FSF3707 Riemann-Hilbert Methods in Asymptotic Analysis 7.5 credits

Riemann-Hilbert metoder i asymptotisk analys

This is a translation of the Swedish, legally binding, course syllabus.

If the course is discontinued, students may request to be examined during the following two academic years

Establishment

Course syllabus for FSF3707 valid from Spring 2019

Grading scale

P, F

Education cycle

Third cycle

Specific prerequisites

A Master degree including at least 30 university credits (hp) in in Mathematics (including SF1628 Complex analysis or equivalent).

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

The overall purpose of the course is to discuss the Riemann Hilbert approach in the asymptotic analysis of special functions/orthogonal polynomials and differential equations.

After the course, the student is expected to explain and work with the following concepts:

- Monodromy for differential equations
- Riemann-Hilbert problems
- Isomonodromic deformations
- Painlevé equations
- Lax pairs
- Discrete Painlevé equations and orthogonal polynomials
- Deift/Zhou steepest descent for Riemann-Hilbert problems

1. g-functions
2. Global parametrix
3. Local parametrices

- Double scaling limits

After the course, the student should have sufficient skills to independently and efficiently read research papers on the topic.

Course contents

- Linear ordinary differential equations and method of stationary phase classical linear steepest descent. (Case study: the Airy equation)
- Riemann-Hilbert problems: Monodromy of ordinary differential equations and the isomonodromy approach
- Riemann-Hilbert problems: general theory
- Riemann-Hilbert problems: The Painleve II equation

1. The Hastings-McLeod Solution
2. Connection formulas
3. The Vanishing Lemma and pole free solutions

- Discrete Painlevé equations and orthogonal polynomials
- Steepest descent for the Riemann-Hilbert problem for orthogonal polynomials
- Double scaling limits
- Small dispersion of KdV (if time permits)

Disposition

Lectures, homeworks and presentation.

Course literature

- .S. Fokas, A. Its, A. A.Kapaev; V.Y. Novokshenov, Painlev transcendents. The Riemann-Hilbert approach. Mathematical Surveys and Monographs, 128. American Mathematical Society, Providence, RI, 2006. xii+553 pp.
- P.A. Deift, Orthogonal polynomials and random matrices: a Riemann-Hilbert approach. Courant Lecture Notes in .Mathematics, 3. New York University, Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 1999. viii+273pp.

We will also use (part of) papers and lectures notes from the arXiv.

Examination

- INL1 - Assignment, 7.5 credits, grading scale: P, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

Homeworks, and presentation/oral exam.

Other requirements for final grade

Homeworks, and presentation/oral exam.

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.