Department of Mathematics

SF1625
One variable Calculus
2014-2015

Problem set for Seminar 3

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do during the seminars. At this seminar you are to hand in a solution to a problem. Solve the problems 1-4 below and write down the solutions, one solution per sheet of paper. Write your name and peronal number. When the seminar begins you will be told which problem to hand in. Before you start working on the problems below you should solve the recommended exercises from the text book:
Kapitel 2.6: 3, 9. Kapitel 2.7: 1, 3, 11, 13, 23, 29. Kapitel 2.8: 5, 13, 21, 27. Kapitel 2.9: 3, 9, 13. Kapitel 2.9: 3, 9, 13. Kapitel 2.11: 5, 7, 13, 16, 17, 18, 19. Kapitel 3.1: 3, 9, 23. Kapitel 3.2: 3, 5, 9, 15, 25, 29. Kapitel 3.3: 3, 5, 7, 9, 19, 21, 31, 33, 43, 51, 59. Kapitel 3.4: 1, 3, 5, 9, 11, 17, 23, 25. Kapitel 3.5: 1, 3, 5, 7, 13, 19, 21, 23, 35. Kapitel 3.7: 1, 3, 5, 7, 9, 13, 15, 21, 25, 29.

Uppgifter

Uppgift 1. Let $h(x) = (x^2 - 1)e^{2x-4}$

A. Find an equation for the tangent line to the curve $y = f(x)$ in the point on the curve with x-coordinate 2.

B. Use linear approximation at $x = 2$ (i.e. the tangent line from A) to estimate the function value $h(2.1)$.

Uppgift 2. Let $f(x) = 4 \arcsin \sqrt{x} + 2 \arcsin \sqrt{1-x}$. Answer the following questions:

A. What is the domain of definition of f?

B. Is f strictly increasing?

C. Explain why a strictly increasing function is injective and therefore invertible. Is f invertible?

D. What is the maximum value of f^{-1}?

Uppgift 3. According to Newton's law of cooling, a warm object cools off in a rate proportional to the difference in temperature between the object and the surrounding room. If the temperature of the object is T and the temperature of the room is T_R the law of cooling can be formulated in the form of a differential equation:

$$\frac{dT}{dt} = -\lambda(T - T_R),$$

where λ is some positive constant.

A. Explain why this differential equation is a mathematical formulation of the law of cooling.

B. If T_0 is the temperature that the object had in the beginning, show that

$$T(t) = T_R + (T_0 - T_R)e^{-\lambda t}$$

solves the differential equation with the initial condition $T(0) = T_0$.

C. On a certain occasion an object is put in a room with room temperature $20°C$. After 10 minutes the temperature of the object is $50°$ and after another 10 minutes $40°$. Find

(i) the initial temperature of the object

(ii) the time it takes for the temperature of the object to drop from 40 till $30°C$.

Uppgift 4. A curve in the plane is (implicitly) defined by the equation

$$\arctan(xy) = \frac{\pi}{4}e^{x-y}.$$

Find the equation for the tangent line to the curve at the point $(1, 1)$.

Discussion problems

Here are some extra problems to discuss at the seminar. You do not have to write down solutions to these problems in advance.

- Find $\arcsin(-1/2)$, $\arccos(-1/2)$, $\arctan\sqrt{3}$ and $\ln(1/\sqrt{e})$
- Find $\arcsin(\sin(3\pi/4))$ and $\cos(\arcsin(1/5))$
- Find $\cos(\arctan x)$, $\sin(\arctan x)$ and $\cos\left(\arccos\frac{4}{5} + \arcsin\frac{5}{13}\right)$
- Is there an x such that $\arctan(\tan x) \neq x$? Find such an x if it exists and explain otherwise why it can't exist.
- Is there an x such that $\tan(\arctan x) \neq x$? Find such an x if it exists and explain otherwise why it can't exist.