{0i\parallel}} = \frac{\tan(\theta_i-\theta_t)}{\tan(\theta_i+\theta_t)} \\ \\ t_\parallel = \frac{E_{0t\parallel}}{E_{0i\parallel}} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_i+\theta_t)\cos(\theta_i-\theta_t)} \end{array} \right|$$

The sign of r indicates the change in direction of the $\vec{E}$-field: a negative sign for $r_\perp$ means that the $\vec{E}$-field has changed direction, a positive sign for $r_\parallel$ means that the $\vec{E}$-field has changed direction (this is due to the choice of coordinate system).
The reflected and transmitted fraction of the intensity given by:
Reflectance $R = \frac{I_r}{I_i} = r^2$
Transmittance $T = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}\frac{I_t}{I_i} = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}t^2$.
For a non-absorbing material it follows that $R + T = 1$. Observe that also $R_\perp + T_\perp = 1$ and $R_\parallel + T_\parallel = 1$.

Total Reflection

Total reflection at a surface happens if: $\theta_i \geq \theta_c = \arcsin(\frac{n_t}{n_i})$.
This means that all light is reflected and nothing is transmitted if the angle of incidence is bigger than or equal to the critical angle θ_c.

This is however not entirely true! If $\theta_i > \theta_c$ one obtains complex values for $r_\perp$ and $r_\parallel$ which actually gives a transmitted wave with exponentially decreasing amplitude. This transmitted wave is called **evanescent** wave and can not normally be observed due to its rapidly decreasing amplitude. However, if two materials are placed close to each other with a very small gap, one can obtain **frustrated total reflection**. This means that a part of the light is tunneling over the gap to the other material even though the condition for total reflection is fulfilled on the first surface. This happens if the gap is so small that the amplitude of the evanescent wave has not been reduced to zero. Frustrated total reflection is used for example in beam splitters.

Geometrical optics: Thin lenses

Geometrical optics is an approximation where the wave properties of light are neglected. Only refraction and reflection of light is taken into account. This approximation works well as long as all dimensions are $\gg \lambda$. One often uses a **Paraxial approximation** assuming that the incidence angles are small. This means that $\sin\theta \approx \theta$.
These approximations combined with Fermat's principle imply that refraction in a spherical surface can be described by:

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

	OS	IS
s_o	+	-
s_i	-	+
R	-	+

OS (object space) is defined as the area to the left of the refracting surface
IS (image space) is defined as the area to the right of the refracting surface
n_1 and n_2 are the refractive indeces in OS and IS respectively
s_o is the distance object - surface (positive if object is in OS)
s_i is the distance surface - image (positive if image is in IS)
R is the radius of curvature of the surface (positive if the center of curvature is in IS)

This formula gives rise to a few relations:

- Thin lens surrounded by different media: $\frac{n_{m,o}}{s_o} + \frac{n_{m,i}}{s_i} = \frac{n_l - n_{m,o}}{R_1} - \frac{n_l - n_{m,i}}{R_2}$
- Thin lens surrounded by air: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2})$
  f = focal length of lens
- Newton's formula: $x_o x_i = f^2$
  x_o and x_i are the distances from front focal plane to object and back focal plane to object
- Transverse magnification: $M_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$
  y_o and y_i are the heights of the object and image (positive if above optical axis)
- Longitudinal magnification: $M_L = \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$

Session 3: Thin lenses, beam limiters

Thin lenses

The following equations hold for imaging using a thin lens:

- Lens maker's formula (thickness → 0): $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2})$
  f is the focal length, s_o and s_i are the distances object - surface (positive if object in OS) and surface - image (positive if image in IS) and R is the radius of curvature of the surface (positive if center of curvature in IS)

- Thin lens surrounded by air: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2})$
  f = focal length

- Newton's formula: $x_o x_i = f^2$
  x_o and x_i are the distances from front focal plane to object and back focal plane to image

- Imaging using mirror: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} = -\frac{2}{R}$

- Transverse magnification: $M_T = \frac{y_i}{y_o} = -\frac{s_i}{s_o}$
  y_o and y_i are the image and objective heights, respectively (positive if above optical axis)

- Longitudinal magnification: $M_L = \frac{dx_i}{dx_o} = -\frac{f^2}{x_o^2} = -M_T^2$

Stops and pupils

An optical systems light gathering capability and imaging constraints are usually described by the following parameters:

- **Aperture stop, AS:** The lens or aperture that limits the bundle of rays emanating from an object on axis and at a given distance from the system. The ray from an object point that passes through the center of AS is called **chief ray**. The ray from an object point that passes through the edge of AS is called a **marginal ray**.

- **Field stop, FS:** The lens or aperture that, together with AS, limits the bundle of rays from an object above/below the axis at a given distance from the system. FS limits the size of objects that can be imaged through the system.

- **Entrance pupil, IP:** The image of AS seen from the object space. IP limits the light cone entering the system.

- **Exit pupil, UP:** The image of AS seen from the image space. UP limits the light cone exiting the system.

For object points above/under the optical axis, the AS in combination with FS can block more light than for objects on axis. This gives a reduced intensity for off axis image points. This phenomenon is called **vignetting**.

Session 4: Think lenses, superposition

Thick lenses

All lenses can not be approximated as thin, i.e. their thickness, d, has to be taken into account. A thick lens optical properties in air are described by two **principal planes**, H_1 and H_2, and an **effective focal length**, f_{eff}:

$$\frac{1}{f_{\text{eff}}} = (n_l - 1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d}{n_l R_1 R_2}\right]$$

Vertex: point where the lens surface intersect the optical axis

The distance from the left vertex to $H_1 = -\frac{f_{\text{eff}}(n_l-1)d}{R_2 n_l}$
(positive direction from OS towards IS)

The distance from the right vertex to $H_2 = -\frac{f_{\text{eff}}(n_l-1)d}{R_1 n_l}$
(positive direction from OS towards IS)

If one *"disregards"* the distance between H_1 and H_2, one can perform calculations and ray constructions according to the same formulas as for thin lenses. One thus pretends that H_1 is the left vertex of a thin lens and H_2 the right vertex. f_{eff} is the focal length of the thin lens, measured as positive from H_1 towards OS and from H_2 towards IS, exactly as for a thin lens with positive focal length.

System of thin lenses

A system of thin lenses can be described by principal planes in the same way as a thick lens. It has a front **principal plane**, H_1 and a back principal plane H_2, and an **effective focal length**, f_{eff}:

$$\frac{1}{f_{\text{eff}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

The distance from the first lens to H_1: $\overline{H_{11}H_1} = \frac{f_{\text{eff}}d}{f_2}$
(positive direction from OS towards IS)

The distance from the second lens to H_2: $\overline{H_{22}H_2} = -\frac{f_{\text{eff}}d}{f_1}$
(positive direction from OS towards IS)
Observe sign! Error in equation 6.10

Superposition of waves

Sum of two harmonic waves:

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 = E_{01}e^{i\varphi_1} + E_{02}e^{i\varphi_2} \text{ where } \varphi = \vec{k}\cdot\vec{r} - \omega t + \epsilon$$

The intensity of the total wave is then:

$$\begin{aligned} I_{\text{tot}} = |\vec{E}_{\text{tot}}|^2 = \vec{E}_{\text{tot}}\vec{E}^*_{\text{tot}} &= E_{01}^2 + E_{02}^2 + E_{01}E_{02}[e^{i(\varphi_1-\varphi_2)} + e^{-i(\varphi_1-\varphi_2)}] \\ = \vec{E}_{\text{tot}}\vec{E}^*_{\text{tot}} &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\varphi_1-\varphi_2) \end{aligned}$$

If the two harmonic waves have the same frequency, i.e. $\omega_1 = \omega_2 = \omega$ (and thus also $k_1 = k_2 = k$), the total wave is also harmonic and can be written as $\vec{E}_{\text{tot}} = E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t+\epsilon)}$. The new phase factor ϵ is given by:

$$\tan\epsilon = \frac{E_{01}\sin\epsilon_1 + E_{02}\sin\epsilon_2}{E_{01}\cos\epsilon_1 + E_{02}\cos\epsilon_2}.$$

The total amplitude is:

$$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos\epsilon$$

The last term in the expression of the amplitude is called the interference term and is the one giving rise to destructive and constructive interference.

Sum of multiple harmonic waves

When two harmonic waves with different amplitudes and phases are added, it results in a non harmonic wave, but the wave will still be periodic. This is exactly what **Fourier's theorem** states: *A periodic function can always be viewed as a superposition of multiple harmonic waves, i.e. it can always be divided into its frequency components.*
Addition of periodic waves in general can therefore be performed by adding the frequency components. The frequency components are found by **Fourier transformation** of the wave. The Fourier transform of $E(t)$ is given by:

$$\widehat{E}(\omega) = \int_{-\infty}^{\infty} E(t)e^{-i\omega t}dt$$

Observe that **convolution** of two functions in time space is a simple multiplication in frequency space and vice versa:

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(t-\tau)g(\tau)d\tau = \widehat{f}(\omega)\cdot\widehat{g}(\omega)$$
$$f(t)\cdot g(t) = \widehat{f}(\omega) * \widehat{g}(\omega)$$

Laser cavity

Frequency difference between the longitudinal modes in a laser cavity: $\Delta\nu = \frac{v}{2L}$ where v is the speed of light in the cavity and L is the cavity length.

Heisenberg's uncertainty principle:

$$\Delta t\Delta E >= \frac{\hbar}{2}$$

where Δt is the spread in time and ΔE the spread in energy ($E = h\nu$).

Session 5: Polarization, birefringence

Polarization

One usually speaks of four different **states of polarization** for light, and they can all be divided into two polarization states orthogonal to each other:

- $\mathcal{P}$ Linearly polarized: $\vec{E} = [E_{0x}, E_{0y}, 0]\cos(\omega t - kz)$
- $\mathcal{R}$ Right circular polarized: $\vec{E} = E_0[\cos(\omega t - kz), \sin(\omega t - kz), 0]$
- $\mathcal{L}$ Left circular polarized: $\vec{E} = E_0[\cos(\omega t - kz), -\sin(\omega t - kz), 0]$
- $\mathcal{E}$ Elliptically polarized: $\vec{E} = [E_{0x}\cos(\omega t - kz), E_{0y}\cos(\omega t - kz + \epsilon), 0]$

When light is reflected by a surface at the **Brewster angle**, only light polarized with its $\vec{E}$-field normal to the plane of incidence will be reflected. The reflected wave is thus linearly polarized, even though the incident wave is unpolarized (this can be understood from Fresnel's equations). Brewster's angle θ_p fulfills the following condition:

$$\theta_r + \theta_t = 90^\circ \implies \tan\theta_p = \frac{n_t}{n_i}$$

Dichroism

Dichroism means that the absorption is different for the two orthogonal states of polarization.

Birefringence

Birefringence takes place in materials that have two different refractive indices for two states of polarization. The reason behind this is an unsymmetrical binding force between the electrons and the atom cores (the material is anisotropic). In a birefringent material the **optical axis** is defined as the direction with the extraordinary refractive index, n_e. All other directions have the ordinary refractive index, n_o. This means that light incident on the material with the $\vec{E}$-field orthogonal to the optical axis will feel n_o, whereas light with the $\vec{E}$-field parallel to the axis will feel n_e. A birefringent material with two different refractive indices has one optical axis and is therefore called **uniaxial**. There are also biaxial birefringent materials with three different refractive indices and therefore two optical axes.

Birefringence can be induced in some common materials by mechanical stress (photoelasticity), magnetic fields (e.g. Faraday effect) or electrical fields (e.g. Kerr effect).

Retarders

Retarders are made of birefringent materials and they can change the state of polarization of a beam by giving one of the $\mathcal{P}$-components a phase difference $\Delta\varphi$. This happens when light is incident orthogonal to the optical axis of the material. The $\vec{E}$-component parallel with the optical axis experiences a refractive index n_e and the component orthogonal to the axis experiences n_o. This induces a phase difference between the polarization states. The phase difference induced between the parallel and orthogonal polarization state, if the material has a thickness d, is:

$$\Delta\varphi = \frac{2\pi}{\lambda_0} d(n_e - n_o)$$

Full-wave plate: $\Delta\varphi = 2\pi$
Half-wave plate: $\Delta\varphi = \pi$
Quarter-wave plate: $\Delta\varphi = \frac{\pi}{2}$

A retarder can have a phase difference that is the desired plus a multiple of 2π, a so called multiple-order retarder.

Optical activity

A material is **optically active** if it refracts the $\mathcal{R}$-component and the $\mathcal{L}$-component differently when light passes through the material. Thus the material has different $n_{\mathcal{R}}$ and $n_{\mathcal{L}}$, and is circularly birefringent. This effect is seen if the molecules are stereoisomeric, i.e. has a righthanded and lefthanded variant, which is the mirror image of the other.

Session 6: Interference

Interference

Interference happens when waves with the same frequency add up. As for the case of superposition one gets:

$$\vec{E}_{\text{tot}} = \vec{E}_1 + \vec{E}_2 = \vec{E}_{01}\cos(\varphi_1) + \vec{E}_{02}\cos(\varphi_2) \text{ where } \varphi = \vec{k}\cdot\vec{r} - \omega t + \epsilon$$

The intensity in the total wave is thus:

$$I_{\text{tot}} = |\vec{E}_{\text{tot}}|^2 = \frac{|\vec{E}_{01}|^2}{2} + \frac{|\vec{E}_{02}|^2}{2} + \vec{E}_{01}\vec{E}_{02}\cos(\vec{k}_1\cdot\vec{r} - \vec{k}_2\cdot\vec{r} + \epsilon_1 - \epsilon_2)$$

The phase difference between the waves is $\delta = \vec{k}_1\cdot\vec{r} - \vec{k}_2\cdot\vec{r} + \epsilon_1 - \epsilon_2$. Observe that the interference term contains $\vec{E}_{01}\vec{E}_{02}$, which means that one has to take the state of polarization into account (if they are orthogonal to each other, the interference term is zero). In case of parallel states of polarization the expression reduce to:

$$I_{\text{tot}} = \frac{|\vec{E}_{01}|^2}{2} + \frac{|\vec{E}_{02}|^2}{2} + E_{01}E_{02}\cos(\delta)$$

If $\delta = 2\pi m,\ m = 0, 1, 2, ...$ **constructive interference** occurs.
If $\delta = \pi(m+1),\ m = 0, 1, 2, ...$ **destructive interference** occurs.

When two waves interfere, they give rise to a fringe pattern called **interference fringes**.

One distinguishes different techniques to create interference:

- Wavefront splitting interferometers, e.g. Young's experiment: Two thin slits in a opaque screen, separated by a distance a, split the light from a point source in two coherent sources. The phase difference between one ray from each slit to a screen is $\delta = a\sin\theta$, where θ is the angle from the optical axis.

- Amplitude splitting thin film interferometers: A dielectric plate with thickness d and refractive index n. A part of the light is reflected at the first surface and the other part transmitted into the plate where it is reflected by the other surface, producing interference between the first and second reflection. The phase difference is $\delta = \frac{2\pi}{\lambda_0}2nd\cos\theta_t \pm \pi$, where the extra term π is due to the phase shift when light is reflected by a denser medium.

- Amplitude splitting mirroring interferometers, e.g. Michelson: A beam splitter divides the light into two parts which are directed by one mirror each towards another beam splitter where they meet and produce an interference pattern on a screen behind the beam splitter. The path for one of the mirror arms, from beam splitter to mirror, is d longer than for the other. The phase difference between the two waves is $\delta = 2d\cos\theta$ where θ is the angle to the optical axis.

- Multiple ray interferometers, e.g. Fabry-Perot: Light is incident between two mirrors separated by d and is reflected back and forth while a fraction is being transmitted through the mirrors. The transmitted waves are caught by a lens and interfere with each other. The phase difference is $\delta \approx \frac{2\pi}{\lambda_0} 2nd\cos\theta_t + 2\phi$ where n is the refractive index of the medium between the mirrors and θ_t is the angle of the transmitted rays with the optical axis. The transmission is given by:

  $$\frac{I_t}{I_i} = \frac{1}{1 + F\sin^2(\delta/2)}, \qquad F = \frac{4R}{(1-R)^2}$$

  When a Fabry-Perot interferometer is used for spectroscopic purposes the **chromatic resolving power**, R, and the **free spectral range**, $(\Delta\lambda_0)_{\mathrm{fsr}}$, are important quantities.

  $$R = \frac{\lambda_0}{(\Delta\lambda_0)_{\mathrm{min}}} \approx F\frac{2n_{\mathrm{f}}d}{\lambda_0}, \qquad (\Delta\lambda_0)_{\mathrm{fsr}} \approx \lambda_0^2/2n_{\mathrm{f}}d$$

Session 7: Diffraction

Diffraction

When light passes an aperture it's diffracted (spread out) due to its wave properties. Diffraction makes the image of the aperture a bit blurred. The mathematical calculation describing how the aperture influences light can be approximated for different distances behind the aperture.

Fraunhofer approximation

The Fraunhofer approximation is adapted to large distances, i.e. when parallel light is incident on the aperture and then propagates to a screen infinitely far away from the aperture. Observe that a lens images infinity to the focal plane, which can be very useful. The Fraunhofer pattern registered is identical to the Fourier transform of the shape of the aperture. These arrangements are therefore called **Fourier optical arrangements**. The Fraunhofer pattern for some common apertures are given in the book, where the Fraunhofer pattern for a circular aperture with diameter d is an Airy disc with radius $q_1 = 1,22\frac{R\lambda}{d} = 1,22\frac{f\lambda}{d}$ where R is the distance between aperture and screen and f is the focal length of an optional lens.

The Fresnel approximation

The Fresnel approximation is valid for distances close to the aperture or when the incident light is not parallel. The mathematical expressions for this case becomes more complicated than for the case with Fraunhofer diffraction.

Diffraction patterns

A few important things that makes it easier to determine what the pattern from a particular aperture will look like:

- A complicated aperture can be divided into a few apertures, each with a simple shape. The sum of the diffraction patterns from each aperture gives the total diffraction pattern. Observe that small details in the aperture give a large diffraction pattern.
- **Babinet's principle** states that the diffraction pattern is independent of whether the aperture is inverted or not, i.e. the pattern from an opaque screen with a hole is the same as that of a transparent screen with an opaque circle.
- The Fourier transform is translation invariant and the Fraunhofer pattern is therefore insensitive to translation of the aperture.

Coherence

The coherence of a light source is a measure of how long (time coherence/longitudinell coherence) and toward which directions (spatial coherence/transverse coherence) the source emits with the same phase. This is especially important when interference is desired. The two interfering waves must be coherent with each other (observe that two separated light sources are not coherent). If this is the case one obtains interference fringes with a certain **visibility**:

$$\mathcal{V} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

The visibility is 1 for a pattern with $I_{\min} = 0$.