

Appendix A:

Cylinderkoordinater

De tre koordinaterna är (r, θ, z) , där r är den radiella koordinaten, θ den azimutala (eller den transverella) koordinaten och z den axiella koordinaten. En vektor $\mathbf{u}$ kan delas upp i komponenter enligt

$$\mathbf{u} = u_r \mathbf{e}_r + u_\theta \mathbf{e}_\theta + u_z \mathbf{e}_z. \quad (1)$$

De båda enhetsvektorerne $\mathbf{e}_r$ och $\mathbf{e}_\theta$ är funktioner av θ , d v s de vrider sig med θ . Följande samband gäller

$$\frac{\partial \mathbf{e}_r}{\partial \theta} = \mathbf{e}_\theta, \quad (2)$$

$$\frac{\partial \mathbf{e}_\theta}{\partial \theta} = -\mathbf{e}_r. \quad (3)$$

Gradienten av en skalär

$$\nabla \phi = \mathbf{e}_r \frac{\partial \phi}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial \phi}{\partial \theta} + \mathbf{e}_z \frac{\partial \phi}{\partial z}. \quad (4)$$

Divergensen av en vektor

$$\nabla \cdot \mathbf{u} = \frac{1}{r} \frac{\partial (ru_r)}{\partial r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}. \quad (5)$$

Rotationen av en vektor

$$\nabla \times \mathbf{u} = \mathbf{e}_r \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right) + \mathbf{e}_\theta \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) + \mathbf{e}_z \left(\frac{1}{r} \frac{\partial (ru_\theta)}{\partial r} - \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right) \quad (6)$$

Laplace av en skalär

$$\nabla^2 \phi = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \phi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2}. \quad (7)$$

Laplace av en vektor

$$\nabla^2 \mathbf{u} = \mathbf{e}_r \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right) + \mathbf{e}_\theta \left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \right) + \mathbf{e}_z \nabla^2 u_z \quad (8)$$

Inkompressibla Navier-Stokes ekvationer utan gravitationskraft

$$\frac{\partial u_r}{\partial t} + (\mathbf{u} \cdot \nabla) u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u_r - \frac{u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} \right), \quad (9)$$

$$\frac{\partial u_\theta}{\partial t} + (\mathbf{u} \cdot \nabla) u_\theta + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r} \frac{\partial p}{\partial \theta} + \nu \left(\nabla^2 u_\theta - \frac{u_\theta}{r^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} \right), \quad (10)$$

$$\frac{\partial u_z}{\partial t} + (\mathbf{u} \cdot \nabla) u_z = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 u_z, \quad (11)$$

där

$$\mathbf{u} \cdot \nabla = u_r \frac{\partial}{\partial r} + \frac{u_\theta}{r} \frac{\partial}{\partial \theta} + u_z \frac{\partial}{\partial z}. \quad (12)$$

Töjningstensorn

$$e_{rr} = \frac{\partial u_r}{\partial r}, \quad (13)$$

$$e_{\theta\theta} = \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \quad (14)$$

$$e_{zz} = \frac{\partial u_z}{\partial z}, \quad (15)$$

$$e_{r\theta} = \frac{r}{2} \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{2r} \frac{\partial u_r}{\partial \theta}, \quad (16)$$

$$e_{\theta z} = \frac{1}{2r} \frac{\partial u_z}{\partial \theta} + \frac{1}{2} \frac{\partial u_\theta}{\partial z}, \quad (17)$$

$$e_{zr} = \frac{1}{2} \frac{\partial u_r}{\partial z} + \frac{1}{2} \frac{\partial u_z}{\partial r}. \quad (18)$$