Department of Mathematics

SF1625
One variable Calculus
2014-2015

Problem set for Seminar 2

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do during the seminars. At this seminar there will be a small test in which you are asked to solve (a variant of) one of the recommended exercises from the text book Calculus by Adams och Essex (8:th ed), namely:
Chapter 1.2: 9, 13, 21, 25, 30, 49, 50, 78, 79. Chapter 1.3: 3, 6, 11, 13, 53. Chapter 1.4: 7, 8, 12, 15, 17, 20, 21, 29. Chapter 1.5: 13, 29. Chapter 2.1: 5, 7. Chapter 2.2: 1, 3, 11, 26, 27, 40, 41, 42, 43, 44, 45, 47. Chapter 2.3: 1, 7, 11, 17, 25, 33, 35, 47. Chapter 2.4: 3, 5, 11, 18, 23, 30, 31, 37. Chapter 2.5: 13, 15, 23, 29, 31, 35, 45, 62.
At the seminar these problems will be discussed.

PROBLEMS

Uppgift 1. Compute the limits:

A. $\lim_{x\to\infty} \dfrac{\sin x}{x}$

B. $\lim_{x\to\infty} \dfrac{x^2+3x+5}{5x^2+2x+3}$

C. $\lim_{x\to\infty} \left(\sqrt{x+\sqrt{x}}-\sqrt{x}\right)$

Uppgift 2. Find the constant k such that the function

$$g(x) = \begin{cases} \dfrac{\sin kx}{x}, & x \neq 0 \\ 4, & x = 0 \end{cases}$$

is continuous at the origin. Is the function then continuous at all x?

Uppgift 3. Differentiate the following functions with respect to x and state for what x these functions are differentiable. Does any of these functions fail to be differentiable in all of the domain of definition?

A. $f(x) = 1 + \tan^2 x$

B. $g(x) = \dfrac{ax + b}{cx + d}$

C. $h(x) = \sin \sqrt{x}$

D. $k(x) = |x| \cos x$

DISCUSION PROBLEMS

Here are some extra problems to discuss at the seminar.

- Show using the intermediate value property that $p(x) = x^5 + x^3 + 1$ has a zero somewhere between -1 and 0. Is the zero closer to -1 or 0? How can you be sure that there are not any other zeros?
- At a car park you are charged as follows: the first hour costs 4 kronor and after that the charge is 2 kronor for each started hour up to the maximal charge 10 kronor. Let $h(t)$ be the cost as a function of time t hours. Sketch the graph $y = h(t)$ for $0 \leq t \leq 24$. Is h continuous?
- Give an example of function with domain of definition $\mathbb{R}$ that is neither continuous nor differentiable at $x = 1$.
- Give an example of function with domain of definition $\mathbb{R}$ that is continuous but not differentiable at $x = 1$.
- Does there exist a function that is differentiable but not continuous at $x = 1$.