



## Problem set for Seminar 2

See [www.kth.se/social/course/SF1625](http://www.kth.se/social/course/SF1625) for information about how the seminars work and what you are expected to do during the seminars. At this seminar there will be a small test in which you are asked to solve (a variant of) one of the recommended exercises from the text book Calculus by Adams och Essex (8:th ed), namely:

Chapter 1.2: 9, 13, 21, 25, 30, 49, 50, 78, 79. Chapter 1.3: 3, 6, 11, 13, 53. Chapter 1.4: 7, 8, 12, 15, 17, 20, 21, 29. Chapter 1.5: 13, 29. Chapter 2.1: 5, 7. Chapter 2.2: 1, 3, 11, 26, 27, 40, 41, 42, 43, 44, 45, 47. Chapter 2.3: 1, 7, 11, 17, 25, 33, 35, 47. Chapter 2.4: 3, 5, 11, 18, 23, 30, 31, 37. Chapter 2.5: 13, 15, 23, 29, 31, 35, 45, 62.

At the seminar these problems will be discussed.

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### PROBLEMS

**Uppgift 1.** Compute the limits:

A.  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

B.  $\lim_{x \rightarrow \infty} \frac{x^2 + 3x + 5}{5x^2 + 2x + 3}$

C.  $\lim_{x \rightarrow \infty} \left( \sqrt{x + \sqrt{x}} - \sqrt{x} \right)$

**Uppgift 2.** Find the constant  $k$  such that the function

$$g(x) = \begin{cases} \frac{\sin kx}{x}, & x \neq 0 \\ 4, & x = 0 \end{cases}$$

is continuous at the origin. Is the function then continuous at all  $x$ ?

**Uppgift 3.** Differentiate the following functions with respect to  $x$  and state for what  $x$  these functions are differentiable. Does any of these functions fail to be differentiable in all of the domain of definition?

A.  $f(x) = 1 + \tan^2 x$

B.  $g(x) = \frac{ax + b}{cx + d}$

C.  $h(x) = \sin \sqrt{x}$

D.  $k(x) = |x| \cos x$

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#### DISCUSSION PROBLEMS

Here are some extra problems to discuss at the seminar.

- Show using the intermediate value property that  $p(x) = x^5 + x^3 + 1$  has a zero somewhere between  $-1$  and  $0$ . Is the zero closer to  $-1$  or  $0$ ? How can you be sure that there are not any other zeros?
- At a car park you are charged as follows: the first hour costs 4 kronor and after that the charge is 2 kronor for each started hour up to the maximal charge 10 kronor. Let  $h(t)$  be the cost as a function of time  $t$  hours. Sketch the graph  $y = h(t)$  for  $0 \leq t \leq 24$ . Is  $h$  continuous?
- Give an example of function with domain of definition  $\mathbb{R}$  that is neither continuous nor differentiable at  $x = 1$ .
- Give an example of function with domain of definition  $\mathbb{R}$  that is continuous but not differentiable at  $x = 1$ .
- Does there exist a function that is differentiable but not continuous at  $x = 1$ .