FSF3945 Advanced Probability 7.5 credits

Avancerad sannolikhetsteori

This is a translation of the Swedish, legally binding, course syllabus.

If the course is discontinued, students may request to be examined during the following two academic years

Establishment

Course syllabus for FSF3945 valid from Spring 2014

Grading scale

Education cycle

Third cycle

Specific prerequisites

A graduate course in probability such as SF3940.

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

After completing the course students are expected to:

- explain the connection between random walks and the heat equation
- explain in detail the properties of the Brownian motion
- have a good understanding of weak convergence in metric spaces
- outline the construction of the Brownian motion (Oonsker's theorem) from random walks
- explain the main results and applications of ergodic theory
- have basic insights in additional topics (that may vary between years) in
- advanced probability be able to solve problems related to the theory

Course contents

- Random walks and the heat equation.Reading: Greg Lawler, Random walk and the heat equation, Chapter 1. Seehttp://www.math.uchicago.edu/-lawler/reu.pdfKey concepts: Simple random walk, boundary value problems, the heat equation, harmonic functions Exercises: 1.5, 1 .12. 1.16. 1.24, 1.26
- Infinite divisibility.Reading: Billingsley. Probability and Measure, Section 28. Sato. Chapter 2.Key concepts: Levy measure, Levy-Khinchine representationExercises: 28.3, 28.5, 28.11, 28.12
- Large deviations.Reading: Durrett, Section 1.9. Dembo and Zeitouni, Chapter 2.Key concepts: Cramer's theoremExercises: Durrett Sec 1.9, Exercise 9.5, 9.6, 9.8, 9.9
- Weak convergence I.Reading: Patrick Billingsley, Convergence of Probability Measures, Chapter 1, Sec 1-3 and 5Key concepts: Prohorov's theorem, the Portmanteau theorem, the continuous mapping theorem, Exercises: 1.10, 2.7, 3.6, 5.4, 5.7, 5.8, 5.9
- Weak convergence IIReading: Patrick Billingsley, Convergence of Probability Measures, Chapter 2, Sec 7-9.Key concepts: Donsker's theoremExercises: 8.2, 8.4, 9.1, 9.3
- Brownian motion.Reading: Patrick Billingsley, Probability and Measure, Section 37.Key concepts: Continuity of paths, Irregularity of paths, the strong Markov property, the reflection principle Exercises: 37.7, 37.11, 37.14, 37.16, 37.18,
- Ergodic theory.Reading: Richard Durrett, Probability: Theory and Examples, Chapter 6, Section 1-7.Key concepts: Birkhoft's ergodic theorem, Benford's law, Subadditive ergodic theoremExercises: 3.5, 6.1, 7.2, 7.3, 7.4

Disposition

The course will consist of roughly bi-weekly discussion meetings (not standard lectures) where students present and discuss the material as well as some weekly exercises.

Course literature

Patrick Billingsley, Probability and Measure, 3"' Edition, John Wiley & Sons, 1995
Patrick Billingsley, Convergence of Probability Measures, 2"" Edition, John Wiley & Sons, 1999.
Amir Dembo and Ofer Zeitouni, Large Deviations Techniques and Applications, 2"" Edition, Springer, 1998.
Richard Durrett, Probability: Theory and Examples. 4"' Edition, Cambridge University Press,

2010.
Greg Lawler, Random walk and the heat equation, American Mathematical Society, 2010
Ken-lti Sato, Levy Processes and Infinitely Divisible Distributions, Cambridge University Press, 1999.

Examination

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

The examination will be done as a combination of homework and oral exam.

Other requirements for final grade

Homework and oral exam.

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.