SF1669 Mathematical and Numerical Analysis II 11.0 credits

Matematisk och numerisk analys II

This is a translation of the Swedish, legally binding, course syllabus.

Establishment

Course syllabus for SF1669 valid from Spring 2015

Grading scale

A, B, C, D, E, FX, F

Education cycle

First cycle

Main field of study

Technology

Specific prerequisites

SF1667 Applied Linear Algebra II and SF1668 Mathematical and Numerical Analysis I, or equivalent courses.

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

An overall goal of the course is that students will develop a good understanding of basic mathematical concepts in multi-variable calculus, and use these for mathematical modelling of engineering and scientific problems.

The student should develop skills to illustrate key concepts and solve applied problems, by using a computer and using standard functions from the programming language library. In addition, the student should be able to visualize and present results in a clear manner.

After completing the course a student with a passing grade should be able to

- Use, explain and apply the terminology and the main basic concepts and problem-solving methods, in particular:
  - interpret graphs of functions and contour/level surfaces and sketch such curves and surfaces in simple cases
  - calculate partial derivatives and using the chain rule for real and vector valued functions of several variables
  - approximate partial derivatives with finite differences of various order of accuracy and be familiar with the error of the approximation.
  - determine and classify critical points
  - use Taylor's formula to approximate functions and estimate the error of the approximation
  - use the Jacobi matrix to perform linear approximation
  - solve non-linear equations using numerical methods
  - use the gradient to calculate the directional derivative and understand how the gradient relates to level curves/surfaces
  - solve certain optimization problems, also with constraints
  - explain how multiple integrals are defined and how they can be approximated using Riemann sums
  - calculate some multiple integrals by means of repeated integration and changes of variables, especially polar, cylindrical and spherical coordinates
  - calculate integrals numerically using techniques with different orders of accuracy and be familiar with the error of the approximation
  - demonstrate an understanding of how to use integral calculus to calculate lengths, areas, volumes, and other quantities such as mass and center of gravity
  - describe how line, surface and flux integrals are defined and carry out calculations of simple integrals using parameterisation
  - solve systems of first order ordinary differential equations with numerical methods of various order of accuracy and demonstrate understanding of the concepts of convergence and stability
  - explain and apply Green's formula and Gauss' theorem (the divergence theorem)
  - explain the concepts of potential and conservative vector fields and use them in calculations
- Set up simple mathematical models of phenomena and processes that can be described by functions of several variables and vector valued functions, and discuss such models and relevance of their solutions, feasability and accuracy, and be familiar with how mathematical software can be used to perform calculations within single- and multi-variable calculus.
- Select the appropriate numerical method for the treatment of a given mathematical model and motivate the choice of method by accounting for advantages and limitations.

- Make numerical experiments.
- Make reliability assessment of numerical results: parameter sensitivity, experimental perturbation, precision, and present the results in a clear manner.
- Read and understand texts on single- and multi-variable calculus and its applications and communicate mathematical reasoning and calculations in this subject verbally and in writing.

Goals for a higher grade. After completing the course a student should moreover

- Demonstrate an understanding of how the Jacobi matrix can be used to determine if a function is locally invertible.
- Apply the implicit function theorem.
- Describe and apply Stokes' theorem
- Evaluate limits of functions of several variables and identify situations when limits do not exist.
- Explain the concepts of limit, continuity, partial derivative and differentiability of real valued functions of several variables.
- Solve problems that require more extensive calculations in several steps.
- Generalize and adapt methods to fit in partly new situations.
- Solve problems that require the synthesis of materials and ideas from the whole course.
- Derive important relations and theorems and algorithms.
- Explain the theory behind the concepts of numerical convergence and stability.

Course contents

The spaces R^n. Functions of several variables and vector valued functions including the following features and concepts: Graph of functions, level curve, level surface. Limits and continuity, differentiability, partial derivatives, chain rule, differentials. Finite difference approximation of different order of accuracy. Tangent plane and linear approximation. Taylor's formula in several variables. Gradient and directional derivative. Jacobian matrix, Jacobian determinant. Newton's method to solve non-linear equations. Inverse functions and implicitly defined functions. Coordinate transformations. Optimization. Multiple integrals. Line integrals and Green's formula. Flux and Gauss and Stokes' theorems. Numerical integration with the trapezoidal rule and Simpson's formula and the existing functions in Matlab. Numerical solution of systems of differential equations using Euler's method and the Runge-Kutta method and the existing functions in Matlab. Applications. Numerical methods for partial differential equations.

Course literature

Calculus (8th edition) by Robert A. Adams and Christopher Essex, 2013, ISBN 978-0-32-178107-9

Numerical Analysis by Timothy Sauer (2nd edition or New International edition)

Hand-out notes

Examination

- PRO1 - Project, 2.0 credits, grading scale: A, B, C, D, E, FX, F
- TEN1 - Examination, 6.0 credits, grading scale: A, B, C, D, E, FX, F
- LAB2 - Lab Assignments, 2.0 credits, grading scale: P, F
- LAB1 - Lab Assignments, 1.0 credits, grading scale: P, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

If the course is discontinued, students may request to be examined during the following two academic years.

Other requirements for final grade

Written exam (TEN1; 6 cr). Computer assignments with oral and written presentation (LAB1 and LAB2; 3 cr) and a project (PRO1; 2 cr)

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.