Department of Mathematics

SF1625
One variable Calculus
2014-2015

Problem set for Seminar 4

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do during the seminars. At this seminar there will be a small test in which you are asked to solve (a variant of) one of the recommended exercises from the text book Calculus by Adams och Essex (8:th ed), namely:
Chapter 4.1: 5, 7, 9, 16, 17. Chapter 4.2: 7, 9. Chapter 4.3: 1, 5, 17. Chapter 4.4: 3, 14, 29, 35. Chapter 4.5: 5, 11, 27, 31. Chapter 4.6: 3, 5, 9, 17, 31. Chapter 4.8: 1, 7, 13, 21. Chapter 4.9: 1, 3, 13, 30. Chapter 4.10: 1, 5, 9
At the seminar these problems will be discussed:

PROBLEMS

Uppgift 1. Find all local extreme values and all asymptotes, sketch the graph and find the range of the function $f(x) = xe^{-x^2/2}$.

Uppgift 2. Let $g(t) = \sqrt{4+t}$. Find the Maclaurin polynomial (Taylor polynomial at the origin) of degree 2 to g and use it to calculate an approximation of $\sqrt{4.4}$. What can you say about the error.

Uppgift 3. You are to construct a cylindrical can with bottom plate and lid. The total area of the material that the can is made of is A. How should you choose the height h and radius R in order to maximize the volume of the can?

Uppgift 4. A steel wire of length 1 meter is split in two parts. One of them is to form a circle and the other one a square. Find the length of that part of the wire which is used to form a square if the sum of the areas of the circle and the square is to be a) maximal b) minimal.

DISCUSSION PROBLEMS

Here are some extra problems to discuss at the seminar. You do not have to write down solutions in advance.

- An aeroplane is flying straight with constant speed 600 km/h and constant altitude 5 km. At a certain occasion the plane passes over a building. How fast does the distance between the plane and the building increase 1 minute later?
- Does there exist a function with domain of definition $\mathbf{R}$ that has an extremal value at the origin without having zero derivative there? Give an example of such a function or show that such a function cannot exist.
- Does there exist a function with domain of definition $\mathbf{R}$ that does not have an extremal value at the origin in spite of the fact that its derivative is zero there? Give an example of such a function or show that such a function cannot exist.
- Does there exist a function with domain of definition $\mathbf{R}$ that has is strictly increasing without its derivative being positive everywhere? Give an example of such a function or show that such a function cannot exist.
- Does there exist a function with domain of definition $\mathbf{R}$ that has is not strictly increasing in spite of the fact that its derivative is positive everywhere? Give an example of such a function or show that such a function cannot exist.
- Find constants a, b and c such that
$$|ae^{bx+cx^2} - 2x^2 - 4| \leq 10^{-4} \quad \text{då } |x| \leq 0.1.$$
- Show that $x((\ln x)^3 - 3(\ln x)^2 + 6\ln x) \geq 6(x-1)$ for all $x > 0$.