



FSF3945 Advanced Probability

7.5 credits

Avancerad sannolikhetssteori

This is a translation of the Swedish, legally binding, course syllabus.

Establishment

Course syllabus for FSF3945 valid from Spring 2014

Grading scale

G

Education cycle

Third cycle

Specific prerequisites

A graduate course in probability such as SF3940.

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

After completing the course students are expected to:

- explain the connection between random walks and the heat equation

- explain in detail the properties of the Brownian motion
- have a good understanding of weak convergence in metric spaces
- outline the construction of the Brownian motion (Oonsker's theorem) from random walks
- explain the main results and applications of ergodic theory
- have basic insights in additional topics (that may vary between years) in
- advanced probability be able to solve problems related to the theory

Course contents

- Random walks and the heat equation. Reading: Greg Lawler, Random walk and the heat equation, Chapter 1. See <http://www.math.uchicago.edu/~lawler/reu.pdf> Key concepts: Simple random walk, boundary value problems, the heat equation, harmonic functions Exercises: 1.5, 1.12, 1.16, 1.24, 1.26
- Infinite divisibility. Reading: Billingsley. Probability and Measure, Section 28. Sato. Chapter 2. Key concepts: Levy measure, Levy-Khinchine representation Exercises: 28.3, 28.5, 28.11, 28.12
- Large deviations. Reading: Durrett, Section 1.9. Dembo and Zeitouni, Chapter 2. Key concepts: Cramer's theorem Exercises: Durrett Sec 1.9, Exercise 9.5, 9.6, 9.8, 9.9
- Weak convergence I. Reading: Patrick Billingsley, Convergence of Probability Measures, Chapter 1, Sec 1-3 and 5 Key concepts: Prohorov's theorem, the Portmanteau theorem, the continuous mapping theorem, Exercises: 1.10, 2.7, 3.6, 5.4, 5.7, 5.8, 5.9
- Weak convergence II Reading: Patrick Billingsley, Convergence of Probability Measures, Chapter 2, Sec 7-9. Key concepts: Donsker's theorem Exercises: 8.2, 8.4, 9.1, 9.3
- Brownian motion. Reading: Patrick Billingsley, Probability and Measure, Section 37. Key concepts: Continuity of paths, Irregularity of paths, the strong Markov property, the reflection principle Exercises: 37.7, 37.11, 37.14, 37.16, 37.18,
- Ergodic theory. Reading: Richard Durrett, Probability: Theory and Examples, Chapter 6, Section 1-7. Key concepts: Birkhoff's ergodic theorem, Benford's law, Subadditive ergodic theorem Exercises: 3.5, 6.1, 7.2, 7.3, 7.4

Disposition

The course will consist of roughly bi-weekly discussion meetings (not standard lectures) where students present and discuss the material as well as some weekly exercises.

Course literature

Patrick Billingsley, Probability and Measure, 3rd Edition, John Wiley & Sons, 1995

Patrick Billingsley, Convergence of Probability Measures, 2nd Edition, John Wiley & Sons, 1999.

Amir Dembo and Ofer Zeitouni, Large Deviations Techniques and Applications, 2nd Edition, Springer, 1998.

Richard Durrett, Probability: Theory and Examples. 4th Edition, Cambridge University Press, 2010.

Greg Lawler, Random walk and the heat equation, American Mathematical Society, 2010

Examination

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

If the course is discontinued, students may request to be examined during the following two academic years.

The examination will be done as a combination of homework and oral exam.

Other requirements for final grade

Homework and oral exam.

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.