FEO3230 Probability and Random Processes 12.0 credits

Sannolikhetsteori och stokastiska processer

This is a translation of the Swedish, legally binding, course syllabus.

If the course is discontinued, students may request to be examined during the following two academic years

Establishment

Course syllabus for FEO3230 valid from Spring 2019

Grading scale

P, F

Education cycle

Third cycle

Specific prerequisites

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

A student who has passed this course should be able to:

- Describe and understand the necessity for adopting measure theory as a foundation for modern probability and random processes, also in cases where the theory is used in a more applied setting
- Describe and understand what parts of the general theory are extra important when pursuing theoretically oriented research in the information sciences
- Understand and present several of the proofs required to provide a foundation for integration, probability,expectation and random processes
- Understand advanced papers in the own field of research that uses tools from measure theoretic probability and ergodic theory
- Use existing results from the general theory, and synthesize new results in the own research field, with proper mathematical rigor

Course contents

The course will start from scratch in the sense that the only required background is calculus-based integration and probability theory. Basic concepts in integration and measure theory will be introduced from first principles, and then the course will explain how these concepts form the foundation for probability and random processes based on measure theory.

A preliminary course outline is provided below.

Lecture 1: Lebesgue measure on the real line

Lecture 2: The Lebesgue integral on the real line

Lecture 3: General measure theory

- Measure spaces and measurable functions
- Convergence in measure

Lecture 4: General integration theory

- The abstract Lebesgue integral
- Distribution functions and the Lebesgue–Stieltjes integral

Lecture 5: Probability and expectation

- Probability spaces
- Expectation
- The law of large numbers for i.i.d. sequences

Lecture 6: Differentiation

- Functions of bounded variation
- Absolutely continuous functions
- The Radon–Nikodym derivative
- Probability distributions and pdf's; absolutely continuous random variables

Lecture 7: Conditional probability and expectation

- Conditional probability/expectation
- Decomposition of measures; continuous, mixed and discrete random variables

Lecture 8: Topological and metric spaces

- Topological and metric spaces
- Completeness and separability, Polish spaces
- Standard spaces

Lecture 9: Extensions of measures and product measure

- Extension theorems
- Product measure

Lecture 10: Random processes

- Process measure, Kolmogorov's extension theorem

Lecture 11: Dynamical systems and ergodicity

- Random processes and dynamical systems
- The ergodic theorem
- The Shannon–McMillan–Breiman theorem

Lecture 12: Applications

- Detection and estimation in abstract spaces
- Coding theorems in abstract spaces

Examination

- EXA1 - Examination, 12.0 credits, grading scale: P, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

Other requirements for final grade

The students will be examined based on mandatory homework problems. A written or oral exam will be offered subsequently in cases where the homework problems do not provide sufficient proof that the learning outcomes have been met.

Ethical approach

- All members of a group are responsible for the group's work.

- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.