

# FSF3619 Elliptic Partial Differential Equations and Harmonic Function Theory 7.5 credits

Elliptiska partiella differentialekvationer och harmonisk funktionsteori

This is a translation of the Swedish, legally binding, course syllabus.

If the course is discontinued, students may request to be examined during the following two academic years

## Establishment

Course syllabus for FSF3619 valid from Spring 2019

## Grading scale

G

#### **Education cycle**

Third cycle

## Specific prerequisites

A Master degree including at least 45 university credits (hp) in Mathematics.

Good knowledge of basic analysis; some introductory partial differential equations course at the undergraduate level.

## Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

## Intended learning outcomes

After completing the course the student should be able to:

- Describe different methods that are used in the study of elliptic partial differential equations
- Explain the difference between different types of solutions to elliptic equations
- Solve problems for elliptic partial differential equations by using tools from functional analysis, operator theory, and potential theory
- Describe fundamental properties of harmonic functions
- Describe and apply important theorems from harmonic function theory.

#### Course contents

List of suggested topics:

- 1. Maximum/comparison principles, Hopf's lemma
- 2. Harnack's inequality
- 3. Poisson kernel, Harmonic measure
- 4. Caratheodory's theorem, Koebe's one-quarter theorem
- 5. Potential theory, Wiener's solution of the Dirichlet problem
- 6. Fundamental solution and Green's function, Green's integral identities
- 7. Elliptic estimates, Alexandroff's estimates
- 8. Barriers, regularity up to the boundary
- 9. Sobolev spaces: Weak and strong convergence in function spaces, imbeddings, compactness arguments
- 10. Notions of solution: viscosity solution, classical solution
- 11. Fractional Sobolev spaces and fractional operators

# Disposition

Lectures and homework.

## **Course literature**

To be announced at least 4 weeks before course start at course home page.

Suggested literature:

- 1. Caffarelli, Luis A.; Cabre, Xavier; Fully nonlinear elliptic equations. American Mathematical Society Colloquium Publications, 43. American Mathematical Society, Providence, RI, 1995. vi+104 pp. ISBN: 0-8218-0437-5
- 2. Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. Reprint of the 1998 edition. Classics in Mathematics. Springer- Verlag, Berlin, 2001. xiv+517 pp. ISBN: 3-540-41160-7 35-02 (35Jxx)
- 3. Pucci, Patrizia; Serrin, James; The maximum principle. Progress in Nonlinear Differential Equations and their Applications, 73. Birkh "auser Verlag, Basel, 2007. x+235 pp. ISBN: 978-3-7643-8144-8
- Garnett, John B.; Marshall, Donald E.; Harmonic measure. New Mathemat- ical Monographs, 2. Cambridge University Press, Cambridge, 2005. xvi+571 pp. ISBN: 978-0-521-47018-6
- 5. Other appropriate literatures/articles.

### Examination

• PRO1 - Project work, 7.5 credits, grading scale: P, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

Homework solutions

Oral presentations.

## Other requirements for final grade

Accepted homework solutions.

Accepted oral presentations.

# Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.