



# FSF3623 Methods in Elliptic and Parabolic PDE 7.5 credits

Metoder i elliptiska och paraboliska PDE

This is a translation of the Swedish, legally binding, course syllabus.

## Establishment

## Grading scale

G

## Education cycle

Third cycle

## Specific prerequisites

A Master degree including at least 30 university credits (hp) in Mathematics.

Lectures and presentation, selfstudy, homework.

## Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

## Intended learning outcomes

After completing the course, students should have a good knowledge of general existence theory, qualitative behavior, as well as geometric approaches to PDEs. Several notions such

as weak, strong, viscosity solutions as well as general tools for handling such problems, including methods from geometric measure theory and Sobolev space theory, will also be required to learn during the course.

## Course contents

The focus will be on various methods, tools and ideas that are used by mathematicians working with PDE.

- Maximum/comparison principle (various forms), Hopf's lemma.
- Harnack's inequality, boundary Harnack.
- Fundamental solution, Green's function, Green's integral identities.
- Elliptic estimates, Alexandroff's, B., P., estimates.
- Barriers, regularity up to the boundary.
- Sobolev spaces: Weak/strong convergence, imbedding, Compactness arguments.
- Notion of solutions:  $W^{k,m}$ , viscosity, classical in  $C^k$ .
- Rearrangements.
- Qualitative theory: Symmetry, Moving plane methods, reflections, inversions, sliding methods.
- Geometric measure theory: Scaling, Blow up, flatness, measure theoretic normal, densities, structure theorems.
- Hausdorff dimension, packing measures.

## Disposition

Good knowledge of basic Analysis, and some introductory PDE course at undergraduate level.

## Course literature

Suggested literatures (which is subject to change depending on the topics chosen)

1. Caffarelli, Luis A.; Cabre, Xavier; Fully nonlinear elliptic equations. American Mathematical Society Colloquium Publications, 43. American Mathematical Society, Providence, RI, 1995. vi+104 pp. ISBN: 0-8218-0437-5
2. Evans, Lawrence C.; Gariepy, Ronald F. Measure theory and fine properties of functions. Studies in Advanced Mathematics. CRC Press, Boca Raton, FL, 1992. viii+268 pp. ISBN: 0-8493-7157-0
3. Gilbarg, David; Trudinger, Neil S. Elliptic partial differential equations of second order. Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001. xiv+517 pp. ISBN: 3-540-41160-7 35-02 (35Jxx)
4. Kawohl, Bernhard; Rearrangements and convexity of level sets in PDE. Lecture Notes in Mathematics, 1150. Springer-Verlag, Berlin, 1985. iv+136 pp. ISBN: 3-540-15693-3

5. Maly, Jan; Ziemer, William P.; Fine regularity of solutions of elliptic partial differential equations. Mathematical Surveys and Monographs, 51. American Mathematical Society, Providence, RI, 1997. xiv+291 pp. ISBN: 0-8218-0335-2
6. Pucci, Patrizia; Serrin, James; The maximum principle. Progress in Nonlinear Differential Equations and their Applications, 73. Birkhäuser Verlag, Basel, 2007. x+235 pp. ISBN: 978-3-7643-8144-82

## Examination

- PRO1 - Project work, 7.5 credits, grading scale: P, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

If the course is discontinued, students may request to be examined during the following two academic years.

- Presentation of a topic with written report.
- Preparation of three homework, with solution, within the chosen topic.
- Solving homework, suggested by other participants.

## Other requirements for final grade

Approved homework assignments, and oral presentation of a project with written report.

## Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.