SI2340 Lie Algebras and Quantum Groups 7.5 credits

Lie algebror och kvantgrupper

This is a translation of the Swedish, legally binding, course syllabus.

If the course is discontinued, students may request to be examined during the following two academic years

Establishment

Course syllabus for SI2340 valid from Autumn 2007

Grading scale

A, B, C, D, E, FX, F

Education cycle

Second cycle

Main field of study

Physics

Specific prerequisites

Recommended prerequisites: Good knowledge of linear algebra. Course SI2380 (5A1385) in quantum mechanics or mathematical methods of quantum mechanics SI2420 (5A1389). Familiarity with abstract algebra, for example algebra course SF2703 (5B1309) or discrete mathematics course SF1630 (5B1203).

Language of instruction

The language of instruction is specified in the course offering information in the course catalogue.

Intended learning outcomes

After completing the course you should

- understand the relation between Lie algebras and Lie groups appearing as symmetries in physics models
- be familiar with the structure theory of semisimple Lie algebras in terms of root diagrams and be able to derive the basic properties of Lie algebras from the root structure
- understand the classification of representations of simple Lie algebras and be able to construct some of the standard representations, especially those which one meets in nuclear and particle physics
- be able to use associative algebra methods (universal enveloping algebra) for constructing representations of Lie algebras
- to understand the basic structure theory of infinite-dimensional affine algebras appearing in quantum field theory, and be able to construct some of their representations
- to understand the structure of quantum groups (Hopf algebras) which are deformations of classical Lie algebras and are relevant in the theory of integrable systems in physics

Course contents

This course starts with a general introduction to Lie algebras with several examples from classical matrix Lie algebras. Next we discuss the classification in nilpotent, solvable, and semisimple Lie algebras. The main part of the course consists of a detailed study of semisimple Lie algebras and their representations. These algebras appear in several applications in atomic, nuclear, and particle physics. Besides, they have a central role in many branches of pure mathematics, in harmonic analysis, differential geometry, algebraic geometry, integrable systems and (symmetries of) differential equations.

We also discuss infinite-dimensional generalizations, including affine Kac-Moody algebras which play an important role in quantum field theory and string theory. Finally, we study quantum groups as deformations of semisimple Lie algebras. These are an important tool in the theory of quantum integrable systems and they also lead to interesting examples in noncommutative geometry.

The course is recommended to students (in F4) specializing either in mathematical physics or in mathematics, and also to interested PhD students. The course is given in English.

Course literature

Written lecture notes. Additional reading:

J.E. Humphreys: Introduction to Lie Algebras a Representation Theory, Springer Verlag, 1980.

V.G. Kac and A.K. Raina: Bombay lectures on highest weight representations of infinite-dimensional Lie algebras, World Scientific Publ. 1987,

C. Kassel: Quantum Groups, Springer GTM 155, 1995.

Examination

- INL1 - Assignments, 4.5 credits, grading scale: A, B, C, D, E, FX, F
- TEN1 - Examination, 3.0 credits, grading scale: A, B, C, D, E, FX, F

Based on recommendation from KTH's coordinator for disabilities, the examiner will decide how to adapt an examination for students with documented disability.

The examiner may apply another examination format when re-examining individual students.

Other requirements for final grade

A combination of hand-in homework exercises (INL1; 4,5 university credits) and of a written or oral examination (TEN1; 3 university credits).

Ethical approach

- All members of a group are responsible for the group's work.
- In any assessment, every student shall honestly disclose any help received and sources used.
- In an oral assessment, every student shall be able to present and answer questions about the entire assignment and solution.