

# Constants and formulas

## Space Physics EF2240

### PHYSICAL CONSTANTS

$c$	speed of light	$3 \times 10^8 \text{ ms}^{-1}$
$e$	elementary charge	$1.6 \times 10^{-19} \text{ C}$
$m_e$	electron mass	$0.91 \times 10^{-30} \text{ kg}$
$m_p$	proton mass	$1.67 \times 10^{-27} \text{ kg}$
$h$	Planck's constant	$6.625 \times 10^{-34} \text{ Js}$
$k_B$	Boltzmann's constant	$1.38 \times 10^{-23} \text{ JK}^{-1}$
$\mu_0$	permeability of free space	$4\pi \times 10^{-7} \text{ Hm}^{-1}$
$\epsilon_0$	permittivity of free space	$8.854 \times 10^{-12} \text{ Fm}^{-1}$
$R_E$	Earth's radius	6378 km
$a$	Earth's magnetic dipole moment	$8 \times 10^{22} \text{ Am}^2$
$\sigma_{SB}$	Stefan-Boltzmann's constant	$5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

$p_d$	dynamic pressure	$\rho v^2$
$p$	thermal pressure	$nk_B T$
$v_D$	drift velocity	$\frac{\mathbf{F} \times \mathbf{B}}{qB^2}$
$R_m$	magnetic Reynolds number	$\mu_0 \sigma v_c l_c$
$n$	index of refraction for plasma	$\sqrt{1 - \frac{f_{pe}^2}{f^2}}$
$\alpha_{fl}$	loss cone angle	$\arcsin \sqrt{B/B_{\max}}$

### CONVERSIONS

1 eV	electron volt	$1.6 \times 10^{-19} \text{ J}$
1 AU	astronomical unit	$1.496 \times 10^{11} \text{ m}$
1 $\gamma$	Gamma	$10^{-9} \text{ T}$
1 G	Gauss	$10^{-4} \text{ T}$
1 LY	light year	$9.461 \times 10^{15} \text{ m}$

### PLASMA ELECTRODYNAMICS

#### PLASMA PHYSICS

$\omega_p$	plasma frequency	$\sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}$
$\omega_c$	cyclotron-/gyro frequency	$\frac{eB}{m_e}$
$r_L$	gyro/Larmor radius	$\frac{mv_{\perp}}{eB}$
$\mu$	magnetic moment	$\frac{mv_{\perp}^2}{2B}$
$\lambda_D$	Debye length	$\sqrt{\frac{\epsilon_0 k_B T_e}{n_e e^2}}$
$v_A$	Alfvén velocity	$\frac{B}{\sqrt{\mu_0 \rho}}$
$p_B$	magnetic pressure/energy density	$\frac{B^2}{2\mu_0}$

$j_{\parallel}$	parallel current density	$\sigma_{\parallel} E_{\parallel}$
$j_P$	Pedersen current density	$\sigma_P E_{\perp}$
$j_H$	Hall current density	$\sigma_H E_{\perp}$
$\sigma_{\parallel}$	parallel conductivity	$\sigma_{\parallel} = \sigma_e + \sigma_i$
$\sigma_P$	Pedersen conductivity	$\sigma_e \frac{1}{1 + \omega_{ge}^2 \tau_e^2} + \sigma_i \frac{1}{1 + \omega_{gi}^2 \tau_i^2}$
$\sigma_H$	Hall conductivity	$\sigma_e \frac{\omega_{ge} \tau_e}{1 + \omega_{ge}^2 \tau_e^2} - \sigma_i \frac{\omega_{gi} \tau_i}{1 + \omega_{gi}^2 \tau_i^2}$
$\sigma_e$	electron conductivity	$e^2 n \tau_e / m_e$
$\sigma_i$	ion conductivity	$e^2 n \tau_i / m_i$
Ampère's law for infinite current sheet		
		$j_z = -\frac{1}{\mu_0} \frac{\partial B_x}{\partial y}$

## RELATIVISTIC FIELD TRANSFORMATIONS

Electric field ( $\perp \mathbf{u}$ ) 
$$\mathbf{E}' = \frac{\mathbf{E} + \mathbf{u} \times \mathbf{B}}{\sqrt{1 - u^2/c^2}}$$

Magnetic field ( $\perp \mathbf{u}$ ) 
$$\mathbf{B}' = \frac{\mathbf{B} - (\mathbf{u}/c^2) \times \mathbf{E}}{\sqrt{1 - u^2/c^2}}$$

The components parallel to  $\mathbf{u}$  remain unchanged.

## RELATIVISTIC MECHANICS

Gamma factor 
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic momentum

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m\mathbf{v}$$

Relativistic energy 
$$E = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mc^2$$

Rest energy 
$$E = mc^2$$

Relation between energy and momentum

$$E^2 = p^2 c^2 + m^2 c^4$$

## OTHER

Dipole magnetic field

$$B_r = B_p \left(\frac{RE}{r}\right)^3 \cos \theta$$

$$B_\theta = \frac{B_p}{2} \left(\frac{RE}{r}\right)^3 \sin \theta$$

where  $B_p$  is the magnetic field strength at the poles

Dipole magnetic field, alternative formulation

$$B_r = \frac{\mu_0 a}{2\pi} \frac{1}{r^3} \cos \theta$$

$$B_\theta = \frac{\mu_0 a}{2\pi} \frac{1}{2} \frac{1}{r^3} \sin \theta$$

where  $a$  is the magnetic dipole moment

Parker spiral 
$$\frac{B_\theta}{B_r} = \tan \psi = \frac{\omega_{sun} r}{u_{sw}}$$

Energy for light quantum 
$$E = hf$$

Wien's displacement law 
$$\lambda = \frac{2.90 \times 10^{-3} \text{ m} \cdot \text{K}}{T}$$

Stefan-Boltzmann's law 
$$J = \sigma_{SB} T^4$$

Dispersion relation for electromagnetic waves in vacuum 
$$c = f\lambda$$