

Problems and solutions for SK2300 Optical Physics

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This document contains all the problems that are treated during the problem classes. The problems consist of old examination questions that have been selected to match the topic of each problem class. All old exams are available at the course homepage (<https://www.kth.se/social/course/SK2300/>).

Solution guidelines are only available for a couple of problems but more might be added later so be sure to check for updates every now and then.

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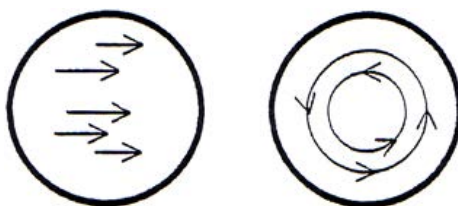
Session 1

Electromagnetic waves

1.1 Problems

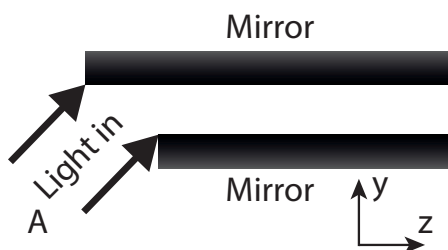
930205:3

The circles in bold in the figure below is meant to illustrate a cross section of a beam of light. The left one illustrates a circularly polarized wave. In this case the electric field vector, E , turns around with constant amplitude. The question is whether the electric field vector can look like the ones in the right circle.



930402:5

Light is reflected back and forth between two lossless mirrors like in the figure below. The incoming light, at A, is a plane wave with the electric field vector in the plane of the paper. The incidence angle of the light is 45° . The main propagation direction of the light wave is in the z -direction. Find the ratio between the longitudinal and the transverse electric field component! What is the phase speed in the z -direction?



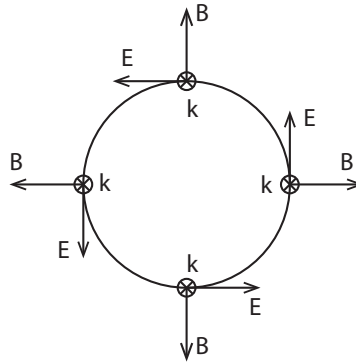
940824:2

A poor student in physics is performing calculations on a problem where the wave equation describing the propagation of light is involved. He/She ends up with a solution where the \mathbf{D} - and \mathbf{E} -field is not in phase with each other, but has a small phase difference ($\Delta\delta \sim 1^\circ$). Confusion arises in his/her head which is why we want you to explain what phenomenon that can give rise to this! Some equations would be helpful to make him/her understand.

1.2 Solutions

930205:3

The propagation direction (\mathbf{k}) is perpendicular to the board. Also, we know that \mathbf{E} , \mathbf{B} and \mathbf{k} are all perpendicular. The only possible configuration for the case in the right circle is then



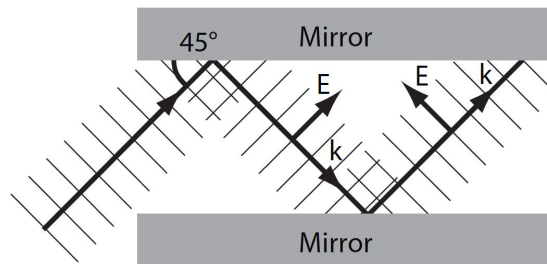
However, from Maxwell's equations we know that

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{no magnetic monopoles})$$

$$\oiint \mathbf{B} \cdot d\mathbf{S} = 0 \quad (\text{no net magnetic flux through a closed surface})$$

A cylindrical surface around the beam would have a net flux
 \Rightarrow The \mathbf{E} -field cannot look like the one in the right circle.

930402:5



Lets start with the expression describing an arbitrary plane wave,

$$\mathbf{E}(\mathbf{r}, t) = \text{Re} \left\{ \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)} \right\}$$

where $\mathbf{E}_0 = [E_x, E_y, E_z]$, $\mathbf{r} = [x, y, z]$ and $\mathbf{k} = [k_x, k_y, k_z]$. It is often easier to perform the calculations with complex waves and then take the real part of the end result to find the physical wave. The total field \mathbf{E}_{tot} between the mirrors is the sum of two fields \mathbf{E}_{\nearrow} and \mathbf{E}_{\searrow} , where the second is a reflection of the first, which means that \mathbf{E}_{\searrow} is shifted π radians compared to \mathbf{E}_{\nearrow} . Since the

problem is in 2D we will only consider the y and z components of the electric field.

$$\begin{aligned}
 \mathbf{E}_{\text{tot}} &= \mathbf{E}_{\nearrow} + \mathbf{E}_{\searrow} \\
 &= [E_y, -E_z] e^{i(k_y y + k_z z - \omega t)} + [E_y, E_z] e^{i(-k_y y + k_z z - \omega t)} \\
 &= \left\{ k_y = k_z = \frac{k}{\sqrt{2}}, E_y = E_z = \frac{E_0}{\sqrt{2}} \right\} \\
 &= \frac{E_0}{2} \underbrace{e^{i(kz/\sqrt{2} - \omega t)}}_{e^{i\varphi}} \left[\underbrace{e^{ky/\sqrt{2}} + e^{-ky/\sqrt{2}}}_{=2 \cos(ky/\sqrt{2})}_{=\theta}, \underbrace{-e^{ky/\sqrt{2}} + e^{-ky/\sqrt{2}}}_{=-2i \sin(ky/\sqrt{2})}_{=\theta} \right] \\
 &= \sqrt{2} E_0 (\cos \varphi + i \sin \varphi) [\cos \theta, -i \sin \theta]
 \end{aligned}$$

Now we find the physical field by taking the real part of this result,

$$\mathbf{E}_{\text{tot}} = \sqrt{2} E_0 [\cos \varphi \cos \theta, \sin \varphi \sin \theta]$$

The ratio between the z and the y component \mathbf{E}_{tot} is then,

$$\frac{E_{\text{tot},z}}{E_{\text{tot},y}} = \tan \varphi \tan \theta = \tan(kz/\sqrt{2} - \omega t) \tan(ky/\sqrt{2})$$

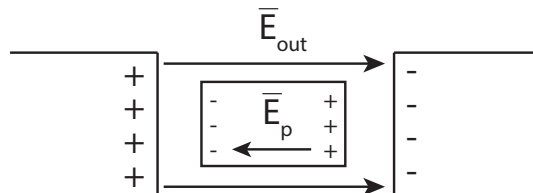
The phase velocity in the z direction is found by calculating how fast a point with constant phase is moving,

$$\varphi = kz/\sqrt{2} - \omega t = C \quad \Rightarrow \quad z = \sqrt{2} \frac{(C + \omega t)}{k}$$

$$\frac{\partial z}{\partial t} = \frac{\omega}{k} \sqrt{2} = c\sqrt{2}$$

This does not mean that information can be transferred faster than the speed of light! The wave fronts are just positions in space where the phase is constant. These positions can move with infinite velocity. Modulations of the initial wave will propagate like a short light pulse would.

940824:2



First we have to know how the outer electric field \mathbf{E}_{out} , the so called displacement field $\mathbf{D} = \epsilon_0 \mathbf{E}_{\text{out}}$, and the electric field inside the dielectric material \mathbf{E}

are related. If an external electric field is applied to a dielectric (not conducting) material the material will be polarized, resulting in a polarization \mathbf{P} . The relation is then,

$$\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} - \frac{\mathbf{P}}{\varepsilon_0} \quad \Rightarrow \quad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

If $\mathbf{P} = 0$, as in vacuum, \mathbf{E} and \mathbf{D} are in phase. If they are out of phase this must be due to the polarization of the material. This depends on the electrical susceptibility χ_e of the material, which describes how easy the material is polarized $\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$.

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} = \varepsilon_0 \underbrace{(1 + \chi_e)}_{\varepsilon_r} \mathbf{E} = \varepsilon \mathbf{E}$$

where ε_r is the relative permittivity (dielectric constant) of the material. If ε_r is real there will be no phase difference between \mathbf{E} and \mathbf{D} , but if ε_r is complex (or negative) $\varepsilon_r = |A|e^{i\varphi}$ it will introduce a phase difference of φ . Now, what is the physical meaning of a complex ε_r ? ε is related to the speed of propagation c , which is related to the index of refraction n .

$$n = \frac{c_0}{c} = \frac{\sqrt{\varepsilon\mu}}{\varepsilon_0\mu_0} = \sqrt{\varepsilon_r\mu_r} = \{\mu_r \approx 1 \text{ for most optical materials}\} \approx \sqrt{\varepsilon_r}$$

So a complex ε_r results in a complex index of refraction $n = \alpha + i\beta$. The real part α of the complex index of refraction is related to the propagation speed and the imaginary part is related to absorption or amplification.

$$\begin{aligned} \mathbf{E} &= \mathbf{E}_0 e^{i(kz - \omega t)} = \left\{ k = \frac{2\pi}{\lambda} = \frac{2\pi n}{\lambda_0} = k_0 n \right\} \\ &= \mathbf{E}_0 e^{i(k_0 z (\alpha + i\beta) - \omega t)} = \underbrace{\mathbf{E}_0 e^{-k_0 \beta z}}_{\text{amplitude}} \cdot e^{i(k_0 \alpha z - \omega t)} \end{aligned}$$

The intensity $I(z)$ will then decrease, or increase, exponentially in the following way,

$$I(z) \propto |\mathbf{E}|^2 = \mathbf{E} \cdot \mathbf{E}^* = |\mathbf{E}_0|^2 e^{-2k_0 \beta z}$$

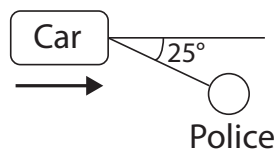
Session 2

Reflection, thin lenses

2.1 Problems

940406:3

A police standing beside the road that wants to study the use of seatbelts in cars has good use for a pair of polarizing sunglasses. How much is the reflection from the windscreen reduced when he wears them, if he is positioned according to the figure below?



960111:1

Assume that we have a 45-45-90-degree prism where a parallel laser beam is incident against one of the cathetus at normal incidence. The refractive index is 1.54 and the prism is surrounded by air. Is it possible to design an AR-coating (anti reflection) such that the hypotenuse has a transmittance higher than 10%?

031024:1

Modern cameras unfortunately often have a fixed objective that can not be changed. However, one can change the focal length of the system by attaching an afocal lens combination in front of the camera objective. The system focal length is thereby changed by a factor equal to the telescope magnification of the lens combination that was attached. Show this by using a ray diagram and conformal triangles! Make a nice figure that is easy to understand!

2.2 Solutions

940406:3

The reflection is reduced by

$$\frac{R_{\perp} + R_{\parallel}}{R_{\parallel}} = \frac{0.11 + 0.003}{0.003} \approx 40$$

given that the police tilts his head by 23° so that R_{\perp} is completely blocked.

960111:1

No, we get TIR.

031024:1

N/A

Session 3

Thin lenses, beam limiters

3.1 Problems

031020

Task 1-3 treat the same problem which is described below:

You have a laser beam which can be treated as circular, i.e. you don't have consider a Gaussian profile, and you want to build a telescope that can be adjusted in such a way that the magnification can be varied between a value smaller than 1 and one which is bigger than 1. You have the following lens package at your disposal.

- a. a $f = -20$ mm lens with a diameter of 10 mm
- b. a $f = 50$ mm lens with a diameter of 20 mm

The distance between the two lenses, d , can be varied from 35 mm to 50 mm. Another lens, with $f = -100$ mm and a diameter of 40 mm, is placed at a distance (for you to calculate) after the lens system.

The laser beam is parallel (not divergent or convergent) both when entering and exiting the telescope.

031020:1-2

Construct the system, i.e. draw a figure with all distances marked for the case where $d = 40$ mm.

Between which values can the magnification be varied?

031020:3

Assume that the beam has a radius bigger than 4 mm, in which case diffraction will occur. What angular spread does this give rise to? Assume d to be 40 mm and that $\lambda = 514$ nm.

040112:1

An ophthalmologic biomicroscope is used for looking into the human eye. It has a special illumination system, the details of which are not given here. It has an objective with a focal length of 50 mm and an eyepiece with 20 mm focal length. The distance between the back principal plane of the objective and the front principal plane of the eyepiece is 220 mm.

The objective has a front vertex focal length of 90 mm.

How far into the eye is it possible to see if the eye is modeled by a sphere with a radius of curvature of 5 mm at cornea and a refractive index of 1.3306?

The distance from cornea to retina is such that a sharp image of a distant object is obtained on the retina.

040112:2

When the previous task is solved you will have found that it is not possible to see the retina since it is approximately 20 mm from cornea.

One way to observe the retina is to use a strong positive lens (40 D) to image the retina into a real image which is in turned viewed by the doctor who at the same time images the pupil of the patients eye onto his/her own pupil using the same 40 D lens. The patient's eye is relaxed, i.e. focused at distant objects.

How large is the field of view, i.e. how much of the retina can be observed in the following case?

Distances:

Patient's eye with pupil in front of cornea

26.7 mm distance

40 D lens

400 mm distance

The eye of the doctor

The lens has a diameter of 25 mm, i.e. the $f\#$ is 1

031024:2

A microscope has an objective with $f = 3.00$ mm and a diameter of 2 mm and use a Huygens eyepiece with $f = 20$ mm. The tube length (distance from objective to the front focal plane of the eyepiece) is 200 mm. What limitations on the size of the field lens and the eyepiece are there if the field of view must be larger than 0.25 mm?

3.2 Solutions

031020

1-2

$$0.5 \geq M \geq 2$$

3

0.094 mrad

040112:1

17.6 mm

040112:2

13 mm

031024:2

$$D_f \geq 16.4 \text{ mm}, D_e \geq 1.64 \text{ mm}.$$

Session 4

Think lenses, superposition

4.1 Problems

031024:6

An optimistic person tries to manufacture a telescope using a lens with $f = 1200$ mm, and a diameter of 50 mm, as the objective. The lens is shaped to minimize spherical aberrations when turned correctly, and it's made of crown glass (use the book for data). He uses a commercially available ocular with $f = 20$ mm. Assume that Jupiter is viewed using this telescope and that it has an angular diameter as seen from Earth ~ 0.3 mrad. How much bigger (%) is the image in blue and red compared with yellow/green (D-wavelength)? Jupiter is placed symmetrically around the symmetry axis of the telescope.

040112:6

Three different situations occur when imaging a detailed image using a slide projector onto a screen.

- Only the central part of the image is sharp. Refocusing makes the peripheral part of the image sharp but blurs the central part.
- Radial structures in the image are sharp. Refocusing leads to structures being orthogonal against lines from the center of the image sharp.
- Straight lines in the periphery of the image are bent.

Which aberrations occur in the three different cases? Motivate!

930825:5

The atoms in a sodium lamp emit light according to $E = E_0 \sin(\omega_0 t) e^{-\gamma t}$ unless they are not disturbed. However, in a high pressure lamp the many collisions force the emission of light to start over. This means that the expression above is changed by multiplying with a rect function with a width equal to the mean time τ between the collisions. Derive an expression for how the spectrum is broadened by this. (Express the spectra in frequency and not in wavelength)

940406:2

Several technologies can be used when the police measure the speed of an approaching car using a laser. The most common method is to measure the change in pulse-travel-time between two pulses in a pulse train from a laser. Assume that one want to generate pulses by letting the modes in a laser interfere thereby creating a beating.

Assume that these are separated in frequency with 300 MHz and that their initial phases are locked against each other (this can be arranged). How many modes must contribute in order to give 100 ps long pulses (full width at half maximum)? The wavelength is 900 nm.

One might find it useful to look at the derivation of the diffraction pattern from a grating!

4.2 Solutions

031024:6

The red/blue image is 120% larger than the yellow/green image.

040112:6

a) Field curvature b) Astigmatism c) Distortion

930825:5

$$I(\omega) = |E(\omega)|^2 = \frac{E_0^2}{4} \frac{|e^{(i\Delta\omega + \gamma)\tau} - 1|^2}{(\Delta\omega)^2 + \gamma^2}$$

940406:2

33 modes.

Session 5

Polarization, birefringence

5.1 Problems

031020:5

Quarter- and half-wavelength plates are described in the book, but there also exist full-wavelength plates in the so called reality. These are commonly of the first order, i.e., the phase difference between o and eo is a wavelength for the design wavelength, and they are positioned between two crossed polarizers. Assume that we have such a full-wavelength plate, with a design wavelength of 550 nm, and that it is illuminated by white light. The optical axis is oriented 45° against the let-through direction of the polarizer. Additional transparent objects are positioned between the two polarizers, which partially cover the field of view with a weak birefringence, i.e., $\sim 1/6$ wavelength for 550 nm. What does the background look like and what does the object look like? Observe that different situations occur depending on the direction of the optical axis of the object.

040112:4

Assume that we have a birefringent crystal that can be controlled optically. It is positioned at a 45° angle between two crossed polarizers. The geometrical path length through the crystal is 5.00 mm. What value on the birefringence ($n_{eo}-n_o$) makes the transmitted light appear:

- a. Yellow
- b. Blue

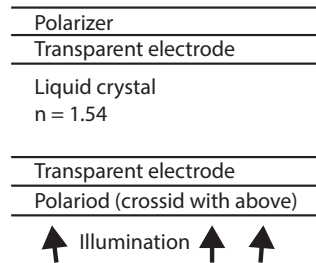
950419:3

Two polarizers can be useful to limit the amount of transmitted light. However, they are seldom ideal but has a limited transmission, T_p , also in the let-through direction, and the transmission T_s is not zero for light polarized orthogonal against it. Assume that we have two polarizers with $T_p = 0.94$ and $T_s = 0.03$.

They are positioned with their let-through directions orthogonal to each other. How much light passes through this combination and how is the transmitted light polarized?

950301:1

Older LCD screens for notebooks use liquid crystals to influence the polarization according to the figure below. A certain color was obtained by adapting the voltage to give transmission maxima for that certain color. However, this produces very pale colors since also surrounding wavelengths are partly transmitted. Assume that one want to produce green (546 nm). Determine analytically and plot the transmitted intensity as a function of wavelength.



5.2 Solutions

031020:5

Only λ -plate: No green transmitted but some blue and red
 \Rightarrow It will look purple.

With an additional $\lambda/6$ -plate:

1. 45° to the λ -plate: No effect
2. 0° : It will look blue
3. 90° : It will look red

040112:4

a) $\Delta n = 1.8 \cdot 10^{-4}$ b) $\Delta n = 1.35 \cdot 10^{-4}$

950419:3

2.8% is transmitted. Same polarization.

950301:1

$$I \propto \frac{E_0^2}{4} \left(1 - \cos \left(\frac{\lambda_g (2m + 1)\pi}{\lambda} \right) \right)$$

Session 6

Interference

6.1 Problems

031020:6

Assume you have a laser based Michelson interferometer (the same kind as in the laboration in this course). What happens to the interference pattern if you turn the beam splitter just a bit (a few mrad)? The interferometer was perfectly aligned from the beginning.

031024:4

In a Mach-Zendner interferometer two interference patterns are obtained which are said to be complementary (from the book). Show this for a simple case, for example a beam splitter with a high index of refraction which has one side AR coated (the effect of the AR coating can be neglected).

930402:4

This Michelson interferometer is based on a laser with a wavelength of 513 nm. The laser beam first passes through a negative lens with $f = -20$ mm and then through a positive lens with $f = 200$ mm. The distance between the lenses is 160 mm. The optical path length from the last lens to the end screen is for the first arm 1200 mm and for the second 1210 mm. How many interference fringes are visible with a radius of 80 mm?

030107:3

The image of a red (620 nm) text display is to be mirrored into the path of light (in the projector mentioned above). This is done with a glass plate in 45° covered with a thin layer with $n=1.80$. How thick should this be in order to get maximum reflectance for red but minimal for blue through green?

6.2 Solutions

031020:6

We get fringes with a period of $30\ \mu\text{m}$.

031024:4

N/A

930402:4

6.9 fringes

030107:3

280 nm thick.

Session 7

Diffraction

7.1 Problems

031024:3

A ring aperture has some advantages over normal apertures when it comes to resolution. Investigate what these advantages can be by plotting (in the same graph) the intensity distribution of:

- a. a circular aperture with diameter D
- b. a circular aperture with diameter $0.8D$
- c. a ring aperture with inner diameter $0.8D$ and outer diameter D

It is important that the zero intensity positions of the different patterns can be seen. In which way is the ring aperture better?

031024:5

Assume you have a phase grating where the amplitude transmission varies between 1 and -1. The period is 1000 nm and the light is incident perpendicular to the plane of the grating. The wavelength of the light is 500 nm. In which angles do you get intensity maxima?

040112:3

One way to make contact lenses is to add zone plates on top of the normal contact lens. Zone plates have a very strong reversed dispersion compared to normal lenses. The power of a zone plate increases with increasing wavelength. What is the dispersion of zone plates expressed in Abbe number?

040112:5

An object consisting of a row of 10 equidistant small holes is imaged using a diffraction limited lens with $NA = 0.20$. For which distance, expressed in

wavelengths, between the holes will the 1st diffraction order fall outside the numerical aperture of the lens? Compare this result with Rayleigh's resolution criteria and comment on differences if there are any.

940824:4

In some electronic components a copper wire grid is used (about 10 wires/mm in a squared grid pattern). It is very important that the wires are intact and that they have the correct position in the grid. It is suggested that the grid will pass by the object plane of a Fourier optical arrangement. The detector placed in the image plane of the arrangement should only detect light if there is an error in the copper grid, not otherwise. Discuss different placements and designs of possible filters and select the one you would choose. Make a drawing with distances marked.

930825:4

Three point-like light sources are placed at a distance B from a double slit. The distance between the point sources is A , the central one being centered in front of the double slit. The distance between the two slits is D . A screen is placed on the other side of the double slit at distance C . If the conditions are correct a diffraction pattern will be visible on the screen. Which is the smallest distance A (except $A = 0$) where the visibility of the pattern is close to 1? The wavelength of the light can be set to λ .

7.2 Solutions

031024:3

N/A

031024:5

$\pm 30^\circ$

040112:3

-3.5

040112:5

Coherent illumination: $d = 5\lambda$

Incoherent illumination: $d = 3.05\lambda$

940824:4

N/A

930825:4

$$A = \frac{B\lambda}{D}$$