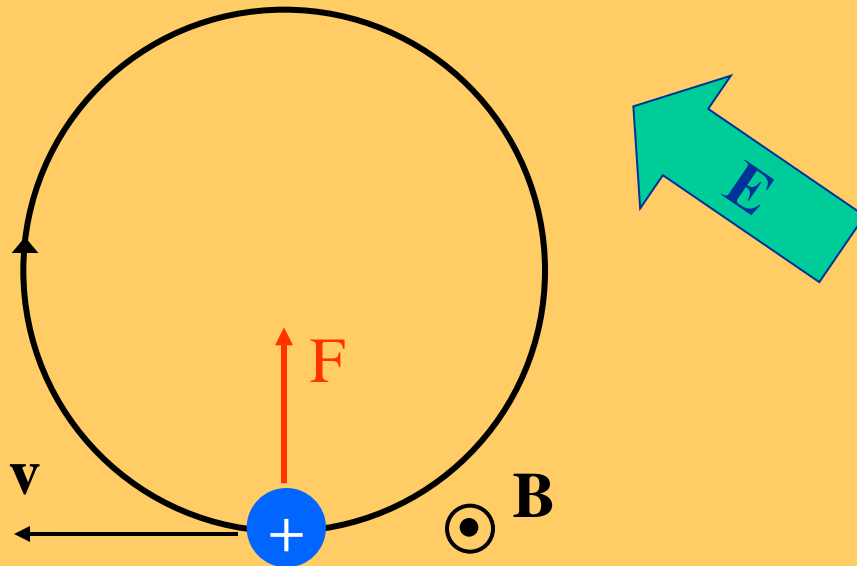


# Think about this:

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B}$$

What happens if you add an electric field  $\mathbf{E}$ ?





# Last lecture (3)

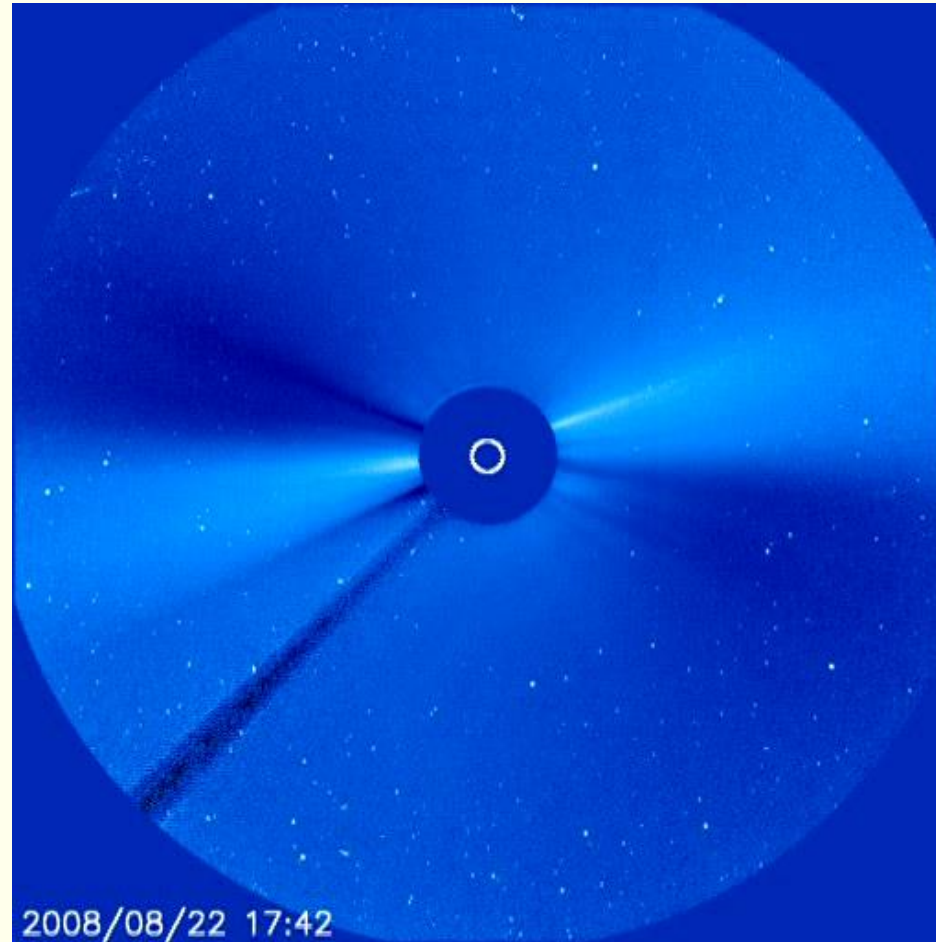
- Solar activity

# Today's lecture (4)

- Solar wind – basic facts
- Solar wind – magnetic structure
- Ionosphere
  - layers
  - radio wave reflection

# Solar wind

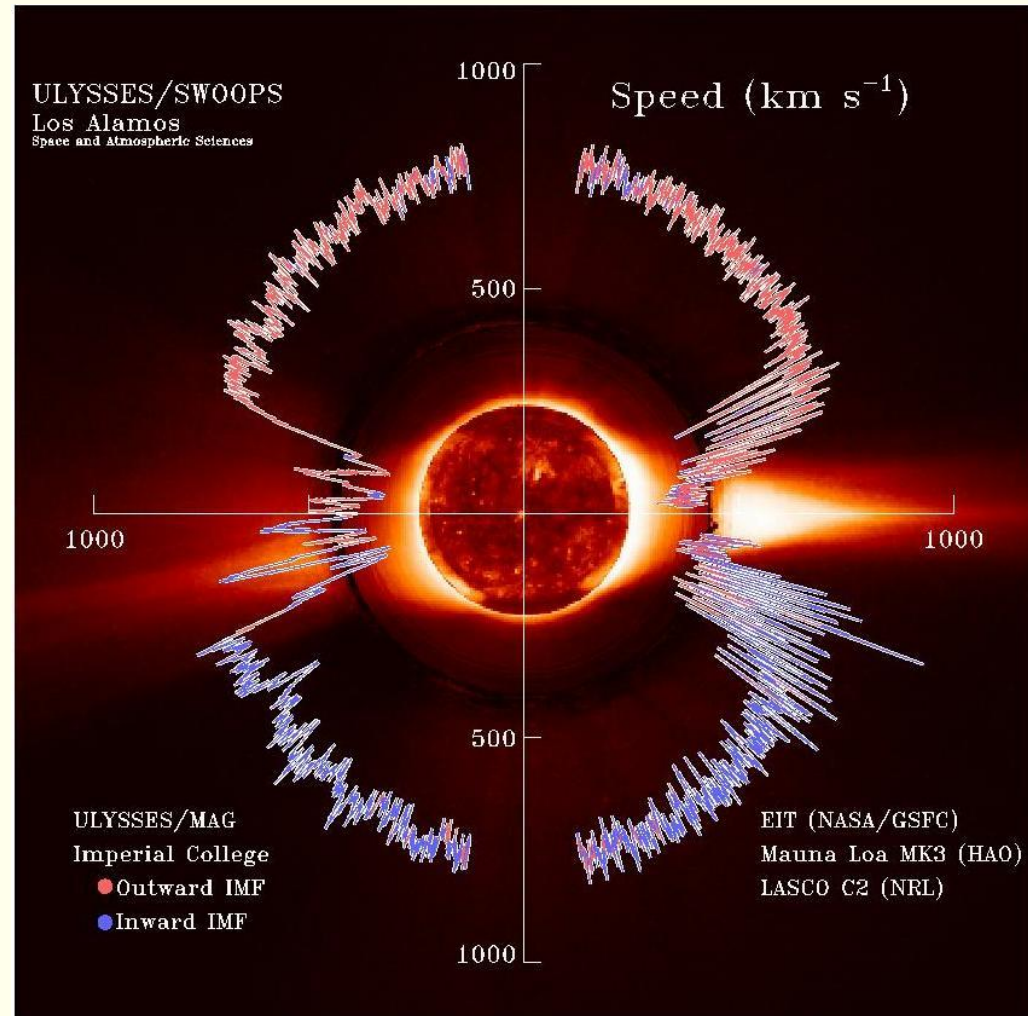
***Corona  
continuously  
merges into  
solar wind***



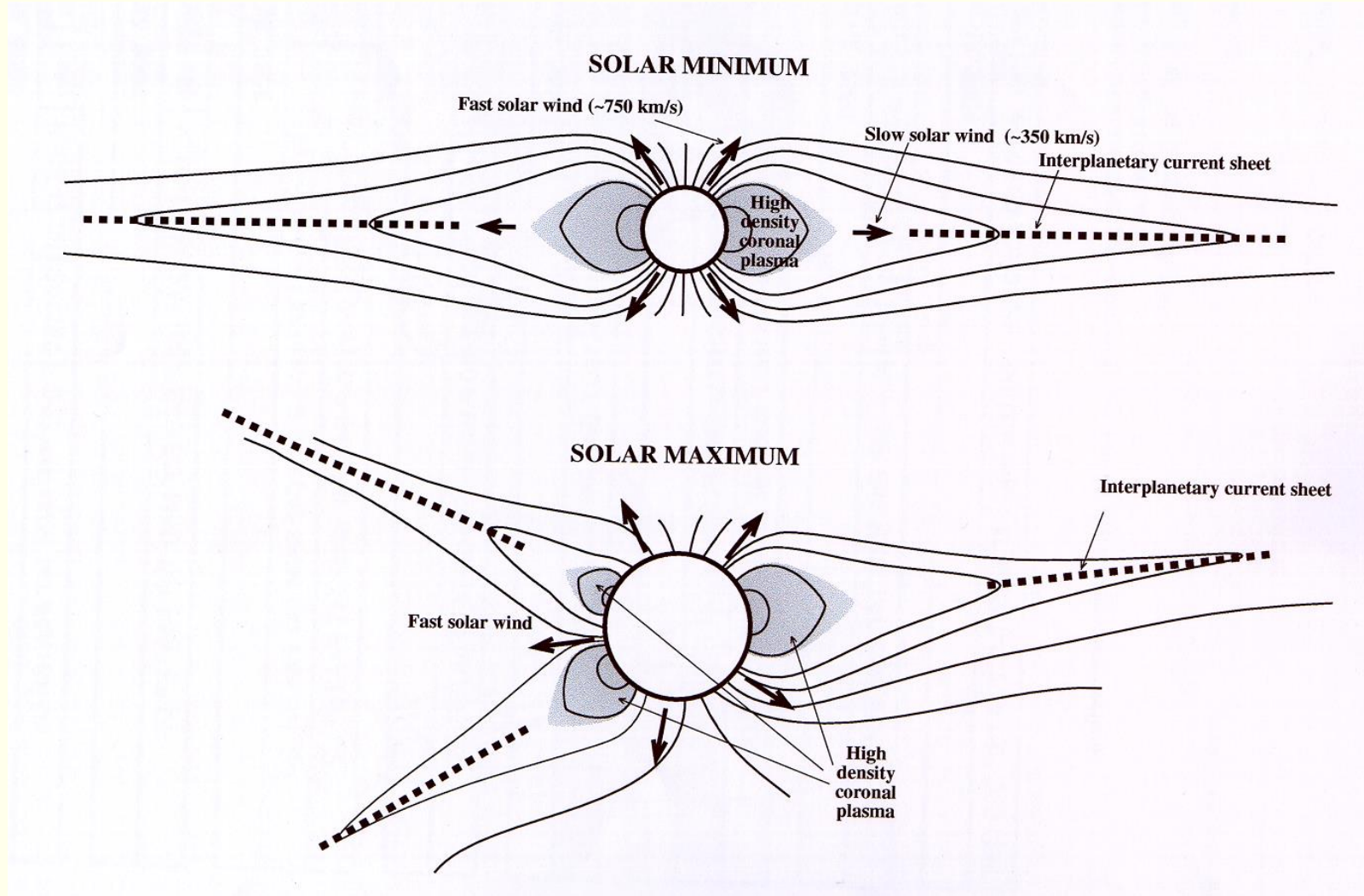
**Solar and Heliospheric Observatory (SOHO)**  
***LASCO C3 Coronagraph Movie***

# Solar wind

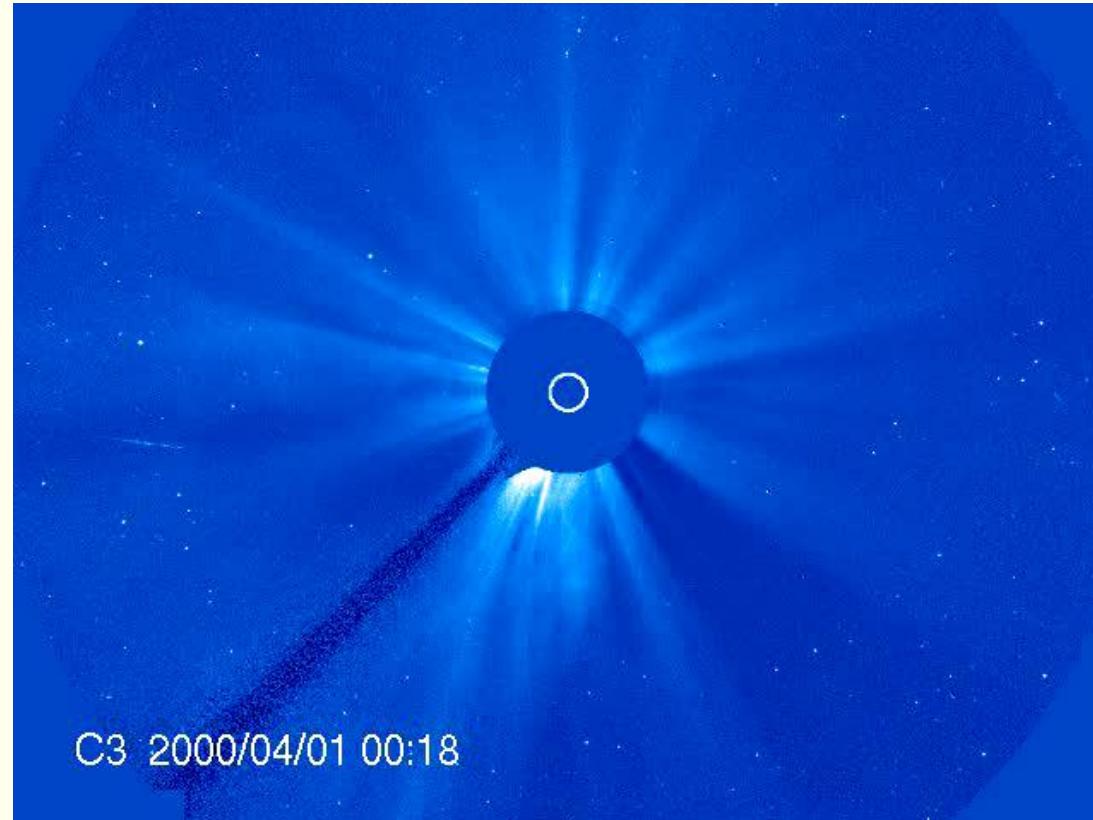
- Fast solar wind in regions closer to poles
- Slow solar wind closer to equatorial plane



# Solar wind

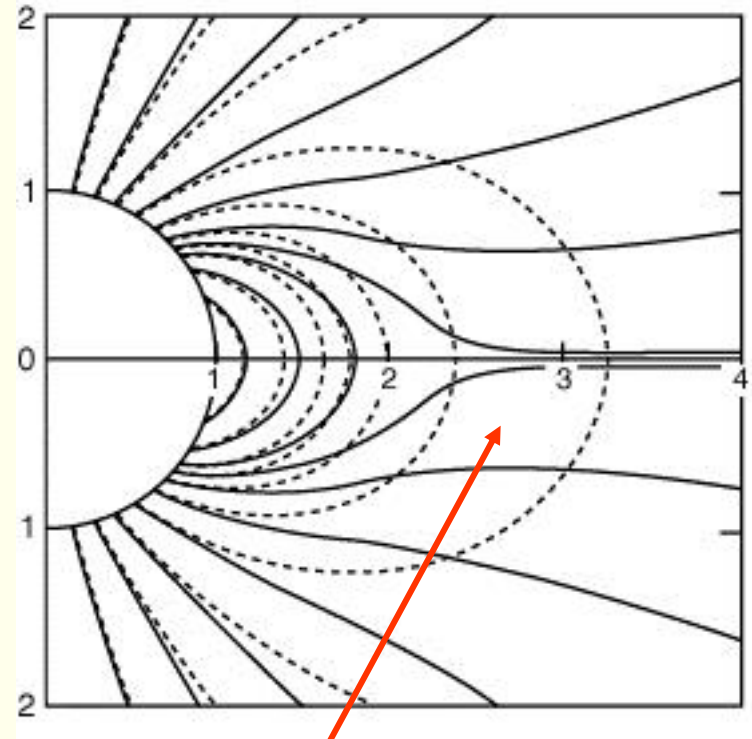
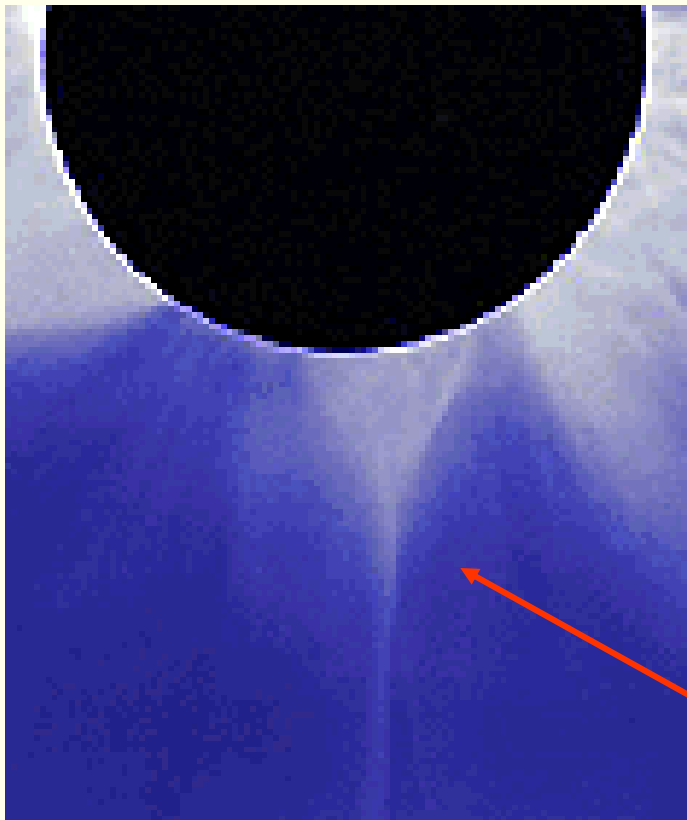


# More active solar wind



**Solar and Heliospheric Observatory (SOHO)**  
*LASCO C3 Coronagraph Movie*

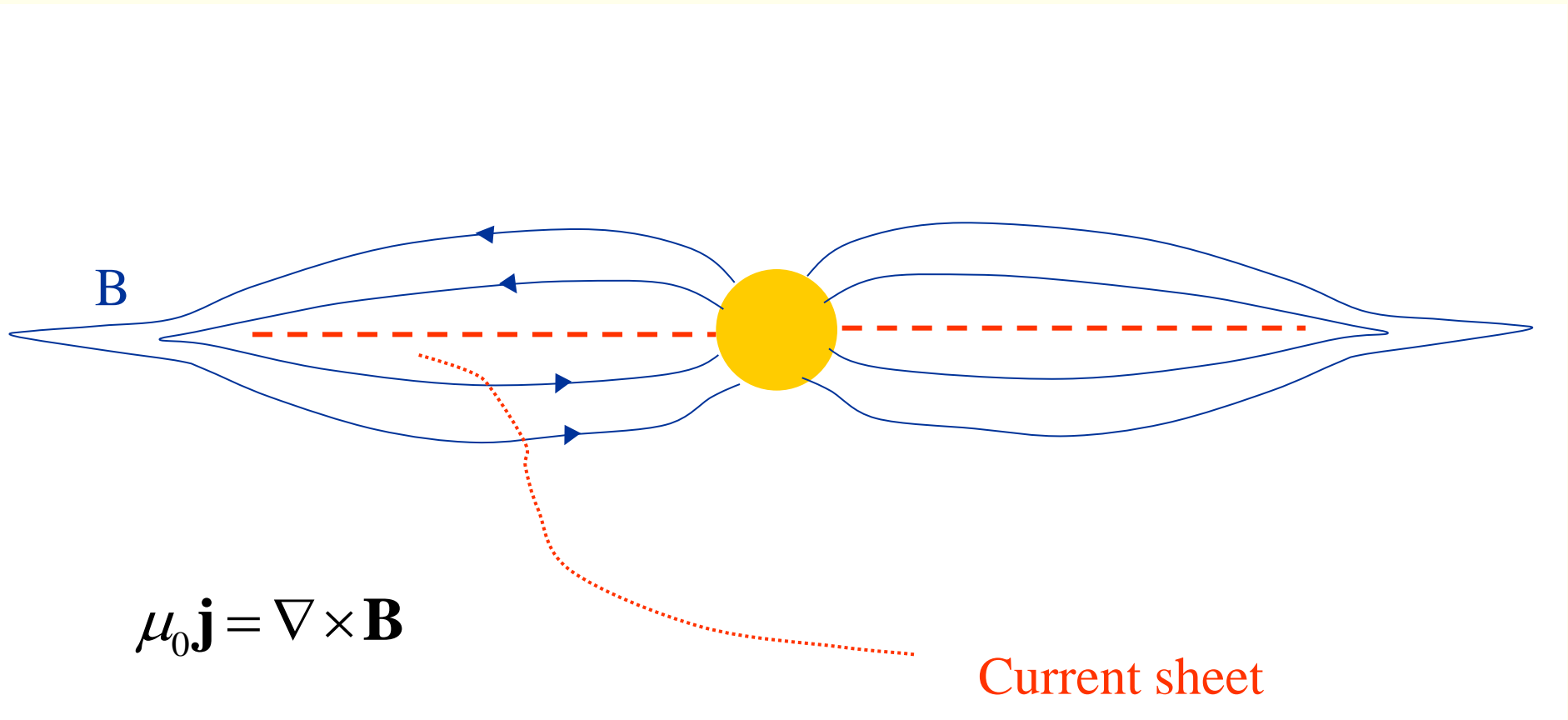
# Helmet streamers



Magnetic field drawn out by solar wind.  
This also brakes the solar wind.

# Solar wind

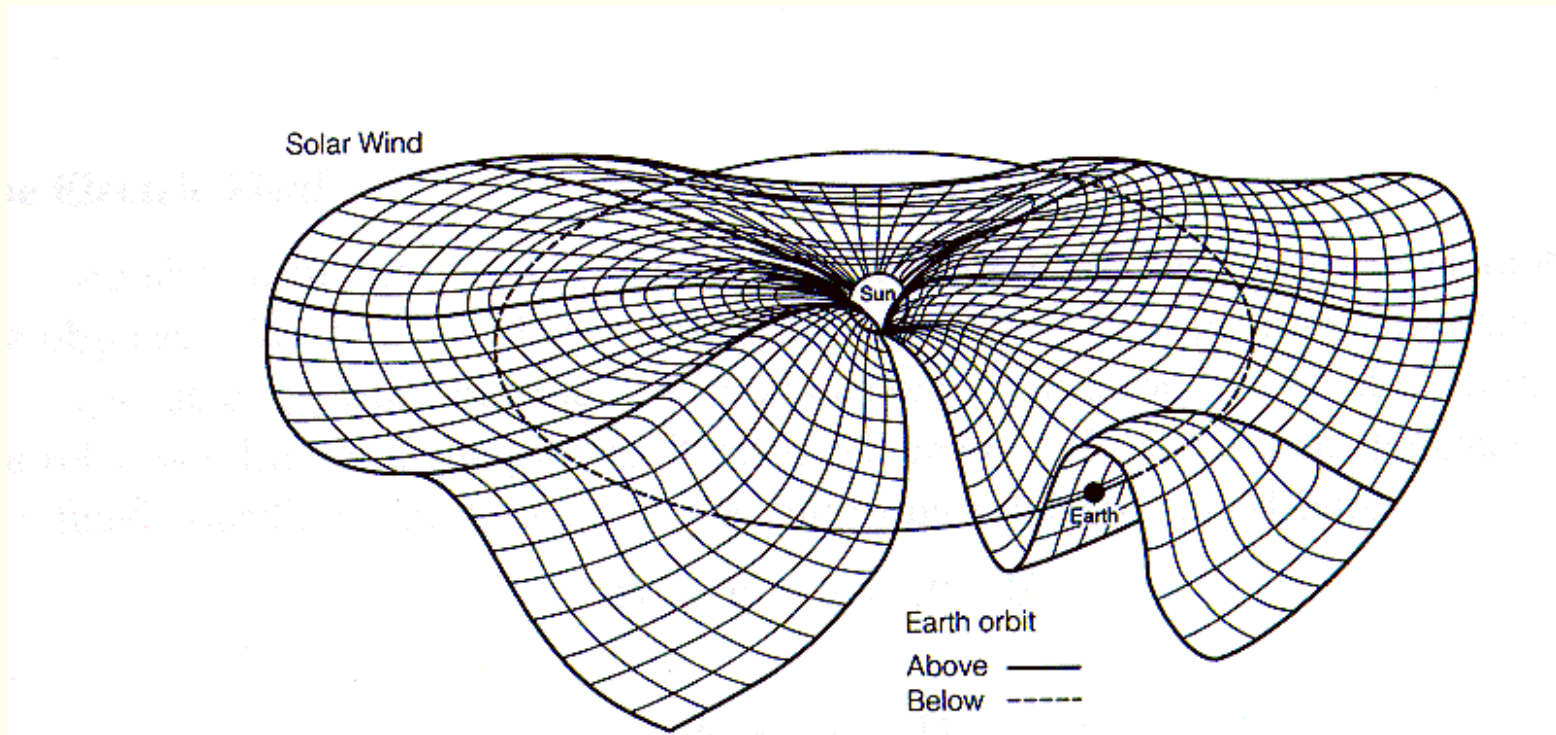
Interplanetary current sheet





# Solar wind

## *Interplanetary current sheet*



Later we will see that the N-S component of the interplanetary magnetic field (IMF is important for the coupling between solar wind and magnetosphere)

# Solar wind

## Some basic facts

### Average values

$$n_p = 8 \text{ cm}^{-3}$$

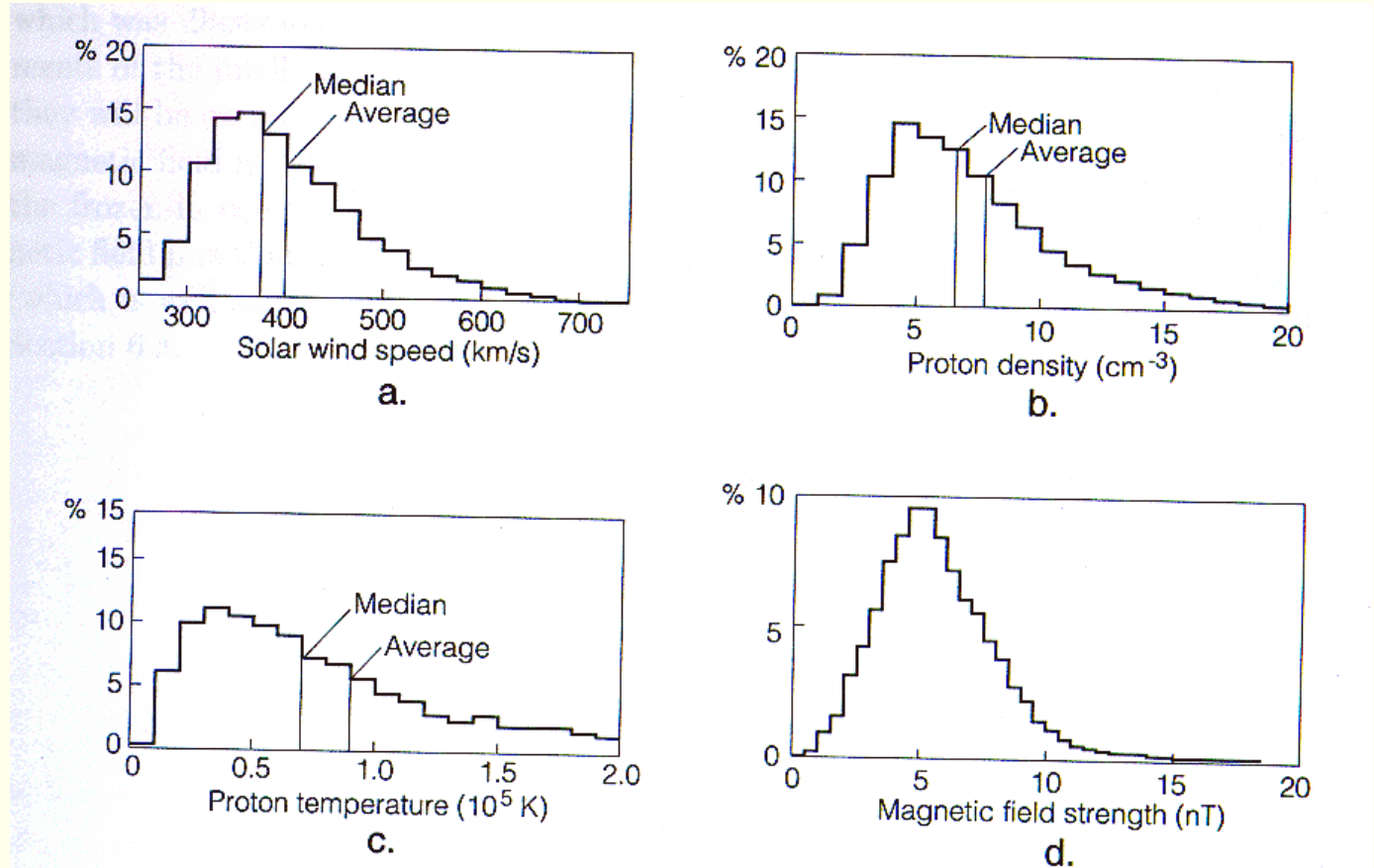
$$v = 320 \text{ km/s}$$

$$T_p = 4 \cdot 10^4 \text{ K}$$

$$T_e = 10^5 \text{ K}$$

$$B = 5 \text{ nT}$$

$$\Phi_K = \rho v^3 / 2 = 0.22 \text{ mW/m}^2$$



# The solar wind today

## Average values

$$n_p = 8 \text{ cm}^{-3}$$

$$v = 320 \text{ km/s}$$

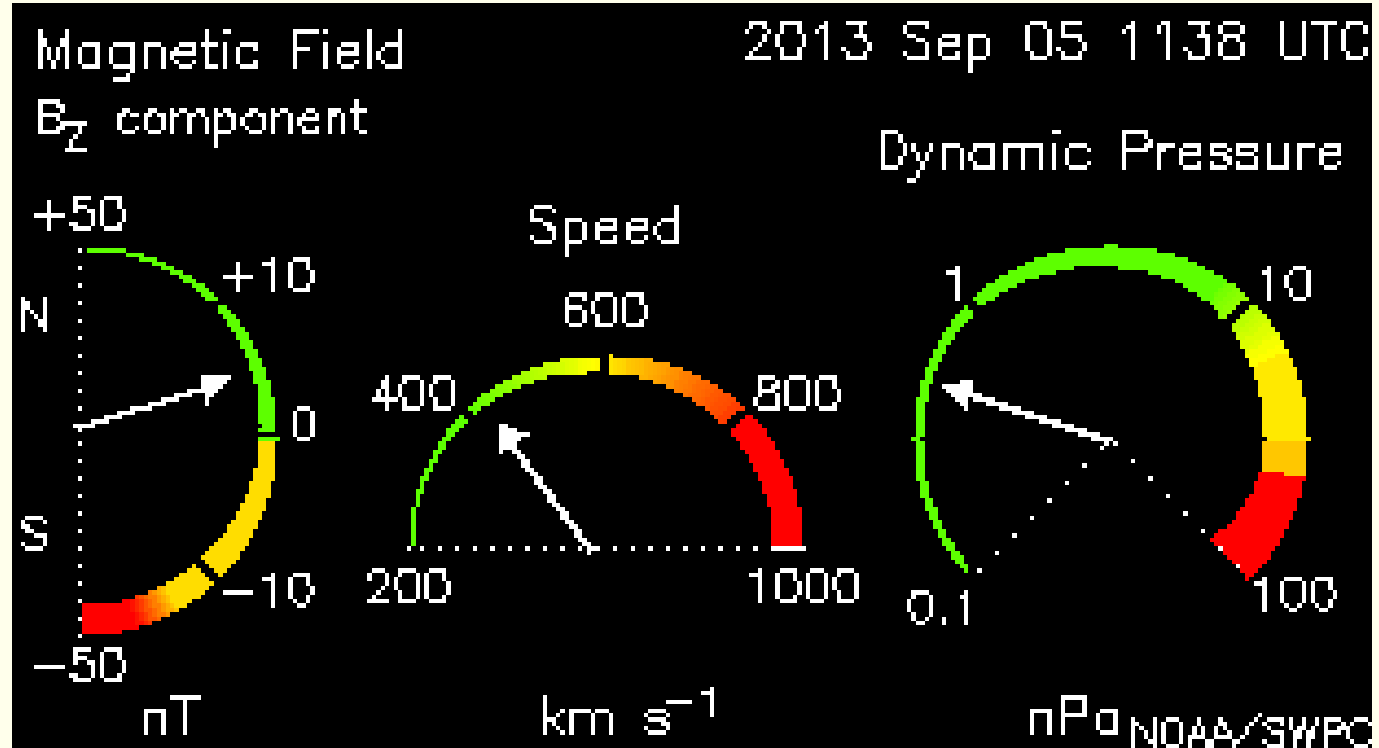
$$T_p = 4 \cdot 10^4 \text{ K}$$

$$T_e = 10^5 \text{ K}$$

$$B = 5 \text{ nT}$$

$$p_D = \rho v^2 / 2 = 0.7 \text{ nPa}$$

$$\Phi_K = \rho v^3 / 2 = 0.22 \text{ mW/m}^2$$



Measurements from ACE spacecraft

<http://www.swpc.noaa.gov/SWN/>

Space Weather Prediction Centre



Guess how long does it take the solar wind to flow from the Sun to the Earth?

Blue

8 min

Yellow

1.5 days

Green

5 hours

Red

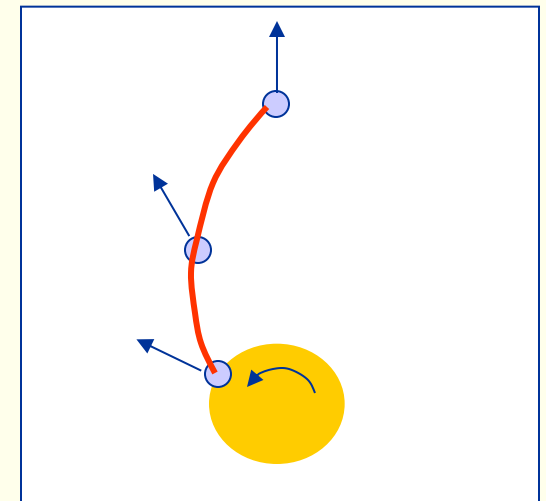
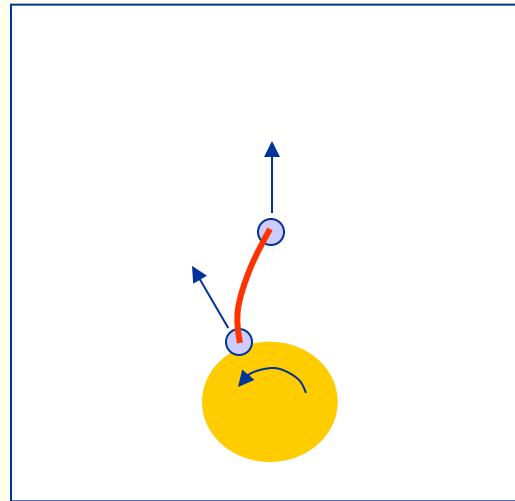
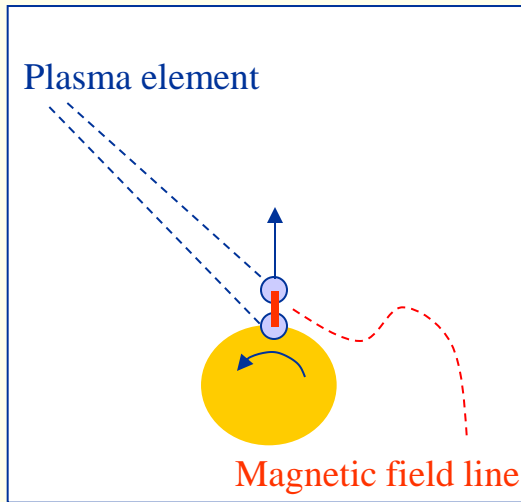
5 days

Does anyone happen to know the mathematical formula for the spiral caused by a rotating garden sprinkler?



# Solar wind

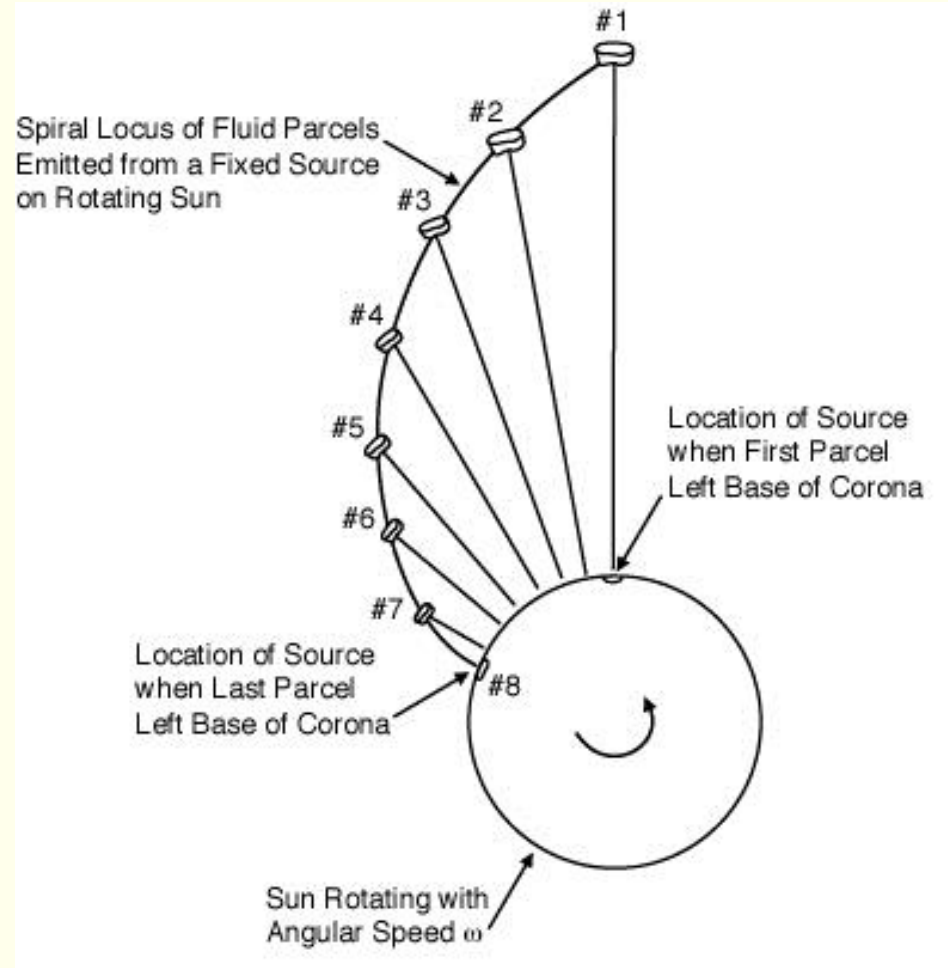
*Magnetic field frozen into solar wind*



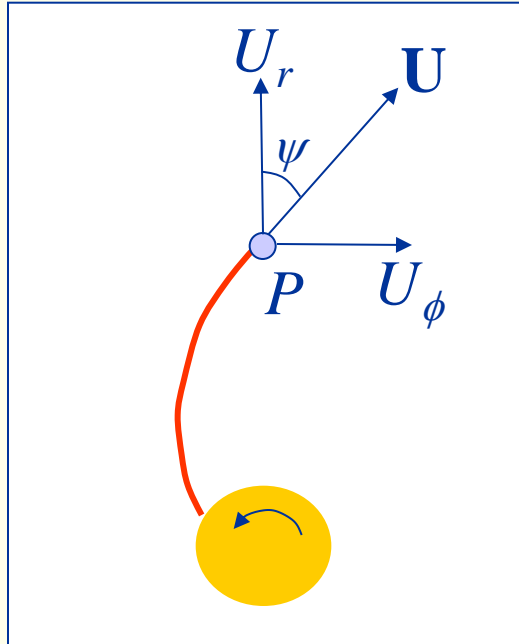
This is now seen from "above"! (Looking down on the ecliptic plane from the pole.)

# Solar wind

## *Parker spiral*



# Parker spiral

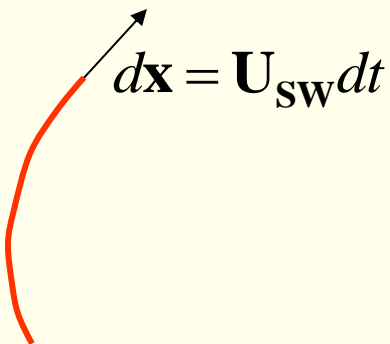


## *Derivation of $\Psi$ (Parker angle)*

Consider a coordinate system rotating with the sun. The plasma element  $P$  in this coordinate system has two velocity components:  $U_r$  and  $U_\phi$ .

Since the magnetic field is frozen into the solar wind, and follows the orbit of the plasma element  $P$ , at any time  $B$  has to be parallel to  $U$ . Then we have:

$$\tan \psi = \frac{B_\phi}{B_r} = \frac{U_\phi}{U_r} = \left( \frac{\omega r}{u_{SW}} \right)$$



$$d\mathbf{x} = \mathbf{U}_{sw} dt$$

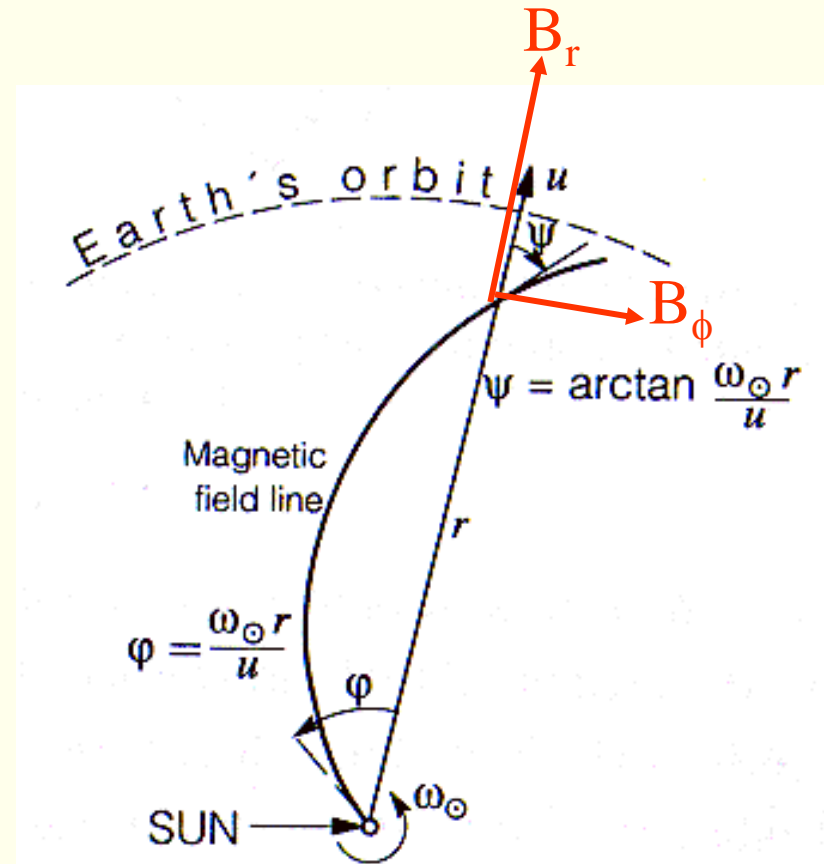


# Solar wind

## Parker spiral

Archimedean spiral:

$$\frac{B_{\phi}}{B_r} = \tan \psi = \left( \frac{\omega r}{u_{SW}} \right)$$



# Archimedean spiral

An Archimedean spiral (also arithmetic spiral), is a spiral named after the 3rd-century-BC Greek mathematician Archimedes; it is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed along a line which rotates with constant angular velocity.

Equivalently, in polar coordinates  $(r, \phi)$  it can be described by the equation (*Wikipedia*)

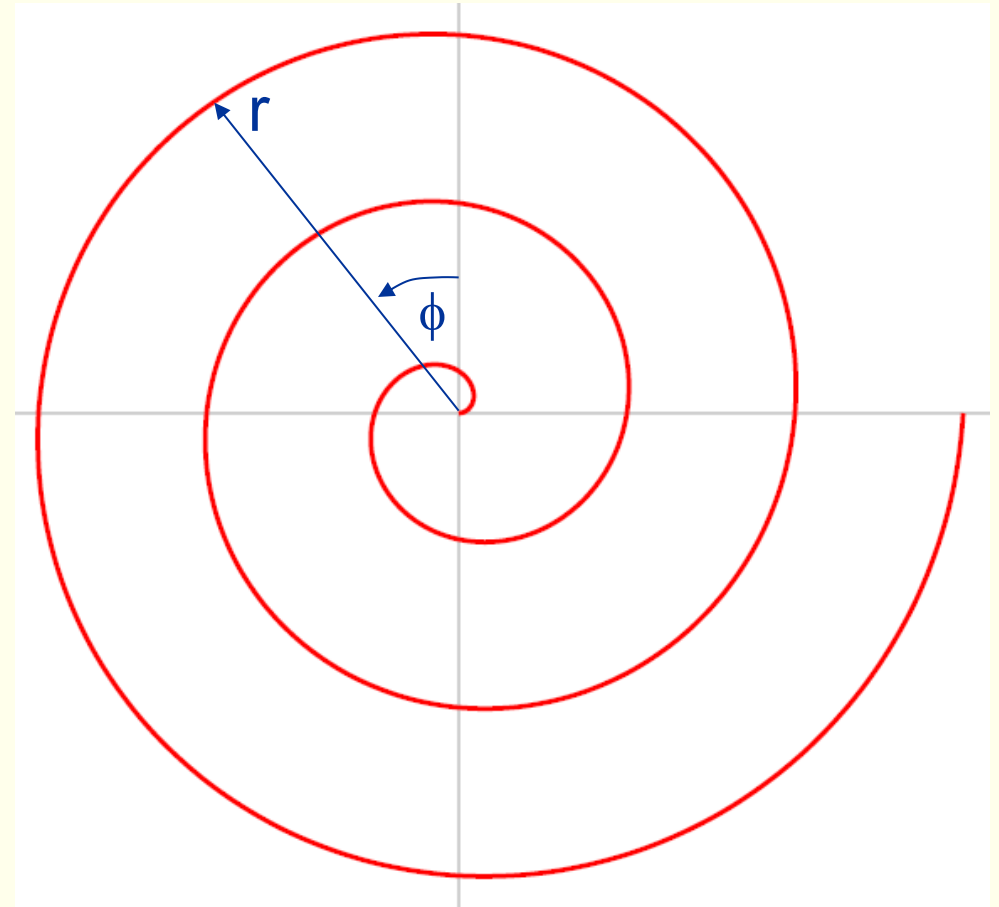
$$r = a + b\phi$$

$$r = a + b\omega t$$

$$\frac{dr}{dt} = b\omega = u_{SW}$$

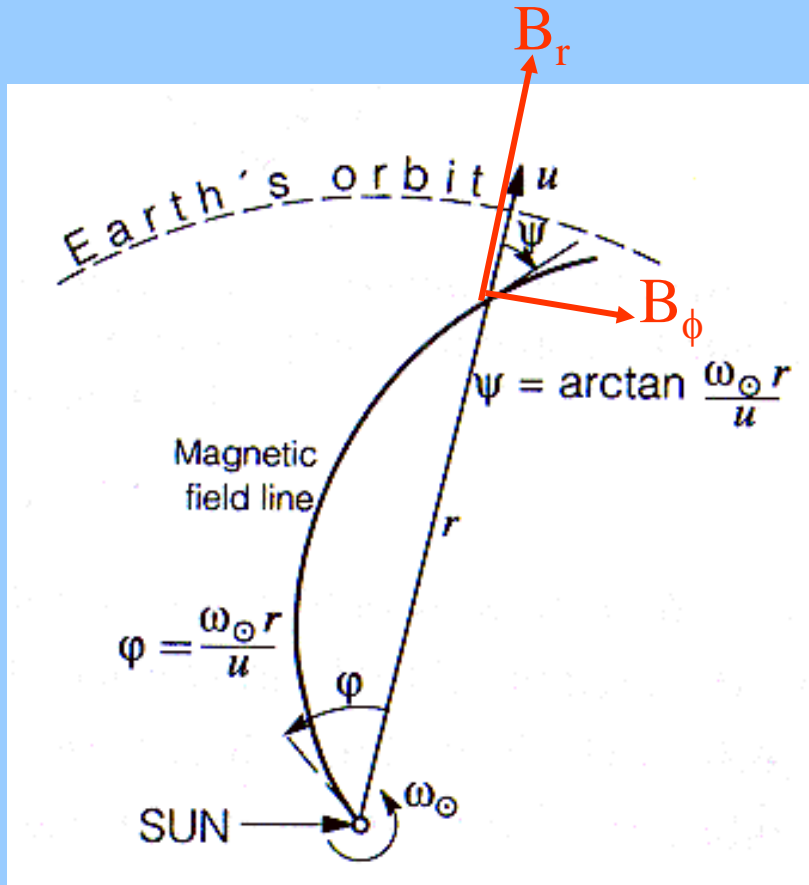
$$b = \frac{u_{SW}}{\omega}$$

$$r = R_{sun} + \frac{u_{SW}}{\omega} \phi$$



Use rotation period  
 $T$  of sun:  $T = 27$  days

What is the angle  $\Psi$   
 at Earth's orbit for a  
 typical solar wind  
 speed?



$r = 1 \text{ A.U.}$

Yellow  $\approx 50^\circ$

Red  $\approx 80^\circ$

Blue  $\approx 1^\circ$

Green  $\approx 10^\circ$



# Classification of plasmas

- **High density plasmas**

- $\lambda \ll \rho$
- *magnetic field not important, collisions dominate, isotropic.*

- **Medium density plasmas**

- $\rho \ll \lambda \ll l_c$
- *magnetic field important, collisions important, anisotropies.*

- **Low density plasmas**

- $l_c \ll \lambda$
- *magnetic field important, anisotropies, uninhibited motion along magnetic field*

$\rho$ : gyro radius

$\lambda$ : mean free path

$l_c$ : dimension of the plasma



# Plasma models/descriptions

- Single particle motion
- Computer simulations of many-particle dynamics
- Generalization of statistical mechanics (kinetic theory)
- Generalization of fluid mechanics:  
*Magneto-hydrodynamics (MHD)*

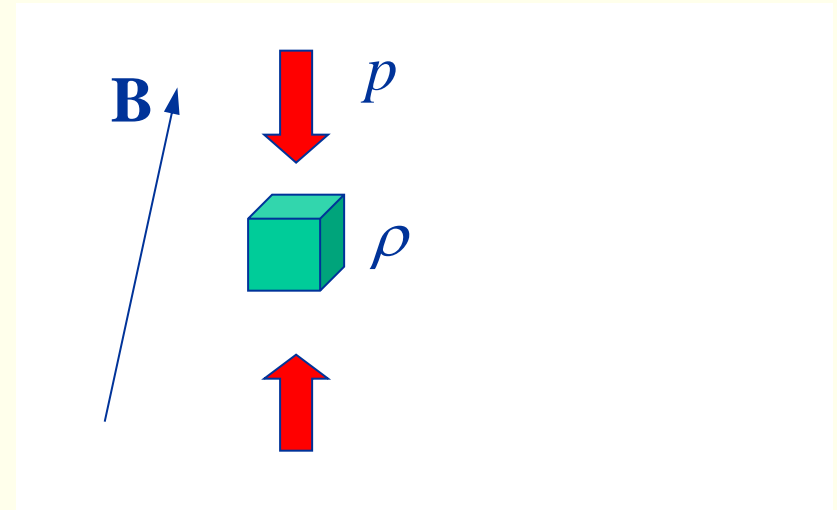
# Plasma physics

## *Magnetohydrodynamics (MHD)*

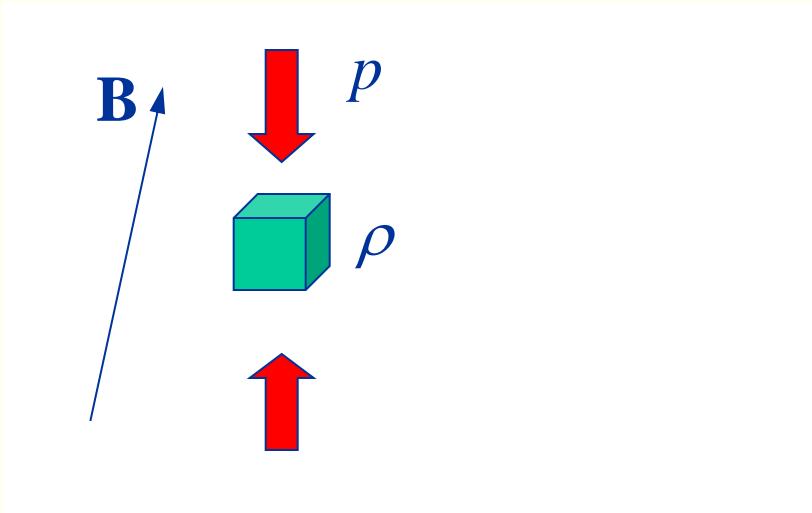
MHD is a combination of

- *fluid-/hydrodynamics* (which is based on Newton's laws of motion)
- *Maxwell's equations* (electrodynamics)

applied on a plasma volume element.



# Magnetohydrodynamics (MHD)



For a volume element of plasma:

$$\mathbf{F} = m\mathbf{a} \quad \Rightarrow$$

$$-\nabla p + n_e q \mathbf{v}_e \times \mathbf{B} + \cancel{\rho q \mathbf{E}} = \rho \frac{d\mathbf{v}}{dt} \quad \Rightarrow$$

quasineutrality

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

# Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

This together with two of Maxwell's equations and Ohm's law make up the most common MHD equations:

$$(2) \quad \mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$(3) \quad \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

Only consider slow variations

$$(4) \quad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$



# Magnetohydrodynamics (MHD)

$$(1) \quad \rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mathbf{j} \times \mathbf{B}$$

In equilibrium:

$$0 = -\nabla p + \mathbf{j} \times \mathbf{B} \quad \longleftrightarrow$$

$$-\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} = 0$$

$$-\nabla p - \nabla \left( \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} = 0$$

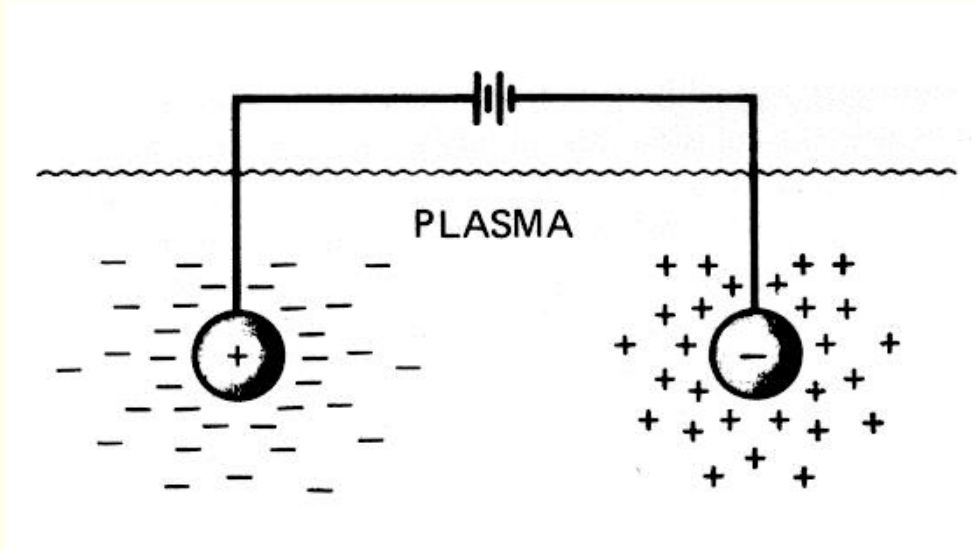
Represents tension in magnetic field

If magnetic tension = 0

$$p + \frac{B^2}{2\mu_0} = \textit{konst}$$

Magnetic pressure

# Quasineutrality



$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

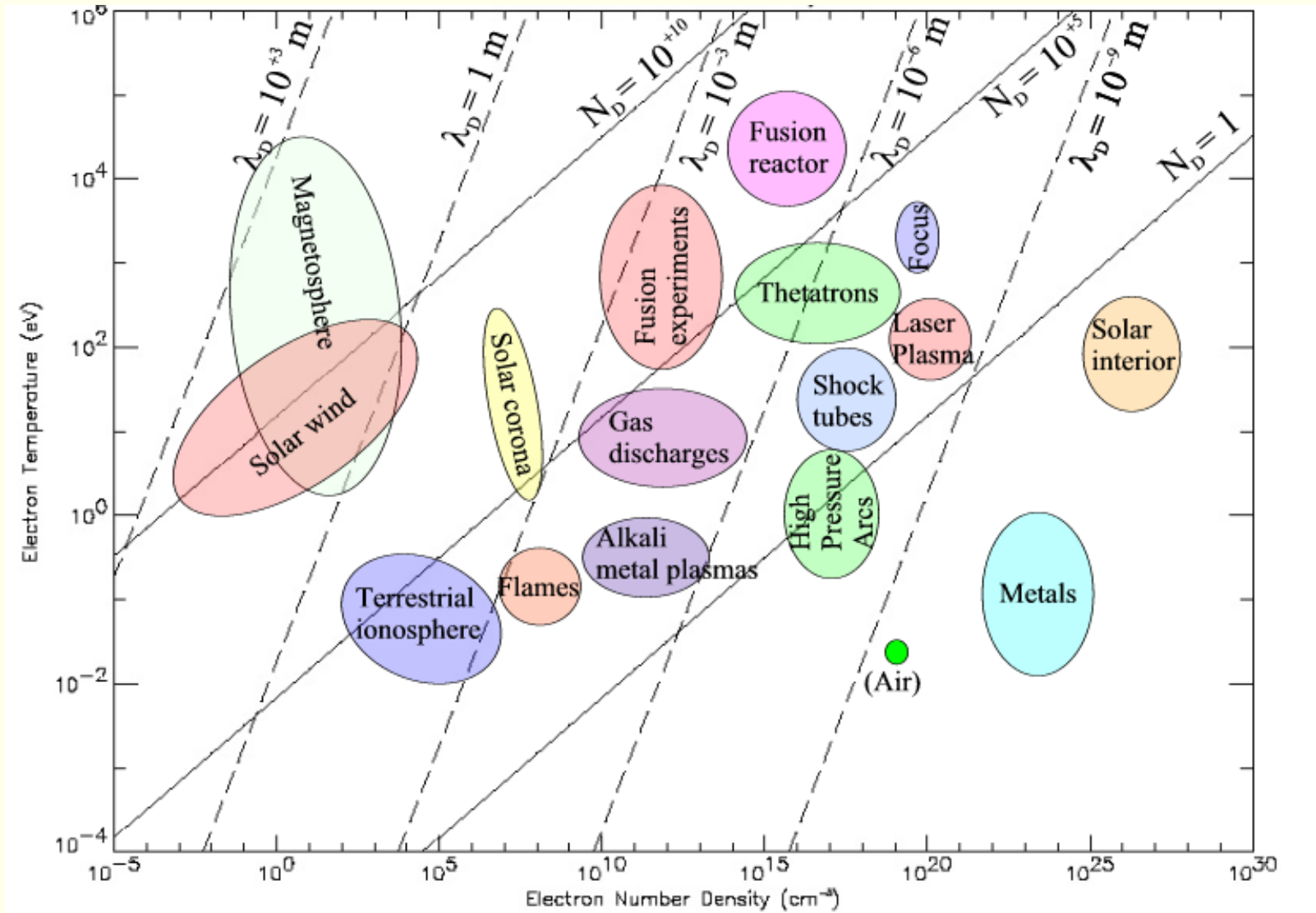
$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left( \frac{\lambda_D}{l_c} \right)^2$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

$$n_e \approx n_i$$

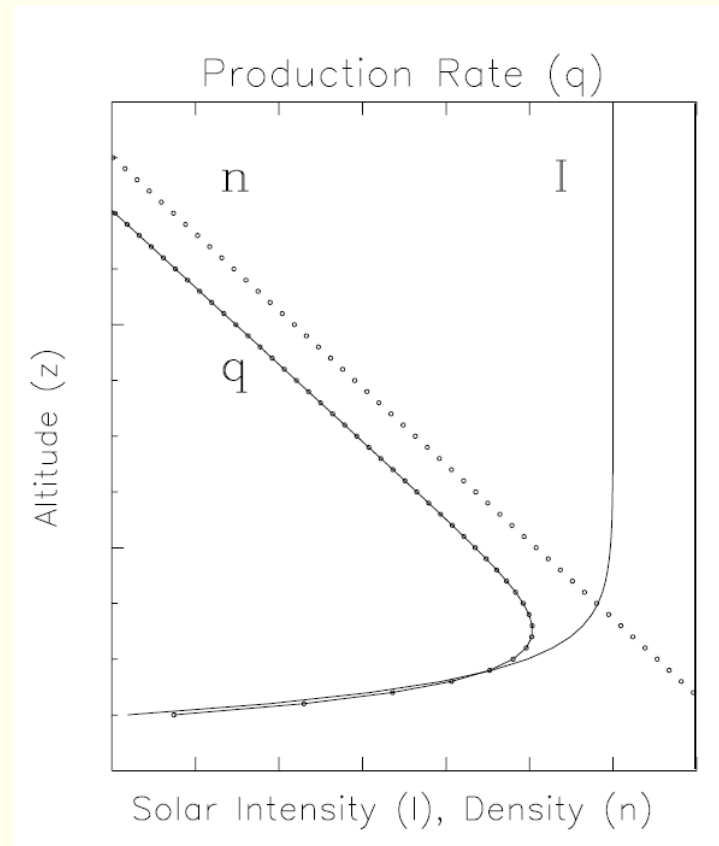
# Debye lengths

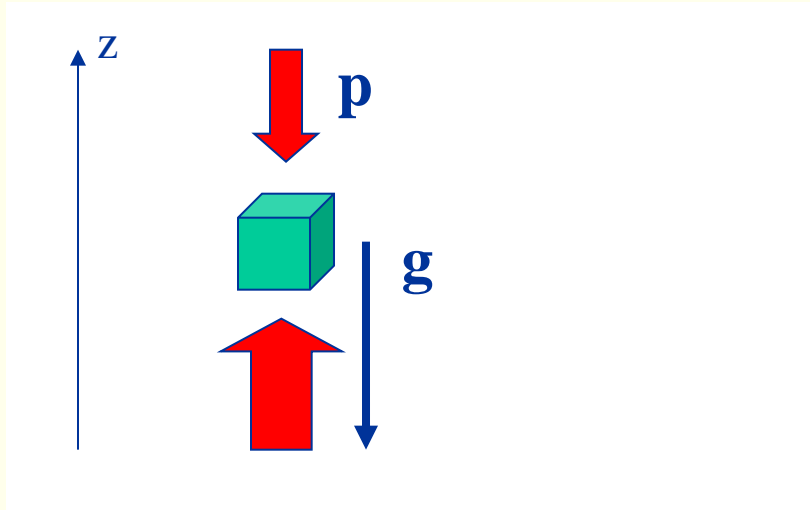




# *The ionosphere*

# Basic principle for creation of ionospheric layer





# Atmospheric scale height

$$-\frac{dp}{dz} = g\rho_m \quad \text{hydrostatic equilibrium for a volume element}$$

$$p = nk_B T = \frac{\rho k_B T}{m} \quad \text{ideal gas law}$$

$$-\frac{k_B T}{m} \frac{d\rho_m}{dz} = g\rho_m \quad \text{if } T \text{ is constant}$$

$$\rho_m = \text{const} \cdot e^{-z/(k_B T/gm)} = \text{const} \cdot e^{-z/H}$$

Scale height

$$H = k_B T/gm$$



## Scale height

$$H = k_B T / gm$$

What is the approximate scale height in the atmosphere right here, right now?

*(0° C = 273 K)*

Blue

1 km

Yellow

30 km

Green

9 km

Red

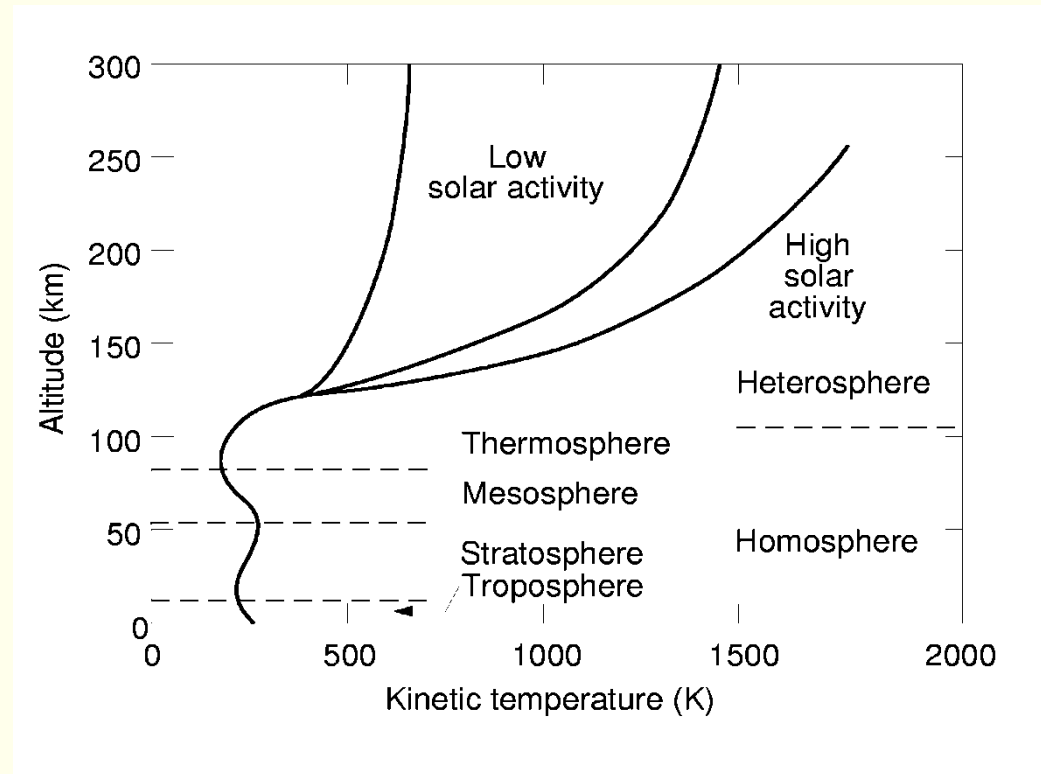
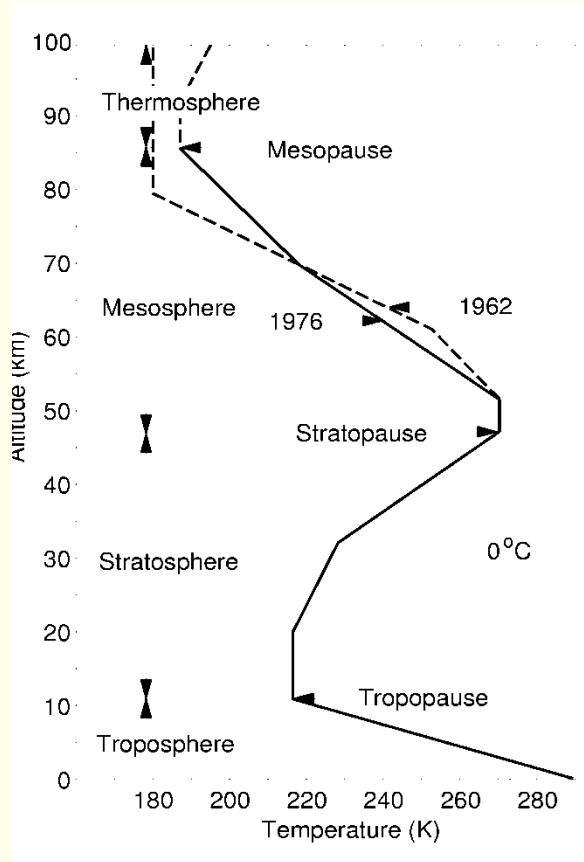
100 km



What did we neglect  
when we derived the  
scale height?

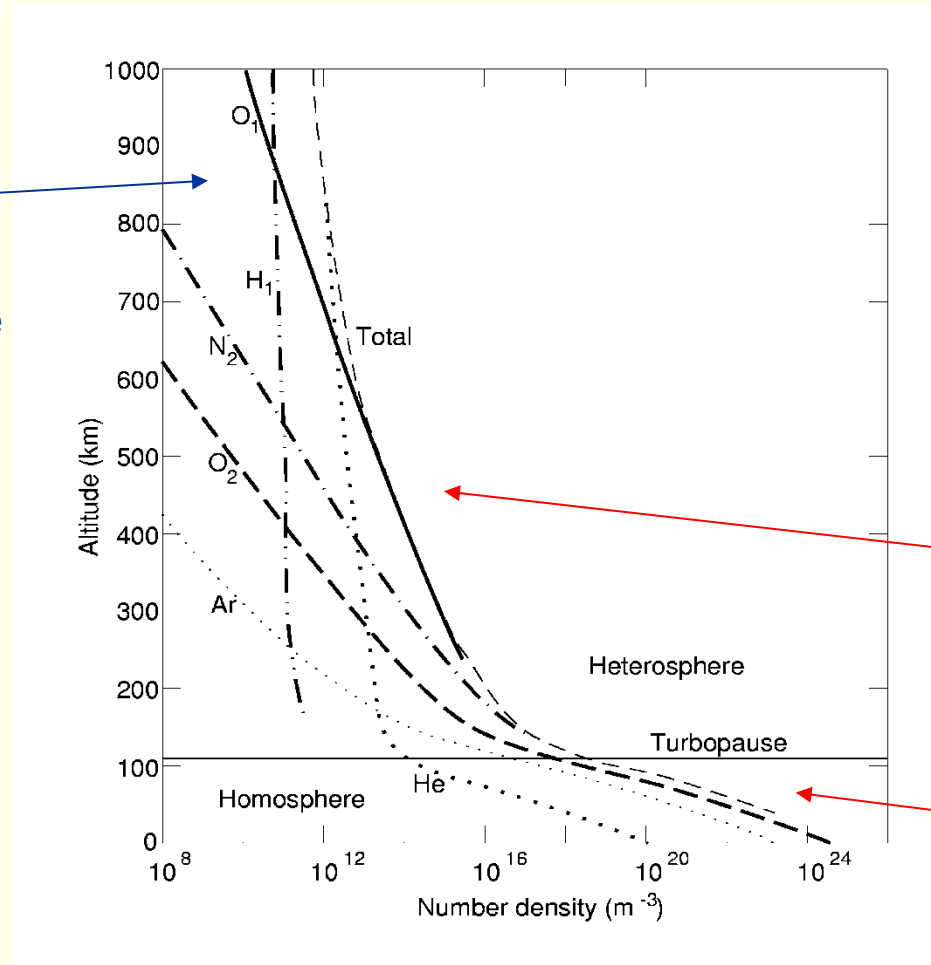


# Temperature profile



# Atmospheric composition

Longer scale height due to higher temperature



Separate scale heights for different components

Turbulent mixing – one scale height



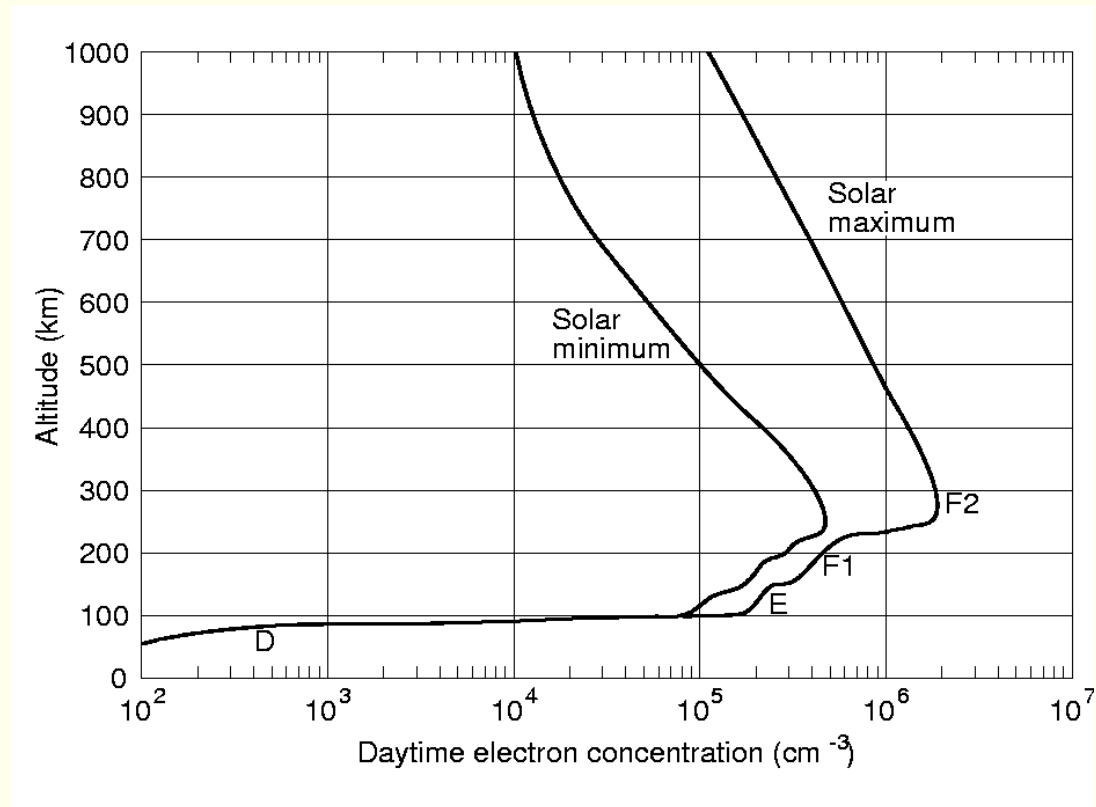
# Ionosphere

- The ionized, electrically conducting part of the upper atmosphere
- The ionosphere is a **plasma**

# History

- Stewart, 1882: Explained variations in the geomagnetic field
- Kenelly & Heavyside, 1902: explained Marconis transatlantic radio communication experiments
- Appleton & Barnett: experimental proof

# Altitude distribution of electron density ( $n_e$ )



# Continuity equation = conservation of ?

$$\frac{\partial n_e}{\partial t} = q - r - \nabla \cdot (n_e \mathbf{v}_e)$$

Ionization ( $\text{m}^{-3}\text{s}^{-1}$ )

Recombination ( $\text{m}^{-3}\text{s}^{-1}$ )

Flow ( $\text{m}^{-3}\text{s}^{-1}$ )

# Continuity equation

$$\frac{dn_e}{dt} = q - r$$

$$q = a_i I n_n$$

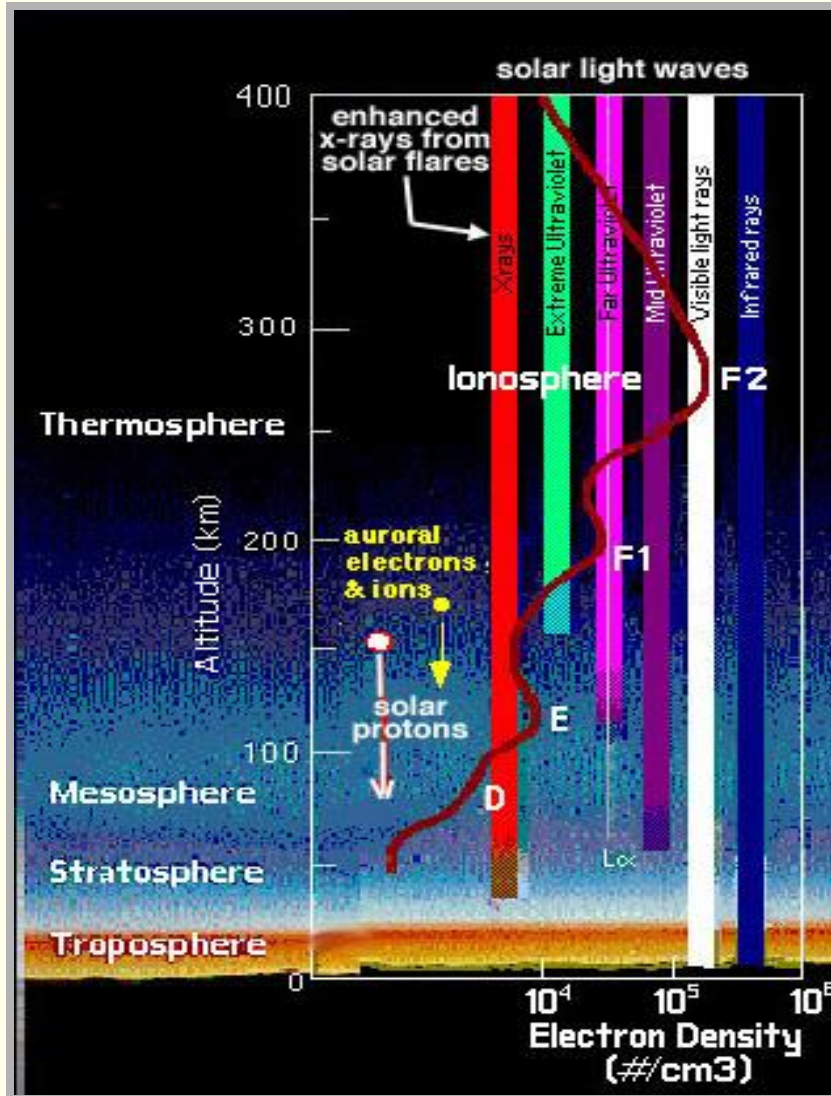
Ionization ( $\text{m}^{-3}\text{s}^{-1}$ )

Recombination ( $\text{m}^{-3}\text{s}^{-1}$ )

$$r = a_r n_e n_i = a_r n_e^2$$

Example:  $e + \text{O}_2^+ \rightarrow \text{O} + \text{O}$  (dissociative recombination)

# UV and X-ray radiation



$$\frac{dI}{dz} = In_n a_a$$



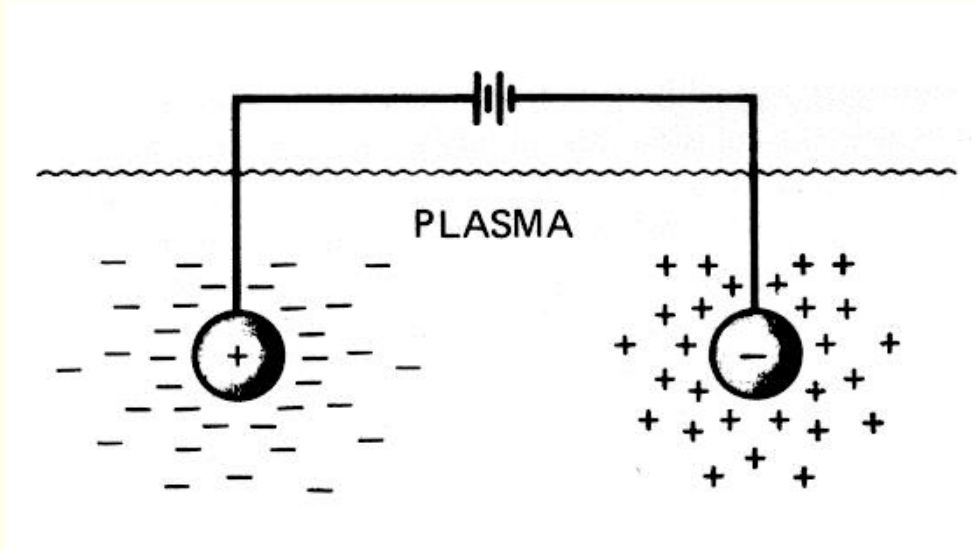
# Derive Chapman layer





# Last Minute!

# Quasineutrality



$$\frac{\Delta n}{n} = \frac{(n_e - n_i)}{n_e} < \left( \frac{\lambda_D}{l_c} \right)^2$$

$$\Phi = \Phi_0 e^{-x/\lambda_D}$$

Debye length

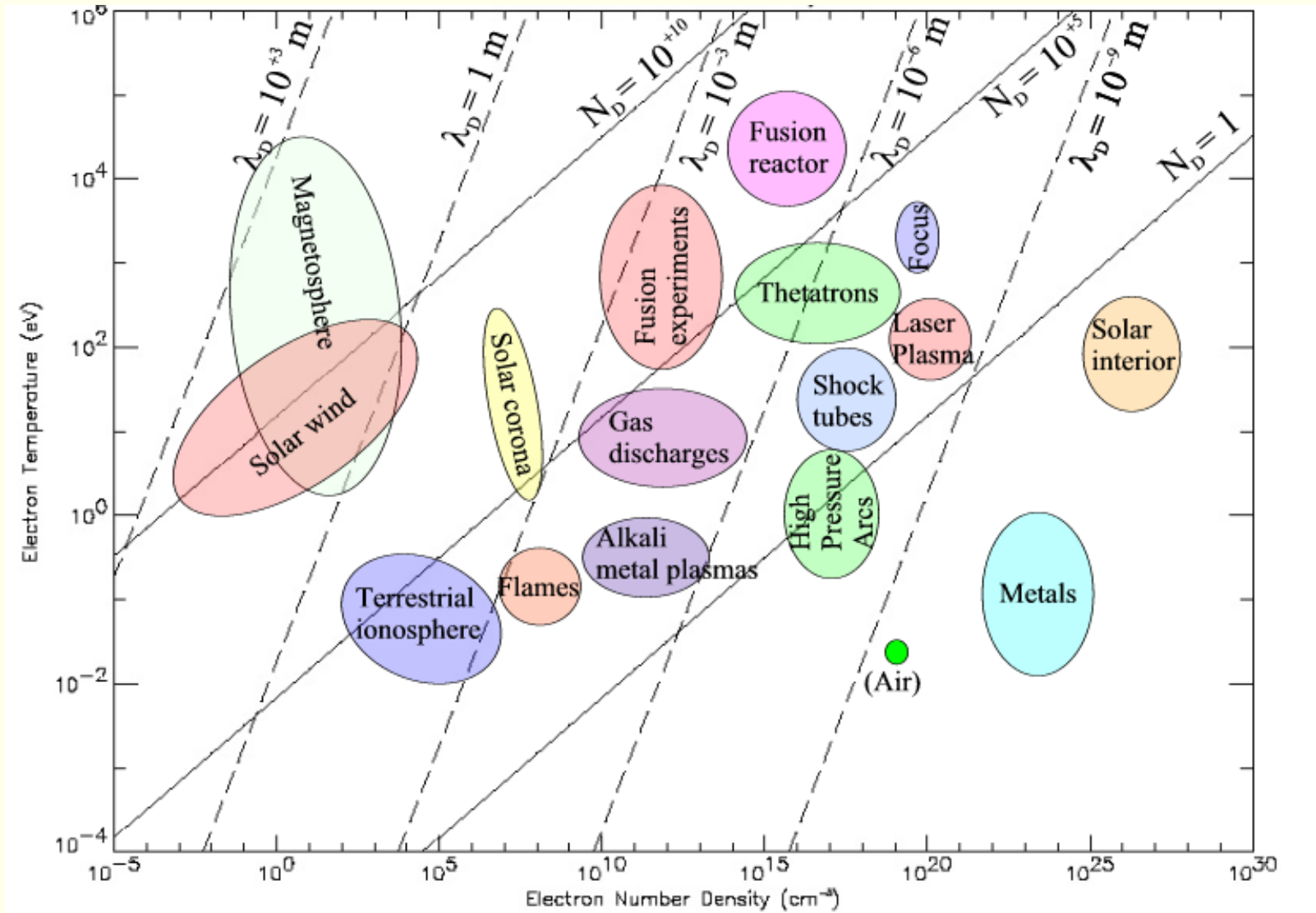
$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n_e e^2}}$$

$$l_C \gg \lambda_D \Rightarrow$$

Plasma close to neutral:

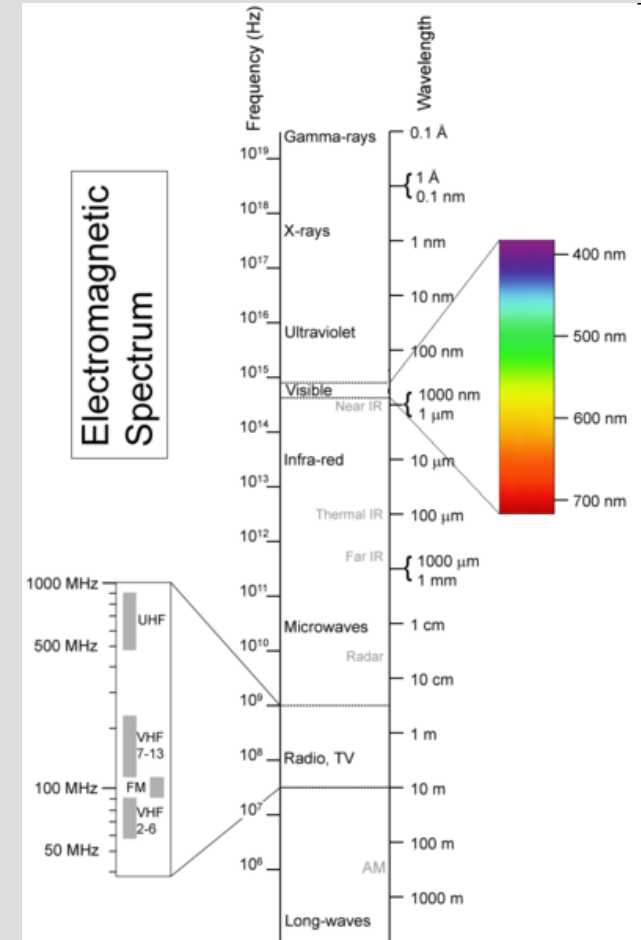
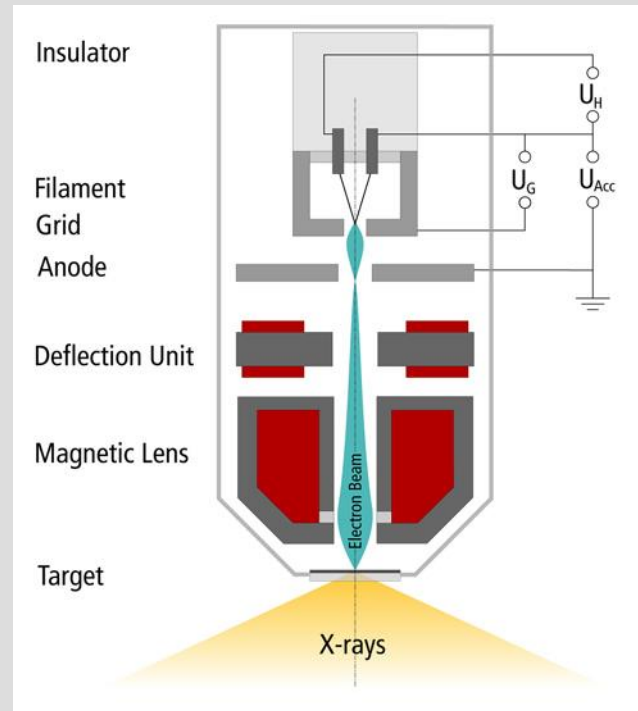
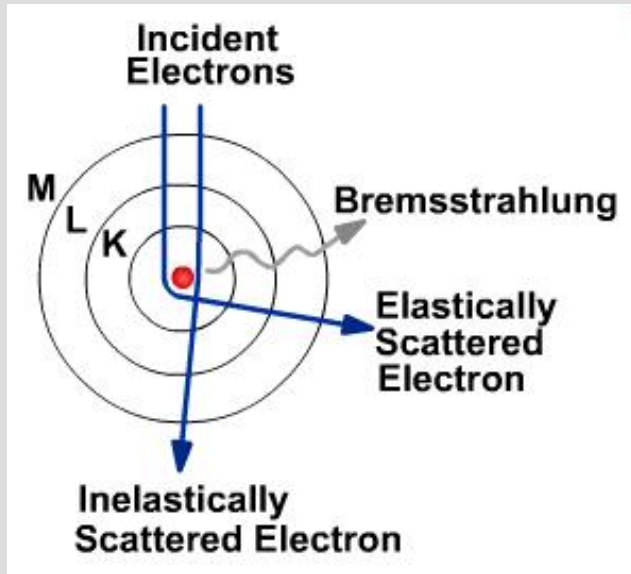
$$n_e \approx n_i$$

# Debye lengths





# X-rays, Bremsstrahlung



$$U_{acc} = hf = \frac{hc}{\lambda} \Rightarrow$$

$$\lambda = \frac{hc}{U_{acc}} = \frac{6.62 \cdot 10^{-34} \cdot 3 \cdot 10^8}{100 \cdot 10^3 \cdot 1.6 \cdot 10^{-19}} = 1.2 \cdot 10^{-11} \text{ m} = 0.012 \text{ nm}$$