Problem set for Seminar 2

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do during the seminars. At this seminar there will be a small test in which you are asked to solve (a variant of) one of the recommended exercises from the text book Calculus by Adams och Essex (8:th ed), namely: Chapter 1.2: 9, 13, 21, 25, 30, 49, 50, 78, 79. Chapter 1.3: 3, 6, 11, 13, 53. Chapter 1.4: 7, 8, 12, 15, 17, 20, 21, 29. Chapter 1.5: 13, 29. Chapter 2.1: 5, 7. Chapter 2.2: 1, 3, 11, 26, 27, 40, 41, 42, 43, 44, 45, 47. Chapter 2.3: 1, 7, 11, 17, 25, 33, 35, 47. Chapter 2.4: 3, 5, 11, 18, 23, 30, 31, 37. Chapter 2.5: 13, 15, 23, 29, 31, 35, 45, 62. At the seminar these problems will be discussed.

PROBLEMS

Uppgift 1. Compute the limits:

A. \( \lim_{x \to \infty} \frac{\sin x}{x} \)

B. \( \lim_{x \to \infty} \frac{x^2 + 3x + 5}{5x^2 + 2x + 3} \)

C. \( \lim_{x \to \infty} \left( \sqrt{x + \sqrt{x}} - \sqrt{x} \right) \)

Uppgift 2. Find the constant \( k \) such that the function

\[ g(x) = \begin{cases} \sin \frac{kx}{x}, & x \neq 0 \\ 4, & x = 0 \end{cases} \]

is continuous at the origin. Is the function then continuous at all \( x \)?
**Uppgift 3.** Differentiate the following functions with respect to $x$ and state for what $x$ these functions are differentiable. Does any of these functions fail to be differentiable in all of the domain of definition?

A. $f(x) = 1 + \tan^2 x$

B. $g(x) = \frac{ax + b}{cx + d}$

C. $h(x) = \sin \sqrt{x}$

D. $k(x) = |x| \cos x$

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**Discussion problems**

Here are some extra problems to discuss at the seminar.

- Show using the intermediate value property that $p(x) = x^5 + x^3 + 1$ has a zero somewhere between $-1$ and $0$. Is the zero closer to $-1$ or $0$? How can you be sure that there are not any other zeros?

- At a car park you are charged as follows: the first hour costs 4 kronor and after that the charge is 2 kronor for each started hour up to the maximal charge 10 kronor. Let $h(t)$ be the cost as a function of time $t$ hours. Sketch the graph $y = h(t)$ for $0 \leq t \leq 24$. Is $h$ continuous?

- Give an example of function with domain of definition $\mathbb{R}$ that is neither continuous nor differentiable at $x = 1$.

- Give an example of function with domain of definition $\mathbb{R}$ that is continuous but not differentiable at $x = 1$.

- Does there exist a function that is differentiable but not continuous at $x = 1$. 