FSF3716 The Atiyah-Singer index theorem 7.5 credits

Atiyah-Singers Indexsats

This is a translation of the Swedish, legally binding, course syllabus.

Establishment

Course syllabus valid from spring 2022 accordning to the school's principal: S-2022-0194.
Date for descitsion: 2022-02-04

Grading scale

P, F

Education cycle

Third cycle

Specific prerequisites

Courses on functional analysis, differential geometry, algebraic topology are recommended as prerequisities, but it is not strictly necessary to have read all these courses.

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

Course description. The Atiyah-Singer index theorem provides a fundamental connection between differential geometry, partial differential equations, differential topology, operator algebras, and has links to many other fields.

The fundamental observation is that the index of Fredholm operators is a homotopy invariant. Examples of such Fredholm operators are elliptic differential operators on manifolds. The Atiyah-Singer index theorem tells us how the analytic index of an elliptic operator is computed in terms of topological data of the underlying manifold. Since elliptic operators on manifolds often appear related to geometric structures on the manifold, the theorem allows for deep conclusions about the geometry and topology of manifolds. Classical results as The Gauss-Bonnet, Hirzebruch, and Riemann-Roch theorems can be concluded as special cases and will be covered in the course.

The goal of this graduate level course is to understand the Atiyah-Singer index theorem and its applications. The goal is also to give a background in the different areas of mathematics involved, such as functional analysis; elliptic differential operators on manifolds; Clifford algebras, spinors, Dirac operators; vector bundles and characteristic classes.

This is a \broad graduate course" which is supposed to be accessible and meaningful for all graduate students in mathematics.

The course goals are:

- understand the Atiyah-Singer index theorem and its applications,
- learn the necessary background in analysis, topology, and geometry.

Course contents

- Index theory in functional analysis.
- Elliptic operators on manifolds.
- Dirac operators.
- Vector bundles, characteristic classes, K-theory.
- The Gauss-Bonnet, Hirzebruch, Riemann-Roch theorems.
- The Atiyah-Singer index theorem.
- First proofs.
- The heat kernel proof of the index theorem in detail.
- Applications to obstructions to positive scalar curvature, dimensions of moduli spaces, Donaldson and Seiberg-Witten invariants.

Examination

- INL1 - Hand in assignment, 7.5 credits, grading scale: P, F

Based on recommendation from KTH’s coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

If the course is discontinued, students may request to be examined during the following two academic years.

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.