FEO3230 Sannolikhetsteori och stokastiska processer 12,0 hp

Probability and Random Processes

Fastställande

Betygsskala

G

Utbildningsnivå

Forskarnivå

Särskild behörighet

Undervisningsspråk

Undervisningsspråk anges i kurstillfällesinformationen i kurs- och programkatalogen.

Lärandemål

A student who has passed this course should be able to:

- Describe and understand the necessity for adopting measure theory as a foundation for modern probability and random processes, also in cases where the theory is used in a more applied setting
- Describe and understand what parts of the general theory are extra important when pursuing theoretically oriented research in the information sciences

- Understand and present several of the proofs required to provide a foundation for integration, probability,expectation and random processes
- Understand advanced papers in the own field of research that uses tools from measure theoretic probability and ergodic theory
- Use existing results from the general theory, and synthesize new results in the own research field, with proper mathematical rigor

Kursinnehåll

The course will start from scratch in the sense that the only required background is calculus-based integration and probability theory. Basic concepts in integration and measure theory will be introduced from first principles, and then the course will explain how these concepts form the foundation for probability and random processes based on measure theory.

A preliminary course outline is provided below.

Lecture 1: Lebesgue measure on the real line

Lecture 2: The Lebesgue integral on the real line

Lecture 3: General measure theory

- Measure spaces and measurable functions
- Convergence in measure

Lecture 4: General integration theory

- The abstract Lebesgue integral
- Distribution functions and the Lebesgue–Stieltjes integral

Lecture 5: Probability and expectation

- Probability spaces
- Expectation
- The law of large numbers for i.i.d. sequences

Lecture 6: Differentiation

- Functions of bounded variation
- Absolutely continuous functions
- The Radon–Nikodym derivative
- Probability distributions and pdf's; absolutely continuous random variables

Lecture 7: Conditional probability and expectation

- Conditional probability/expectation
- Decomposition of measures; continuous, mixed and discrete random variables

Lecture 8: Topological and metric spaces

- Topological and metric spaces

• Completeness and separability, Polish spaces
• Standard spaces

Lecture 9: Extensions of measures and product measure

• Extension theorems
• Product measure

Lecture 10: Random processes

• Process measure, Kolmogorov's extension theorem

Lecture 11: Dynamical systems and ergodicity

• Random processes and dynamical systems
• The ergodic theorem
• The Shannon–McMillan–Breiman theorem

Lecture 12: Applications

• Detection and estimation in abstract spaces
• Coding theorems in abstract spaces

Kurslitteratur

The main text for the course is Robert Gray (Stanford): Probability, random processes and ergodic properties (1st edition available from Gray's web-page, 2nd edition printed by Springer). As a complement, parts of the first half of the course will be based on lecture notes only. The lecture notes essentially follow McDonald andWeiss: A course in real analysis, and students who plan to dig deeper are advised to acquire this textbook too, as a complement.

Examination

Examinator beslutar, baserat på rekommendation från KTH:s handläggare av stöd till studenter med funktionsnedsättning, om eventuell anpassad examination för studenter med dokumenterad, varaktig funktionsnedsättning.

Examinator får medge annan examinationsform vid omexamination av enstaka studenter.

När kurs inte längre ges har student möjlighet att examineras under ytterligare två läsår.

Övriga krav för slutbetyg

The students will be examined based on mandatory homework problems. A written or oral exam will be offered subsequently in cases where the homework problems do not provide sufficient proof that the learning outcomes have been met.

Etiskt förhållningssätt

- Vid grupparbete har alla i gruppen ansvar för gruppens arbete.
- Vid examination ska varje student ärligt redovisa hjälp som erhållits och källor som använts.
- Vid muntlig examination ska varje student kunna redogöra för hela uppgiften och hela lösningen.