Department of Mathematics

SF1625
One variable Calculus
2014-2015

Problem set for Seminar 6

See www.kth.se/social/course/SF1625 for information about how the seminars work and what you are expected to do during the seminars. At this seminar there will be a small test in which you are asked to solve (a variant of) one of the recommended exercises from the text book Calculus by Adams och Essex (8:th ed), namely:
Chapter 6.1: 1, 3, 5, 7, 13, 21. Chapter 6.2: 1, 5, 9, 11, 13, 23. Chapter 6.3: 1, 3, 9. Chapter 6.5: 1, 3, 5, 15, 23, 33, 34, 35. Chapter 7.1: 1, 3, 5, 13, 19, 21. Chapter 7.2: 1, 3.
At the seminar these problems will be discussed:

UPPGIFTER

Uppgift 1. Compute the integrals:

A. $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx$ (use $x = \cos t$)

B. $\displaystyle\int_e^{e^2} \frac{\ln(\ln x)}{x}\,dx$ (use $\ln x = t$)

C. $\displaystyle\int_0^{1/2} \arcsin x\,dx$ (start by using integration by parts)

D. $\displaystyle\int_0^1 \frac{x-1}{x^2-5x+6}\,dx$ (use partial fractions)

Uppgift 2. A ball with radius r has its center on the surface of a ball with radius R, where $R > r$. Compute the volume of that part of the smaller ball which is inside the larger ball. (Use volume of revolution techniques)

Uppgift 3. During the early 1970's, the world's yearly rate of consumption of oil $f(t)$ grew exponentially with growth constant 0.07 (per year). In the beginning of the year 1970 the rate of consumption was 16.1 billion vats of oil per year. This yields

$$f(t) = 16.1e^{0.07t}, \qquad t \geq 0,$$

where t is the time (measured in years) since the beginning of the year 1970.

The oil crisis 1974 led to a dramatic increase of the prize of oil and the growth constant dropped from 0.07 to 0.04. A fairly accurate model for the worlds yearly rate of consumption of oil between 1974 och 1980 is given by

$$g(t) = 21.3e^{0.04(t-4)}, \qquad t \geq 4,$$

where $t = 4$ is the beginning of the year 1974.

Compute the total amount of oil that was saved from consumption during the years 1976 through 1980 due to the change in rate of consumption caused by the oil crisis.

DISCUSSION PROBLEMS

Here are some extra problems to discuss at the seminar. You do not have to write down solutions in advance.

- Give an example of a function that is not integrable and explain why it is not integrable.
- The function $f(x) = e^{x^2}$ is continuous on the closed and bounded interval $[0, 1]$ and is therefore integrable over that interval (by theorem 2 in Chapter 5.3). But it is not possible to write down an anti-derivative to f in the normal way. Is there a contradiction here?
- Does the integral $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+1}\,dx$ converge or diverge?
- Compute $\displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}e^n} \int_n^{n+1} \sqrt{x}e^x\,dx$